Mathematics and Science Fiction

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Introduction

This curriculum unit is a series of lessons that use science fiction stories to enhance the teaching and learning of some topics in mathematics. Students do not usually see mathematics in literature. The majority of the textbooks that the students use, present the mathematics in what seems like the dullest way possible. The students get adept at skipping the text and looking for the formulas, equations, and theorems. There is a wealth of expository material about mathematics available, but the students seldom read it and teachers seldom have enough time to find the appropriate reading material and then develop lessons to go with it.

This unit grew out of a seminar offered by the Pittsburgh Teachers Institute given by Dr. Richard Holman called "Learning Physics Through Science Fiction". As I read the science fiction stories and listened to the physics related to those stories, as presented by Dr. Holman, I began to wonder if there were science fiction stories from which mathematics could be developed. As I searched the literature, I found several books containing mathematical science fiction stories. These stories have a mathematical basis rather than a physics basis. For each story I chose I include a brief synopsis of the story, the mathematics involved and, a list of problems that naturally develop from the story.

You may ask, "Why use science fiction to teach mathematics?" Science fiction has had a long history of exciting people's minds. From the days of Jules Verne to the modern writers of today, the stories force the reader to think, imagine and stretch his mind. When Verne took me around the world, under the sea, and to the moon I had to think about other cultures, other forms of transportation, and other logistics that the stories required. Many questions occurred to me. How did Captain Nemo keep the air breathable in the submarine? How did they breathe while traveling to the moon? Did the moon have air and if not how did they breathe on the moon? When Arthur C. Clarke gave us an alien in his novel Stranger in a Strange Land, questions abounded about the alien, how he arrived on earth, and how did he adapt to life here? Star Trek presented us with warp drives, beaming, phasers and alien cultures. I always wondered which of these were possible and which were not. Indeed, even in the Star Wars trilogy, we saw aliens beyond imagination and applications of science and mathematics that boggled the mind. These tales gave us new science and technology to wonder about. The science in these stories showed us the necessity for learning mathematics and science, and from these stories many young scientists and mathematicians got the first glimmerings of their future. These stories force us to speculate on the future and what we have to learn to get there. The science fiction stories of today talk about black holes, gravity, quantum mechanics, and alternate time lines. The authors give us speculations about current science applied to future situations. We must always keep in mind that we are teaching for the future as well as the rapidly changing culture of today.

The stories that are used in this unit will show students that science fiction writers must have knowledge about science and mathematics to make their stories plausible. They can serve as jumping off points for research and discussion about mathematics and science. The stories can be used one after another as a unit or each one can be used separately. Each story and its accompanying mathematics should take 2-5 days to complete depending on the depth of mathematics the teacher wishes to teach.

The stories that I have chosen for this unit can be, for the most part, used for students in grades 9 to 12. Some of them, such as "*A. Botts and the Moebius Strip*" require little prior knowledge of mathematics to enjoy them and could be used in middle school if desired. Others, such as "*The Devil and Simon Flagg*" require some mathematical ideas that are simple to understand, but whose proofs are quite beyond the high school students' ability, and I think most college professors. The teacher must decide by reading the stories what is appropriate for their students.

Rationale

The students in today's classroom are much different from the ones that were in my classroom when I started teaching. In the 'old' days there was a reasonable expectation that the students would do the homework, pay attention in class, and actually pretend to study once in a while. We had none of the distractions of CD players, cell phones, DVD players that can hide behind a book, and pagers that distract today's student. Instead, we had sit-ins, race confrontations on a larger scale than today, and all the usual ills of society that seem to plague schools. Pittsburgh, at that time, didn't even have cable.

Today's student has many distractions that vie for his time and these distractions, some of which are mentioned above, are very powerful. The teacher, in turn, must find a way to present mathematics in ways not imagined while attaining his or her Bachelors and Masters Degrees. Textbook publishers have not helped the situation. Many of them are just now realizing that the students today are expecting to use technology to do for them what paper and pencil did for former students. The text of these books reads the same today as they did 30 years ago and teach many of the same topics in the same way. This forces the teacher to find more inventive ways to present topics to these distracted, technologically more sophisticated students.

Early in my teaching career, I discovered that students remembered more if there were silly stories attached to them. I told them a story about a little boy, playing in the sand by a pyramid, who discovers the Pythagorean Theorem. When asked by a student one day how a microwave oven worked, I told them that the microwave got the particles of food excited. This in turn caused the food to be happy and the food molecules clapped their little hands for joy. This in turn heated and cooked the food. This explanation is not too far from the truth and is certainly more interesting than the scientific one. As I progressed in my career I made up more of these stories. I never wrote them down and they always changed each year to match the students I was teaching that year. I was always looking for new material, but I never, until this year, thought about using short stories to teach mathematics.

I decided to take the seminar "Learning Physics Through Science Fiction" offered by the Pittsburgh Teachers Institute not because I could foresee myself teaching physics, but rather because I liked science fiction, and I had a huge lack of knowledge about physics. As more and more applications creep into the curricula of mathematics, it became obvious to me that my meager knowledge of science would no longer serve me. I knew nothing beyond very elementary ideas about gravity and motion. The concepts of the physics I was taught in college were basic and are probably now outdated since much has happened in science since the sixties. By reading the stories chosen by Professor Holman, and then listening to the explanations of the physics that supported the stories, I learned some of the ideas of modern physics. I then began to wonder if there were stories to do the same for mathematics. A search on the internet found a website that contained an outline of a course offered jointly by an English professor and a mathematics professor at Dartmouth College on mathematics and science fiction. Here was the information I needed to start the unit. Although some of the stories listed on the site were beyond the scope of high school mathematics, I found some that were appropriate. The stories that I wanted had to be engaging as well as have a sound basis in mathematics. They either had to strengthen mathematics already taught to the students or introduce them to new mathematical ideas. The stories also had to lend themselves to problems and research ideas. The stories I used in this unit, I believe, fit the criteria.

The Pittsburgh Public Schools places an emphasis on literacy. Each teacher was told to develop ways to support the literacy standard. Having the students read, write reports, and present problems and reports to the class support this goal. The biggest problem I found was having the students read the textbooks and the problems in them. The reading in many of the books contains so much jargon that the students have extreme difficulty reading and making sense of the materials. Some of the problems were so vague that the students had trouble explaining what the problem was about. Because I believe that literacy is so important and is, indeed, the corner stone of all educational endeavors, I began searching for ideas that would introduce reading in the mathematics classroom. I found little about how to teach students to read technical writing. I actually found more about how to make technical writing more readable. The writing in one textbook that is used is so difficult that many teachers have difficulty reading and interpreting the material. The stories in this unit provide one way to introduce literacy into the mathematics classroom. By having the students read the story either in class or as an assigned reading outside of class, we can begin to introduce the student to reading about mathematics without the threatening material of the mathematics textbook. Discussing the story will provide practice in speaking and listening. You could even have the students write about the stories, make up a story of their own based on some concept of mathematics they have learned, and write problems based on the story. To help the students understand the stories you could have them try to illustrate it. Thus through reading, writing, speaking and a tie to the visual arts, you would be engaging many of the students' senses in the learning process.

Reading the stories on the list, provided by the Dartmouth website, was my first job. I was looking for stories that contained mathematics accessible to high school students of all levels and abilities. I found stories about large numbers, probability, geometry, and limits or, in the words of our elementary functions' text, end behavior. Knowledge of probability is one of Pennsylvania's standards that the class of 2003 must reach to graduate. The story, *"The Tale of Happiton"*, gives an interesting introduction to probability. The other story, *"Inflexible Logic"* gives the student more practice in finding probabilities in a situation that is not difficult to understand. The stories about large numbers deal mostly with huge numbers that are difficult to imagine. In Clarke's story, *"The Nine Billion Names of God*" the student is introduced to the idea of counting to 9 billion in a relatively short time. Through the other story, the classic *"The Universal Library"*, the student contemplates how big a library would be if it contained every book ever written. In the 2 stories about geometry the students are introduced to some basic ideas in topology by using Moebius bands. In the story, *"The Devil and Simon Flagg"*, the students are introduced to Fermat's Last Theorem that states that $\sqrt[n]{\quad}$ has no integer solutions for $\sqrt{\quad}$, n an integer. Larry Niven's story, *"Convergent Series"* gives a reason for learning about convergence that is unusual to say the least. I hope that the students will never have to use convergence in this way but, if they do, they will be prepared.

There are several ways the stories can be used in the classroom. One way would be to give the students the story before starting the mathematics mentioned in the story. Then as you teach the mathematics, refer to the story, and have the students answer the appropriate questions about the story. Of course, you could also use the story at the end of the appropriate mathematical instruction as a way to review the concepts you have taught. A third way could be to give the story to the students and have them research the mathematics and answer questions independently. You may also want to have the students write stories that involve mathematics you are teaching. This, of course, should only be done after the students have read several of the stories.

For example, to teach about probability I would have the students read the story *"The Tale of Happiton"* for homework. The next day I would discuss the story with class and give a brief introduction to probability. Over the next few days I would discuss in more detail the ideas of sample spaces and probability. I would have the students solve the problems related to the story as we discuss the mathematics perhaps giving extra credit to the first student to solve the problem. In addition, of course, I would have the students solve exercises and simple problems about probability especially those dealing with probability in other situations. As a final project I would have the students write a brief story involving probability.

The standards that this unit addresses come from both the mathematics and communication standards. As I have been teaching these many years I have come to the conclusion that you cannot teach mathematics without the communications. It is useful to have students write about the mathematics they are learning. Whether the writing is in journals, in requiring students to write their answers in complete sentences, in research, or in writing stories, the process of writing helps students to focus their thoughts. Reading the stories supports all areas of educational endeavor since students must be able to read to succeed. From a mathematical viewpoint the stories use mathematics that is often presented in a dry computational context. The problems that I have suggested will help the students to meet the problem solving standards of the Pittsburgh Public Schools. Please see the complete listing of the standards addressed at the end of this unit.

Stories about counting and numbers

Arthur C. Clarke, *"The Nine Billion Names of God"*

Synopsis of story

Tibetan monks are writing down all the names of God. They hire a computer company to alter the circuits of their computer so that it will print out all the names of God according to the monks' rules for naming God. (Note that the story was written in 1952 and, at that time, computers were slow and could usually only print numbers.) The computer experts that were in the monastery eventually find out why the monks are compiling this list.

Mathematics in the story.

- Permutations

You will have to use permutations for this story since order matters. Most calculators can handle this easily. The monks' rules state that you must use 9 letters for each name and no letter can be used more than 3 times in succession so you will have to devise a way to eliminate the non names.

- Large numbers

Students traditionally have a difficult time imagining the size of large numbers. Indeed, most of us have little concept of much beyond a million. Therefore it might be wise to discuss such things as how far would a stack of one billion pennies reach or how many times would a billion dollars laid end to end stretch around the world. The national debt is in trillions of dollars. Is this number even meaningful to most people?

Problems and questions

1. How big is 9 billion?
2. How long would it take to count to 9 billion if one number is said every minute? Every 10 seconds? Every 2 seconds? Every 0.5 seconds?
3. What if 6 numbers were said each minute? Why is this answer the same as the answer for 10 seconds?
4. If we wanted to count to 9 billion in one year how many numbers would we have to say each minute? Each second?
5. The monks state each name of God can be written with 9 letters from an alphabet they devised, as long as no letter occurs more than three times in succession. Suppose

that they used our alphabet. How many 9-letter names can be written following the rule above?

6. The lama states "What would have taken us fifteen thousand years it (the computer) will be able to do in a hundred days." How fast would the computer be working?

Kurd Lasswitz, *"The Universal Library"*

Synopsis of story

A mathematician decides to figure out how many volumes are needed to contain all works of writing and literature that were ever and can ever be written. The story states a method for computing this.

Mathematics in the story

- Permutations

Again, the story looks at permutations of letters to form words. It might be interesting to ask the students to try to solve this problem before they read the story.

- Large numbers

The number here is so huge that it would be very difficult to find any real meaning in it.

Problems

1. If the books in the story were placed beside each other, how long would it take to walk from one end to the other if the books were closed? Opened?
2. There are 2 numbers in the literature of mathematics that are very large. One is the googol and the other is the googolplex. Find out what these numbers are, and how they were named.

A story about number theory

Arthur Porges, *"The Devil and Simon Flagg"*

Synopsis of the story

Simon Flagg makes a deal with the devil. If the devil can answer Simon's question then the devil gets his soul. If the devil fails to answer the question, Simon and his wife will receive a sum of money and health and happiness for the rest of their lives.

Mathematics in the story

- Fermat's Last Theorem

Fermat stated that $\sqrt{\quad}$ has no integer solutions for n>2, n an integer. At the time that this story was written no proof of this had been found. However it now has been proven.

Problems

1. For the case n = 2, what famous theorem is it? Give several examples of Fermat's Last Theorem for the case n = 2.
2. Research the history of proving this theorem. Who finally did prove it? When? Write a report about Fermat and his famous theorem.

Stories about probability

Hofstadter, Douglas, *"The Tale of Happiton"*

Synopsis of the story

The people of Happiton find that after their town hall bell rang, there was a strange scratching sound. Upon climbing the belfry, the people found that there were five strange-looking dice that were being rolled into a pan by a mechanical arm attached to the bell. A TV camera was aimed at the pan and was connected to a microcomputer. There was a note left for the townspeople from Demon # 3127 telling the people what the dice were for and what they had to do to escape doom.

Mathematics in the story

- Permutations
  In this story permutations can be used to determine how large the sample space is.

- Probability
  It will be useful here to discuss the probability of rolling 1 standard die, then 2 standard dice. Independent and dependent events need to be discussed.

- Exponential functions
  We meet exponential functions in many situations. This is just one of them. Discuss with the students how the function in the story is arrived at.

Problems and questions

1. The dice are twenty-sided, with each number 0 – 9 appearing twice on each die, once on a face and once on the face opposite it. Make a model of one of these dice.

2. What is the probability of a seven appearing on top of one die after 1 roll?

3. On a single die, what is the probability of not rolling a seven on the top of the die?

4. Suppose we rolled two dice, what is the probability of rolling a seven on each one?

5. Does it matter whether we roll the dice one at a time or both at once?

6. When we roll five dice, what is the probability that we roll 5 sevens? Explain your work.

7. The demon states that the probability of rolling five sevens is exactly 1 in 100,000. Is he telling the townspeople the truth?

8. The demon tells the people of Happiton that for each postcard they send him, he will slow down the clock the next day by a factor of 1.00001. How long would it be between the hourly rings of the clock? How many times would the bell ring in one day?

9. Sketch a complete graph of $\lfloor \sqrt{\quad} \rfloor$. What is the domain and range of this function? Is the function increasing or decreasing?

10. If the townspeople wanted the bell to ring once every 1.5 hours how many postcards how many postcards would they have to write? How many would that be for each person?

11. According to Mrs. Doobar, there are 11 chances out of twelve that they will make it through the year. Is she correct? Explain her reasoning if it is correct and your reasoning if it isn't.

12. The story doesn't tell us what happened. Write an ending for the story that tells what happened. Be sure to include the mathematics of why this ending makes sense.

Russell Malony, *"Inflexible Logic"*

Synopsis of story

The main character, Mr., Bainbridge, contacts a mathematicion about the famous story about 6 chimpanzees typing all the books that ever written and ever will be written. The consequences were disastrous.

Mathematics in the story

- Probability
  The students will need to calculate sample spaces and simple probabilities.

Problems and questions

1. Using the letters d,o,g how many one-letter words would there be? (Note: A word is considered to be one letter whether it is a real word or not.) How many are actually 'real' words? If the 3 letters are put in a box, what would the probability be on drawing a letter and it being a real word?
2. Suppose you use 2 letters at a time. How many 2-letter words can you make? How many are actual words? What is the probability of drawing 2 letters and getting an actual word?
3. Suppose you use the 26 letters of the alphabet, how many 3-letter words could you form? What is the probability that the word starts with a 't'? Starts with 'th'? Is the word 'the'?
4. Write an expression that represents the total number of words that can be made using the alphabet and 1,2,3,4, and 5 letter words.

5. Write an expression that represents the total number of words that could be formed using 1-15 letters.
6. Would it be possible to figure out how many words of all lengths that could be formed?
7. Would it be possible to figure out how many real words could be formed by using 4 letters of the alphabet? If so, do it. If not, why?
8. Why do you think Professor Mallard did what he did at the end?
9. The chimpanzees never made a mistake in the story. What words told you this? Do you think it would be possible to do this? Why or why not?

Stories about geometry

William Hazlett Upson *"A. Botts and the Moebius Strip"*

Synopsis of the story

A clever soldier found a way to confound an uncooperative officer. Because of his plan soldiers were saved.

Mathematics in the story

In this story the students will only need to know about how to determine sidedness. The branch of mathematics that the Moebius strip comes from is topology.

Problems and questions

1. Using adding machine tape or some other type of strips of paper, make a model of a Moebius strip.
2. How can we determine how many sides it has? How many sides does it have?
3. How many edges does a Moebius strip have? How do you know?
4. Can you think of a way that a Moebius strip may be useful in real life?
5. Cut the strip in half lengthwise. What happens? How many sides do these strips have? Why do you think this happens?

Gardner, Martin. *"No-Sided Professor"*

Synopsis of the story

A professor twists himself into a knot and disappears later reappearing in a most unusual way. A proof is argued about, resulting in another disappearance.

Mathematics in the story

This story gives hints about the topology of three-dimensional objects.

Problems and questions

1. If you have not already done so, construct a Moebius strip following the directions in the story.
2. Cut the strip in half lengthwise. Describe what happens.

3. Now cut each of the strips in half. Describe what happens. Why do you think it happens? If you cut the strips again what do you think will happen? Cut the strips to prove your conjecture.
4. Research Klein's bottle. What makes it interesting? Would it be possible to build a model of this bottle?
5. The story was written in 1946. What has happened since then in the theory of knots? Research this and write a report to present to you class.
6. Find a picture of Ballantine's trademark. Demonstrate what happens when you remove one ring, Why does this happen?

A story about the infinite

Niven, Larry *"Convergent Series"*

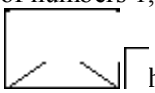
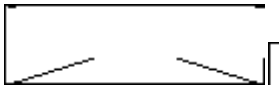
Synopsis of the story

A student becomes interested in magic and attempts to conjure up gentle demons. He succeeds in conjuring a bad one and has to figure a way to get rid of him.

Mathematics in the story

- Convergence
  A discussion about a series converging is necessary either before or after the story. It may be helpful to look at geometric series and the conditions of convergence.

Problems and questions

1. Draw a sequence of 5 pentagrams that illustrate the demon's dilemma.
2. Would this process ever stop? Why or why not?
3. Does the sequence of numbers 1,2,3… have a limit? Why or why not?
4. Does the sequence  have a limit? If it does, to what number, and if it doesn't tell why.
5. Investigate the sequence defined by  . Does it have a limit? If so, to what number? Graph the equation  . Does the limit of the sequence have anything to do with this function?
6. Investigate the sequence defined by  . What is the limit of this sequence? Investigate the function  . Does the limit of the sequence have anything to do with this function? What is it? How do mathematicians indicate this limit?

Standards

This unit incorporates the following standards:

Mathematics

- All students use numbers, number systems, and equivalent forms (including numbers, words, objects and graphics) to represent theoretical and practical situations.
- All students compute measure and estimate to solve theoretical and practical problems, using appropriate tools, including modern technology such as calculators and computers.
- All students formulate and solve problems and communicate the mathematical processes used and the reasons for using them.
- All students understand and apply basic concepts of algebra, geometry, probability, and statistics to sloe theoretical and practical problems.

Communications

- All students read and use a variety of methods to make sense of various kinds of complex text.
- All students respond orally and in writing to information and ideas gained by reading narrative and informational texts and use the information and ideas to make decisions and solve problems.
- All students write for a variety of purposes, including to narrate, inform, and persuade, in all subject areas.

Bibliography

Clarke, Arthur C. Nine Billion Names of God. Harcourt, Brace and World, 1967.
A group of early science fiction stories written in the fifties and republished. These stories are interesting for their context as well as the mathematics and science.

Egan, Greg. Axiomatic. HarperCollins Publishers, Inc., 1997.
A group of stories some of which would not be appropriate. Mr. Egan is known for writing stories about mathematics.

Fadiman, Clifton, Editor. Fantasia Mathematica. Simon and Schuster, 1958.
Mr. Fadiman has collected stories, poems and limericks about mathematics.

Rucker, Rudy, Editor. Mathenauts: Tales of Mathematical Wonder. Arbor House Publishing Co.,1998.
This is a series of tales about mathematics. These stories are more complicated than those collected by Mr. Fadiman.

Sheffield, Charles. Borderlands of Science. Baen Publishing Enterprises,1999.
This book is a good reference for teachers about science, science fiction and the line that separates them. It explains the science that is found in the stories in an accessible way.

Internet sites

http://emmy.dartmouth.edu/~c18w99/
This site is an outline for a college course in science fiction co-taught by an English and mathematics professor. It contains a bibliography and story list that could be adapted for high school use.

http://www.dot.net.au/~krenon/PhysicsinSF.html
This site contains a talk by Andrew E. Love Jr. about physics and science fiction. It contains brief discussions of stories as well as the pertinent physics' formulae.