# The Death of Common Sense?

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## INTRODUCTION

***It is God's glory to conceal things but the glory of kings is to search things out… Proverbs 25***

Any explanatory model advanced in today's intellectual climate is subject to close critical even cynical scrutiny. Experiments are carefully designed and conducted. Data is gathered and statistically analyzed. References are cited and experts are polled. All of this is to try and convince others as to the validity of the proposed explanatory model: to prove one's point as it were. But why is proof so important? American culture is rich in reference to proof and its implications. Our justice system assumes innocence unless guilt is proven beyond a reasonable doubt. "I'm from Missouri, show me", "Seeing is believing", "Don't judge a book by its cover", "Where is the smoking gun?" "Pictures don't lie", and "Put it in writing" are all about making arguments more believable or convincing. Something in the human condition craves the certainty that a good, transparent proof provides. In fact, if arguments are convincing enough, there can develop a compulsion to believe. In the television series the X-Files FBI agent Mulder has a sign in his office that states "I want to believe!" The underlying theme of the show is his quest to prove the existence of alien beings by the accumulation of indisputable evidence. Of course some things are easier to prove than others and, coincidentally, in the same office is a sign proclaiming "Trust No one!" We do trust things that have been proven, and as a science teacher, I try to convince or prove to my students things about our physical universe. To a mathematician, proof is a rigorous and thorough exercise in logic and reason and can be defined as an argument put forth to be accepted beyond an even unreasonable doubt. In a more archaic sense, proof means to test or to try an argument in order to establish its validity. In some ways I think that a mathematician, if he examined my teaching, would look at me as an annoying neighbor who was always borrowing his tools, using them inappropriately, and returning them dirty and in need of repair. This is partly why I am writing this paper. I want to examine and prove some scientific concepts so that they become transparent and convincing to my students. If I can work in appropriate mathematical pedagogy, this will be even better. Hopefully, this will sharpen my teaching skills and any other interested science teacher will be able to glean useful materials or ideas from my experience that can help his or her teaching.

In order to explain our environment we constantly construct explanatory models of the universe. These models are more or less useful depending on the thoroughness of the observations and tests we make and of the predictive value that results. Certainly early peoples formed opinions of their surroundings and interacted with their environment on the basis of their observations no less than we do today. In fact, because humans have the power of reason,"reasonable" explanations of our world occur throughout history. What is the difference between early man upon hearing growls believing that supernatural monsters lurked in the forest and scientists observing that waves need a medium in which to travel, believing in the existence of invisible, undetectable ether through which light waves propagate? Both are based on interpretations of observations that were later superseded by more insightful explanations. In this paper I intend to examine some models reasonable people have put forth to explain the way the world works and to subject them to logical and mathematical analysis in order to show the shortcomings of "reasonable" common sense techniques in evaluating our environment. This paper is entitled "The Death of Common Sense" because it is the feeling of this writer that all persons use common sense to a greater or lesser degree and that what opinions or conclusions we form as a result can be incomplete or at times completely misleading. I'd like to point out that my title is a tongue-in-cheek reference to a controversial article in the October 1993 Scientific American by John Horgan entitled "The Death of Proof." Discussions that follow are written to be used in a middle school science classroom.

A definition of common sense is in order at this point. Several definitions of common sense can be found in dictionaries. Webster's 9th New Collegiate Dictionary defines common sense as the "unreflected opinions of ordinary men or sound and prudent but often unsophisticated judgment." The Random House dictionary defines common sense as "sound practical judgment that is independent of specialized knowledge or normal native intelligence." Finally, Webster's Third New International Dictionary defines it as a translation of the Greek "koine aisthesis or Latin sensus communis or something that is evident by the natural light of reason and hence common to all men." Indeed it is unarguable that humans have the capacity of reason and the ability to communicate their reasoning to others. The problem arises when shallow or unexamined reason is accepted and perpetuated by large groups of individuals and put forth as common sense to the exclusion of rigorous examination.

Prior to writing I decided to ask students and adults on the staff of the gifted center where I work questions relevant to some of the topics covered below. The first two questions concerned falling bodies. Another referred to the circumference and shape of the Earth. Questions on floating and sinking objects, calculating the lengths of sides of a triangle, and a problem on determining a country's honest and dishonest mints were asked. The notion of a vacuum was explored as well. These topics and more are explored below with middle school students in mind. Lesson ideas and suggestion are in *italics* for ease in identification and use by interested teachers

**Falling Bodies**

***My Lord, what a morning, When the stars begin to fall… spiritual***

One question asked of students and adults was: If two objects of differing weight were dropped simultaneously from the same height, which would hit the ground first? One hundred and sixty two seventh and eighth grade students were polled and nineteen adults. The results are as follows:

**Student Answers Adult Answers**

**The heavier object lands first: 29 8**

**The lighter object lands first: 5 1**

**They land at the same time: 128 10**

I conclude from this that most people believe that weight is not important in respect to the rate at which an object falls. I suspect that most science classes teach about this subject but still some cling to the common sense notion that heavier objects fall faster. The second question was a rephrasing of question one and the contrasts in answers are revealing. The question was if a bullet is fired from a gun set horizontally and a bullet of the same size is dropped simultaneous with firing, which would hit the ground first? The answers:

**Student Answers Adult Answers**

**The bullet fired drops first: 32 2**

**The dropped bullet hits first: 87 10**

**They hit at the same time: 34 4**

**No answer: 10 3**

Among students and adults the introduction of a horizontal component of motion is confusing. Only 34 of the students and 4 of the adults chose the correct answer. Even though most students and adults got the first question correct, it did not carry over to the second question. In the case of the bullet fired from the gun, the law of gravity is still in effect. The bullet will drop at the same rate as the dropped object in spite of its horizontal motion. Both adhere to the formula for freely falling bodies; $s = 1/2\ at^2$ where s = distance, a = acceleration, and t = time. As the formula shows, if the time is equal for both bullets, the equation solved for distance dropped will be the same. This is not obvious and common sense seems to suggest, based on the answers above that the dropped object would hit first. When I was conducting my survey, one of the teachers insisted that the bullet would travel in a straight line until it was spent and then fall straight to the ground. It is nice to see that Aristotelian ideas still persist in some minds.

The examination of falling bodies has an interesting history and illustrates well how common sense can lead well-meaning people astray. The problem unfolds as a question. When a heavy object and a light object are dropped from the same height at the same time, which hits the earth first? Common sense indicates that the heavy object should hit first. Indeed, that most ancient advocate of observation, reason, and common sense, Aristotle, drew a definitive conclusion that was accepted for nearly 2000 years. In dividing all matter into four components, fire, earth, air, and water, he laid the framework for answering the question. The heavier object naturally fell faster because it contained more "earth" and naturally sought the center of the earth more than the lighter one. An Aristotelian syllogism might be stated as; the natural resting-place of earthly matter is the earth's center. A heavy object contains more earthly matter than a lighter one. The heavier one therefore seeks the center of the earth more (faster) than the lighter one. The error of this logic is evident in this thought (gedanken) exercise. Join a heavy and a light object together with a rigid stick. Drop them from a high point such as a building. The heavier object should fall faster but the lighter object should fall slower, thus slowing the total fall rate. Taken together, the weight of the two objects is greater than either one, so logically it should fall faster! In spite of this contradiction Aristotle's belief that heavier objects fall faster than lighter ones went unchallenged for almost 2000 years until Galileo Galilei finally set the record straight. Legend has it that Galileo dropped objects from the leaning tower of Pisa but no real records exist of these experiments. His notes do contain references to observations of hailstones. He noted that large and small hailstones landed simultaneously. Using the Aristotelian model, this could only happen if the larger hailstones were falling from a lower height in the clouds than the smaller ones. He doubted this and made detailed observations of objects (billiard balls) rolling down inclined planes and concluded that the acceleration of an object is uniform due to a constant gravitational force. One problem I have observed with this experiment is that round objects of the same diameter may roll at differing rates. If a round, hollow object such as a hoop and a solid disk of the same size are rolled, the solid disk rolls faster. This does not resolve the issue as to why different objects fall at the same rate. Obviously, other variables beside gravity, one of which is angular momentum, is at work. More analysis is needed to resolve the problem.

*An interesting demonstration can be conducted for students by collecting several disks. All should be the same diameter and two should have the same mass. Some should be solid and the others should be hollow. Before they are rolled down a ramp simultaneously, students can be asked to predict the order in which they arrive at the bottom. An interesting discussion can arise in attempting to explain the results.*

*A demonstration, which convincingly suggests that objects accelerate uniformly as they fall, can easily be carried out in the classroom. Two pieces of cord, each 3 meters in length, can be obtained. To string one attach 5 heavy washers at equal intervals of about 50 centimeters. To string two attach 5 similar washers but at 5, 25, 70, 125, and 250 centimeters. The distances on string two are chosen to reflect the acceleration due to gravity by using the formula* $s = \frac{1}{2} at^2$ *where* ***a*** *has been rounded off to 10 meters per second per second. The instructor or student can stand on a table and release string one. Students can be asked to describe the spacing of the sounds as the washers hit the floor. Because the washers are equally spaced and are accelerating, they should hit with increasing frequency. String two should be held so that the 5 centimeter washer is at the bottom. Hold the string about 5 centimeters from the floor. When the string is released, the sound of the falling* *washers should be equally spaced reflecting the uniform acceleration of gravity. Students supplied with the formula for freely falling bodies, $s = ½ at^2$, should be able to predict the spacing of any number of additional washers placed on string two to keep the washer cadence equal.*

*Galileo addresses the problem of freely falling bodies by describing the actions of pendulums. In easily verifiable experiments or demonstrations it can be shown that the period (swing) time of a pendulum is independent of the pendulum's mass. It depends instead on the length of the pendulum. This would suggest that objects fall at a rate independent of mass. This also shows that Galileo combined observation, experiment, and theory to prove his hypotheses.*

Students may be asked to experiment with pendulums. They can be furnished with string, modeling clay or various weights, a ring stand, a ring stand clamp, and a timing device. Students should develop theories on falling bodies and pendulum periodicity by constructing pendulums of varying length and mass.

Galileo died in 1642, the same year in which Isaac Newton was born. It was Newton who settled definitively the issue of falling bodies. Essentially Newton explained gravitational force as a property inherent in any object in the universe. The earth's gravitational pull is noticeable because the earth is so massive compared to most things we commonly encounter. It is true, however, that an object attracted to the earth is at the same time attracting the earth due to its own gravitational field. The mass disparity between the earth and object is usually so large that the object's gravity is undetectable. In the case of the planets, however, the gravitational fields are noticeable. The orbit paths of the planets are due to gravitational interactions. Ocean tides are also affected by the moon's gravitational pull on the earth.

Newton's Laws nicely explain mathematically why objects accelerate uniformly when dropped and why they fall at the same rate. The proof is as follows: Newton's Second Law of Motion states that the force (F) an object exerts is equal to its mass ($M_o$) times its acceleration (A). In his Universal Law of Gravitation Newton explained that the force of earth's gravity on any object ($F_g$) is equal to the Mass of the earth ($M_e$)times the Mass of the object being attracted ($M_o$) times the Universal Gravitational Constant (G) divided by the square of the distance between them (r). I will discuss the experimental derivation of the gravitational constant later in this paper. The equations can be developed as follows:

$$F_g = M_oA$$

Also $F_g = GM_oM_e/r^2$

By substituting: $M_oA = GM_oM_e/r^2$

$$A = GM_oM_e/M_or^2$$

By simplifying: $A = G\cancel{M_o}M_e/\cancel{M_o}r^2$ Which becomes: $\underline{\mathbf{A = GM_e/r^2}}$

This shows that the masses of the falling objects from rest are not relevant to the equation. Objects dropped from the same height will accelerate and fall at the same rate! Video clips of objects falling in vacuo are readily available which support Newton's conclusion.

If a proof were an argument that is intended to be convincing beyond an even unreasonable doubt, the problem of freely falling bodies would appear to be resolved. There is one exception to the explanation. What if the falling object's mass was near to that of the earth? It would then pull the earth noticeably towards it as it fell and would arrive before a much less massive falling body. This is an unlikely scenario but not impossible when one considers that the North American land mass reportedly rises six inches during a full moon. The irony is that we have come full circle and perhaps Aristotle was not completely wrong in describing the more massive body falling faster albeit for objects near to the mass of the Earth.

**The Earth's Measure**

***Where were you when I laid the foundations of the earth? Tell me, if you have understanding. Who determined its measurements-* … Job 38, 4-5**

I asked my students and adult participants in my survey this question. When Columbus sailed west looking for a route to India in 1492, what did most people in Europe believe concerning the shape of the earth? Was it A. flat B. round? The results are below.

**Among students Among adults**

**A. Flat: 159 A. Flat: 16**

**B. Round: 12 B. Round: 3**

The results show that students and adults overwhelmingly believe that people in Columbus' era thought the earth was flat. Several students made the statement that people told Columbus that he would sail off the edge of the earth. Historical evidence indicates that people at this time actually believed the earth was **not** flat but spherical. When I asked why Columbus sailed west in order to reach the orient, which was geographically east, the reply was that Columbus believed the earth was round but every one else didn't.

Regardless of what Columbus did or did not believe, the results of this question indicate that some historic housekeeping is in order. Certainly lunar eclipses have been noted for thousands of years. It is obvious that the shadow that the earth is casting on the moon is round not square! When Aristotle put forth his model of the universe in about 330 BC, he placed the earth at the center and depicted it along with all of the other heavenly bodies as spheres. In fact he gave a value for the circumference of the earth at 40,000 stadia, adding: "From their (mathematicians) supposition, it follows that the shape of Earth must be a sphere and also that its size must be small relative to the distance of other celestial bodies." The stadium referred to by Aristotle corresponds to about 185 meters. Converting 40,000 stadia to meters yields a value of 74,000 km, which is much more than today's accepted value of about 40,000 km.

Eratosthenes, (275-194 BC) a Greek mathematician, designed and carried out an experiment about 150 years later that yielded a value close to the actual circumference of the earth. His methods were ingenious and surprisingly accurate. As Director to the Great Library at Alexandria he had access to great stores of knowledge. His readings of Posidonius revealed that on the day of the Summer solstice the bottom of a well situated at Syene (today known as Aswan) in Upper Egypt was illuminated by the Sun. At Alexandria an object placed in the sun always cast a shadow. In other words, the sun was always at a slant and never directly overhead when it was overhead at Syene. Believing that the Earth was a sphere, and that Syene and Alexandria were on the same longitude (meridian) and that the sun's rays were parallel, he reasoned that by knowing the distance between the two locations it was possible to determine the circumference of the earth if the angle the sun made with the upright at Alexandria were determined. Extend a line vertically from the sun into the well in Syrene to the center of the sphere (Earth). Draw a parallel line that in one part of its path strikes the upright at Alexandria. Continue the line through the earth keeping it parallel to the line in the well. The angle (a) formed because of the Earth's curvature by the beam on the upright at Alexandria is congruent to the angle (b) formed at the center of the earth when the upright line intersects the line drawn through the well. Eratosthenes determined this angle to be about 7.2 degrees or about one fiftieth of the 360 degrees in a circle.

Sun

Angle a

Angle b Earth

He estimated that the distance between Syene and Alexandria was about 5,000 stadia or the distance a camel caravan covered in 50 days at 100 stadia per day. This translates to about 787 km. We can now determine the circumference by proportions:

$$7.2 / 360 = 787 / x$$

Solving for x: x = 39350 km which is very close to the accepted value of 40,000 km.

It should be noted that the stadia used by Erastothenes and Aristotle were different.

*Rice University offers a nice student lab activity for repeating Eratosthenes' experiment on the Internet and the website is**http://www.mathRice.Edu/~ddonovan/Lessons /eratos.html**. It involves locating a city about five or six hundred miles directly north of your school's location. At noon on June 21 calculate the tangent of the angle cast by a meter stick placed vertically in the ground at your school, the tangent of the angle formed when the shadow forms on the ground on a vertical meter stick is also found at the more northern location. (The tangent in both cases is determined by dividing the length of the shadow cast by the length of the meter* *stick.) Subtracting the more southern angle from the northern one and by using the same proportional technique above but using your measured values it is possible to determine your own value for the circumference of the earth! This is an excellent opportunity for students to establish a link with another school and to carry out a historically significant experiment.*

Several others including Eudoxus of Cnidus (ca. 370 B.C.), Dicaearchus (died 296 B.C.), and Aristarchus of Samos (died 230 B.C.) either calculated the Earth's circumference or proposed a heliocentric view of the universe.

Another significant advance in establishing the notion of the Earth's shape and dimensions came in the area of cartography or mapmaking. Eratosthenes constructed a map of the known world, which was a remarkable achievement and influenced perceptions of the world in spite of its many inaccuracies into the middle ages. Claudius Ptolemy improved on Eratosthenes' map by marking it with grid lines of longitude and latitude. *(The choice of $15^0$ for each space between a line of longitude was no accident. Twenty-four longitude lines times fifteen represents the $360^0$ in a circle).* This helped navigation by dividing the world into manageable sections. Even when barbarians sacked Rome in the fifth century A.D. and plunged the world into the so-called dark ages, knowledge did not disappear. As roads the Romans had constructed deteriorated and communities in Europe languished in ignorance, feudalism, and isolation, monks and scholars kept alive the lamp of learning by guarding and copying ancient manuscripts. Commerce, travel, interaction, and ideas were not dead but continued at a reduced rate until the rebirth of learning in the $12^{th}$ and $13^{th}$ centuries in Europe. This rebirth was certainly stimulated by the fall of Constantinople in 1453 and the release of books and maps that certainly added to the sum total of knowledge in Europe. Also, the Portuguese in the $13^{th}$ and $14^{th}$ centuries mapped routes on the west coast of Africa as far south as the gold coast in ever widening circles of exploration.

Christopher Columbus learned the art of seamanship by shipping out to sea as a teenager. At the time of his first voyage to the new world he was a seasoned seaman with considerable skills developed first hand. Columbus was also an avid reader and surely availed himself of scholarly texts of the period. In 1476 he and his brother established a business in Genoa making maps and charts. Not satisfied with this business, he returned to the sea and while sailing the African coast developed theories about the size of the Earth and the distance to Asia by sea. He grossly underestimated the distance and didn't realize that the North American continent lay between him and Asia. Nowhere in the historical records is a flat earth discussed and in fact every evidence indicates that not only did Columbus think that the world was a sphere but so did everyone else.

The notion of the historic belief of a flat earth is taught in elementary schools in the United States. As a result it is a pernicious popular conception clung to by vast numbers of people. Where it arose is anybody's guess. The notion was certainly popularized in a biography of Christopher Columbus by Washington Irving in the nineteenth century. He wrote that Columbus defended his round Earth theory against flat-Earth believers at Salamanca University. This never happened and it is doubtful that Irving believed it himself, probably inserting it for dramatic purposes. It has been suggested that sails disappeared on the ocean's horizon because of the Earth's curvature. This might give the impression that a ship is dropping off the edge of the Earth. But this would suggest a spherical earth to a careful observer because the ship would disappear bow first and then the sail. Later the ship would reappear as the two ships closed distance showing that the ship hadn't dropped off the Earth after all. I think the idea persists because it is easy to remember and it serves as a convenient way to dismiss and compartmentalize an era in human history only dimly remembered. It is a way of saying that people in those days weren't very well educated and look how much superior we are today with our vast knowledge and superior technology. Whatever the reason, it is an excellent example of how shallow thinking misleads and shortchanges our understanding of the accomplishments of our ancestors.

**Floating and Sinking**

***Non teneas aurum totum quod splendet ut aurum***

***Do not hold as gold all that shines as gold…Parabolae***

The story of Archimedes, that most venerable Greek of antiquity, running naked through the streets of Syracuse crying, " eureka!" or " I have found it!" is emblazoned on the consciousness of our culture. When asked about the incident, students who had heard of Archimedes vaguely referred to a gold crown and a bathtub but couldn't give many details. The story is, in fact, rich in material that a teacher might use in a science classroom and can involve common sense techniques as well as methodology of proof in spite of paucity of hard facts. Simply stated, Archimedes was commissioned by Hiero II, the king of Syracuse, to determine if a gold crown was actually pure gold or alloyed with a baser metal such as silver. The king apparently suspected that the goldsmith might have been less than honest. As the story goes, Archimedes was puzzled for some time but upon relaxing in a bath noticed that his body displaced water as he settled in the tub. If the crown were submerged in water, it too would displace its volume just as had his body in the tub. Since silver is less dense than gold, the crown, if alloyed with silver or any other base metal of a lesser density than gold, would occupy a greater volume than a crown of pure gold of the same weight and thus displace more water. Indeed, the crown was adulterated and the king had the dishonest goldsmith executed for his misdeeds.

Continuing with the survey, I asked the following question with the accompanying results:When an object is submerged, it displaces its own weight, volume, or mass. The results are below.

**Among Students: Among Adults:**

**A. Weight 47 3**

**B. Volume 51 9**

**C. Mass 26 3**

In regards to this question it is evident that students are almost equally divided between weight and volume in what is displaced when an object sinks. A common sense explanation is that two objects cannot occupy the same space at the same time so an object must displace its volume when submerged in water. An object submerged in an overflow can, which is readily available from science supply houses, will quickly and convincingly show students empirically that an object does indeed displace its own volume when submerged. The procedure is as follows*: Determine the volume of a metal cylinder by the formula V=pi times $r^2$ /h where r is the radius of the cylinder and h is its height. One cubic centimeter is equal to one milliliter of water for all practical purposes. The overflow can is filled with water to the maximum amount and the cylinder is submerged in it. The water that overflows is collected in a graduated cylinder and compared to the volume of the cylinder. Within a small amount of experimental error, they should be equal.*

It would appear that common sense would serve a person well in this case. A deeper examination of the supposed procedure reveals some problems, however. A nice discussion of this problem can be found on the Internet at www.mcs.drexel,edu/~crorres/Archimedes/Crown/CrownIntro.html. The Internet discussion is as follows. Historical records indicate that the largest crown or wreath known at the time of Archimedes had a rim diameter of 18.5 centimeters and a mass of 714 grams. Let the mass of Hiero’s wreath, for argument’s sake, equal 1000 grams and the circular diameter of the container it is submerged in equal 20 centimeters. The container would then have a cross-sectional area of 314 square centimeters. Gold has a density of 19.3-grams/cubic centimeter. 1000 grams of gold would have a volume of 1000/19.3 or 51.8 cubic centimeters. If 51.8 cubic centimeters were divided by the 314 square centimeter cross-sectional area, the level of water at the opening would be raised only 0.165 centimeters.

If the goldsmith was exceptionally greedy and substituted 40% of the gold with silver, the amount of water raised can also be calculated. The density of silver is 10.6 grams/cubic centimeter. The gold-silver crown would have a volume of 600/19.3 + 400/10.6 = 68.81 $cm^3$. This crown would raise the water level in the container by 68.81/314 = .219 centimeters.

The change in water level would be .219 centimeters minus .165 centimeters or .54 millimeters. This is a very small amount of water to measure. Considering sources of experimental error such as surface tension and air bubbles trapped on the crown, it is very unlikely that this was the method used by Archimedes to expose the crown adulteration. In short once again common sense is not enough to fully explain the problem.

The author of the Internet source mentioned earlier gives a plausible alternative explanation. He proposes that Archimedes had a balance with the crown on one arm and an equal mass of gold hanging from the other arm both submerged in a tank of water. If the arms balanced, the crown was pure gold because both would have the same volume. If it tipped in the direction of the gold, it meant that the crown had a greater volume than the gold thus increasing its buoyancy. Because its density is less than that of the gold, it had to be an alloy. As in the problem earlier, if the crown was an alloy of 60% gold and 40% silver, it had a volume of 68.81 $cm^3$. Because water has a density of 1.00 $gm/cm^3$, the crown would displace 68.81 grams of water. 1000 grams of pure gold has a volume of 51.8 $cm^3$. It would displace 51.8 grams of water. The difference is 68.81 grams minus 51.80 grams or 17.01 grams. This difference in mass could easily be detected with scales of that era. If the crown and the gold mass it was compared to were not the same, the balance arms could be adjusted in length until they balanced before placing them in water. This explanation is plausible and also utilizes levers, which Archimedes used at length in his investigations. Although the true method Archimedes used is not clearly known it makes a good discussion topic and one in which students can experiment.

I asked another question relating to floating in my survey. When an object is floating, it displaces its own:

**Among Student Among Adults**

A. Weight 35 4

B. Volume 64 2

C. Mass 14 4

It is evident from the results that students need instruction in floating objects. A thought exercise easily clears up their misconceptions. When an object is floating, what force is acting on it to make it sink? The answer is gravity. If the object is floating, the force of gravity must be equaled by an upward buoyant force. Another term for the force of gravity on an object is its weight. The upward buoyant force then equals the weight of the floating object. *Experimentally this is easily confirmed by weighing an object that will float and by placing it in an overflow can filled to capacity with water. The weight of the water displaced when the object floats can be compared to the weight of the object. They should be equal.*

It is hoped that the discussion on floating and sinking will clear up misconceptions and emphasize the importance of logic and reason in approaching a problem. I think that students of middle school age are capable of understanding the concepts and realizing the fallacy of superficial thought in explaining what occurs. A deeper appreciation of Archimedes and his thought processes can also emerge from the teaching of these topics.

***A participle is not the only thing that can dangle***

*What follows is another good test of common sense and how to present a convincing argument to students. Consider a triangle whose greatest angle is not too obtuse. The upper limit to its size might be about* $75^0$. *Find the point inside the triangle whose distance from the three vertices is least. Practically stated the problem* *might be a triangular piece of land in which a developer wants to pave a road from the three corners using as little asphalt as possible.*

The solution may be approached with a pencil and paper or as a construction of a triangle using poster board or plywood. If a wooden or poster board triangle is constructed, it can be laid flat on a table so that the three vertices are over open space. Holes can be drilled in each vertex just large enough to let a string through them. Take one length of string, XY, and pass it through two vertices so that 10 centimeters or so hangs from each end. Loop another string, Z, over XY and feed its end out of the remaining vertex of the triangle. Attach equal weights to each string. One hundred to five hundred grams should suffice. The three strings will have equal tension on them since the weights are equal. The resultant force vectors at the three angles formed by the strings will also be equal since the tension on each string is the same. This point will be the minimum distance from the three vertices because with equal weights at equilibrium the maximum amount of string will be outside the vertices. Students can test this empirically by measuring the total length of the three strings from the vertices to the common point. If a heavier weight is attached to any string, the resultant string length as measured from the vertices to the new common point will be greater than that as a result of equal weights.

*The solution can also be a paper and pencil construction. If the lengths of 2 of the internal sides are known, the third can be determined by constructing an ellipse whose foci lie on the vertices of the triangle from which the two known lengths are drawn. The ellipse itself can be constructed by placing the paper with the triangle containing the two known lengths on a larger piece of foam board or cardboard. Stout pushpins are placed at the aforementioned vertices and pushed into the foam board. Loop a piece of string around the two upright pins and tie a knot so that the string becomes a loop. The loop should extend beyond one of the pins just enough to allow an ellipse to be drawn with its upper extension just touching the vertex of the angle formed by the convergence of the two known lengths. A perpendicular line drawn through the tangent line on the ellipse from the third vertex of the triangle to the intersection of the two known lengths represents the shortest path to these two lines.*

***Are we comparing apples and oranges?***

Students can be asked this seemingly innocuous question: Imagine a sphere that just fits inside a cylinder. What is the ratio of the volume of the sphere to the volume of the cylinder? Common sense would suggest that the volume of the sphere is less that that of the cylinder because it is contained in the cylinder. Just how much was carefully determined by Archimedes 2200 years or so ago in a blinding tour de force of logic, insight, and imagination. Essentially, he constructed a cone with its radius the height of the cylinder and its height the diameter of the sphere. By slicing the three structures in plane sections and comparing the ratios of their section areas he was able to extrapolate to the ratios of their volumes by imagining those three sections located appropriately on a balance. If the ratio of sections is shown to be constant for all sections, then the ratio of volumes as a whole is the same constant. As it turns out the ratio of the volume of the sphere to the cylinder is 2/3. This proof, although not rigorous as Archimedes himself acknowledged, does show the value of imagination and ingenuity in approaching a problem. Archimedes does prove the problem rigorously at another time.

***A vacuum speaks volumes…anonymous***

*One of my favorite demonstrations is to place a small amount of water in a 500 ml round-bottomed flask. Holding the flask firmly with metal tongs, I heat the flask over a Bunsen burner flame. I ask students to tell me when they see water vapor issuing from the neck of the flask. At this time I place a deflated balloon over the flask neck and continue heating the flask until the balloon inflates. I then place the flask-balloon system on the demonstration table and ask the students to predict what will happen. A few say that the balloon will get smaller but all are surprised when the balloon enters and practically fills the interior of the flask. The process can be accelerated by placing the bottom of the flask in a battery jar of cold water or by running cold water onto the flask itself. When pressed, some student will invariably state that the balloon was sucked into the* *flask. For me this is where the fun begins. I ask, "If this is true, what is doing the sucking?" For some reason adolescents love it when I ask this question. They also are about to learn how using common sense can paint one into a corner from which there is no easy escape. Before I reveal the answer, I perform another demonstration using a similar flask half-filled with water. I heat the water to boiling and then carefully place a stopper in the neck of the flask. I then ask my by now wary students what will happen when I plunge the stoppered flask into the battery jar of cold water. A few usually guess that the stopper will be blown out but most wait and see. What does happen is that the water begins boiling again. In fact the flask can be plunged into the water any number of times and boiling will commence. This usually surprises and puzzles my students. I add to the confusion by asking them how the balloon in the flask and the boiling flask are related.*

The answer is that in both flasks a partial vacuum is created and unbalanced pressures produce the surprising results. Students are aware that air pressure exists and that it is caused by the attraction of gravity on the atmosphere. At sea level this amounts to about 15 pounds on every square inch of the Earth's surface. In the case of the balloon and the flask I make sure that students observe the water vapor leaving the flask before the balloon is placed on it. The air in the flask has been replaced by the water vapor. When the water vapor is cooled, it condenses into water leaving a near vacuum in the flask. The greater outside air pressure then pushes the balloon into the flask. Students initially have a hard time accepting this explanation. They cling stubbornly to the notion that the balloon is being sucked into the flask until asked to explain what is doing the sucking. Nothing is sucked into a vacuum cleaner for the same reason. Dirt is pushed into the bag because of the pressure imbalance created by the partial vacuum inside the bag.

The water boils in the second flask for exactly the same reason. Most students haven't done the necessary thinking to realize that boiling is a pressure-related phenomenon. Most students can be convinced of what causes water to boil by an argument that begins by asking what boiling means. Eventually someone will volunteer that water is being turned to vapor. Vapor represents molecules that have overcome the forces that hold water in the liquid state. These include the force of gravity, surface tension, and most importantly, air pressure. Fifteen pounds on every square inch is a lot of force and water molecules have to gain enough energy from some source to overcome these forces. Heating water supplies the energy necessary to overcome the forces that hold the water in the liquid state. Conversely, if these forces can be reduced, it stands to reason that water will be able to escape as vapor in proportion to the reduction of force. That is exactly what happens when the flask with the stopper is placed in cold water. The water vapor, which has forced out the air, condenses leaving a partial vacuum above the confined water surface. The water temperature is already near the boiling point and has the energy necessary to allow it to escape as vapor in this environment of reduced pressure so boiling occurs. Eventually the boiling stops because the water vapor pressure increases to the point where it stops rapid vaporization. Plunging the flask back into the cold water starts the cycle all over again.

To test their understanding of the problem of boiling I then ask if it would take longer or less time to hard boil an egg in Denver Colorado as compared to sea level. Careful thought would suggest that the atmospheric pressure is less on a high mountain than at sea level. Denver is a city approximately one mile higher than sea level and thus water should boil at a lower temperature so it would take longer to boil the egg in question. It is helpful to have a cake box on hand. On the baking instructions it is suggested that the cake be left in the oven for longer periods of time because of decreased pressure in mountainous regions.

*If the instructor has access to a vacuum pump, additional demonstrations using a vacuum are possible. I have been able to boil water at temperatures as low as* $60^0$ *Celsius by placing a beaker of hot water under a bell jar and evacuating the air. It is possible to place ones hands on the top of the bell jar and feel the heat given off by the evaporated water as it condenses when it encounters the colder bell jar surface. Marshmallows and balloons swell appreciably as air is removed from the bell jar. As balloons swell dangerously near the breaking point students will often cover their ears. I ask them why they are doing this since sound waves don't travel in a vacuum. Indeed, the occasional popping balloon makes a slight ping as it bursts, not the loud bang that they expect.*

*It is possible to show the tremendous crushing power of air pressure by pouring an ounce or two of water in a gallon can with a screw-top lid. The can is then heated on a hot plate or over a Bunsen burner until water vapor is visibly issuing from the opening. At this point the cap is screwed on the can and the can is left to cool. Within a short period of time the can's sides will be crushed. An unbalanced force is created as the water vapor inside condenses and creates a partial vacuum. Running cold water on the sides of the can will cause the can to crush faster. Students like to crush 12 ounce soda cans by placing a small amount of water in the bottoms of the cans. The can is then held with metal tongs and heated over a Bunsen burner. When steam is issuing from the top, the can is quickly inverted with just the top placed in cold water. The can crushes dramatically and instantly.*

The demonstrations above suggest that something other than suction is afoot. The logical explanation is that if nothing but a little water is inside, the greater pressure on the outside must be producing the effects. Given a little time and their observations, students usually accept the possibility of the existence of a vacuum. In fact, this is an easy concept to accept today but historically this was not always so. Until the seventeenth century in Europe the universally accepted Aristotelian model of the universe precluded the existence of a vacuum. The Earth was at the center of this model, the planets circled it in circular, crystalline spheres, and the stars had their place in another perfect sphere. Every space was filled with earth, wind, water, fire, or ether. A favorite saying of time was "Nature abhors a vacuum." In defiance of the Aristotelian view Robert Boyle proposed that a vacuum could indeed exist and experimentally showed its existence. He believed that any theory that could not be experimentally tested or observed was not proven. This view underlies modern science. A theory that can't stand up to observation or testing won't last too long. Like a house of cards the Aristotelian model of the universe collapsed as observation and experiments by the likes of Galileo, Tycho Brae, Copernicus, Boyle, and Newton removed its underpinnings.

***If not common sense, what, then…?***

This paper began with the premise that humans use their powers of reason to construct models of their universe. The Aristotelian model lasted for 2000 years only to be supplanted by the Newtonian mechanistic view. So complete was the Newtonian model that many believed that the entire universe was explained. In this model space was infinite in scope and contained all matter, which interacted according to strict natural laws. Events occurred simultaneously in the same inertial reference frame. Time contained space since all events occurred in time and continued inexorably and forever. Man's horizons were limitless as long as natural laws were obeyed and this optimism buoyed humanity into the nineteenth century. Newton had ignored or overlooked one thing, however, and this thing undermined the mechanistic universe as surely as his model had Aristotle's. That thing was electricity.

*Students can be advised or assigned to research some of the pioneers of electricity. From the late 1700's to the early 1900's their contributions made possible the lifestyle we take for granted today as well as relativity and quantum theory. Some of these names are Oersted, Volta, Faraday, Maxwell, Michelson , Morley, Lorentz, FitzGerald, and of course, Einstein.*

To Newton electricity was a curiosity at best. He certainly did not connect it to light a phenomenon he had studied. Newton subscribed to the light model as described by the ancient Greeks. He described light as particulate in nature, emitted by hot objects and observable when reflected off of an object to the observer's eye. *Students can readily observe that light does travel as well as reflect in straight lines thus suggesting a particle. An index card with well-defined edges casts a clean, dark shadow on a wall fifteen or so feet away from the light source. If, however, the same index card is held close to a light source such as a slide projector, the shadow projected on the screen does not have crisp, black edges as would be expected if light were discrete particles. The fuzziness observed in the shadow suggests wave action as the light is bending around the edges.* The dual nature of light acting both as a wave and a particle led to intense investigation as to the nature of light and the investigation continues today. James Clerk Maxwell in the middle of the $19^{th}$ century described light as an electromagnetic wave that had a definite speed of 300 million meters per second. The connecting of light to electricity led to numerous experiments and to the invention of the concept of the aether or ether in which electromagnetic waves were thought to propagate through space. The invisible uncompressible ether was an Aristotelian construct and caused no end to efforts on the part of its adherents to prove its existence. *Students can do research on the famous experiments of Michelson-Morle, which not only showed the speed of light but suggested strongly that the ether did not exist.* Albert Einstein in his theories of relativity showed not only that ether to carry light was unnecessary but that a single inertial frame of reference as suggested by Newtonian mechanics was nonexistent. Frames of reference change with the observer, as does the concept of time. In short the universe is not as "cut and dry" as the Newtonian model would suggest.

The Biological Sciences have also shown changes as a result of deep observation and experimentation. Aristotelian writings in Biology are extensive and spurious. In one account Aristotle attributes to the female human, sheep, goat, and pig fewer teeth than the male of each respective species. Why no one bothered to correct such a mistake by actually counting the teeth is hard to say. For centuries it was also assumed that the male of the species was responsible for genetic traits. The female role was receptacle, incubator, and nurturer. The male "homunculus" theory reigned supreme until investigations by the likes of Gregor Mendel, Thomas Hunt Morgan, and others showed that hereditary material and thus traits were contributed by both sexes. The discovery of chromosomes, genes, and the structure of DNA have led to cloning, genetic engineering, and the genome project. As Biology has advanced the expense has increasingly been borne by private enterprise whose understandable action has been to patent what formally was publicly shared research. To those who believe that science advances when information is freely shared this is seen as an alarming development.

***Honesty is one policy…***

*Very interested, highly motivated* students *might be challenged by the following problem whose common sense solution proves false on closer examination. Imagine a country whose money is coined in private mints. The government whose best interest lies in having "honest" currency suspects that some mints are producing coins with less than prescribed ratios of precious metals. All dishonest coins are of the same weight and the government has in its possession a scale and the weight of an "honest" coin. The government also can demand any number of coins for examination from any mint. Let us further assume that the cost of weighing coins is so prohibitively expensive that it is in the government's interest to solve the problem in as few weighings as possible. How can the dishonest mints be determined with only two weighings?*

The answer to this problem might not be obvious to middle school students but some may choose to tackle it. The dishonest mints may feel secure in their anonymity but they have left a footprint. ***Their scheme can be exposed by using a base 2 number system with the digit 1 in every place there is a bad mint. Vis.*** Let us assume that the country has 7 mints. Weighing one would be as shown in the table below.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 1 bad + | 1 good + | 1 good + | 1 good + | 1 bad + | 1 good + | 1 bad |
| -(1 good + | 1 good + | 1 good + | 1 good + | 1 good + | 1 good + | 1 good) |

1 1 1

The three ones represent the as yet unknown bad mints and results from subtracting seven honest coins from the combined weight of the dishonest and honest coins. Bear in mind that we do not know at this point what the weight of a dishonest coin is. All we have is a composite weight that could be plus or minus depending on whether the good coins weigh more or less than the bad ones. The second weighing is as follows. Let each of the seven mints be assigned a value to the base two and that many coins from that mint will be placed on the scale. Mint one has the value of $2^0$ or 1; mint two is $2^1$ or 2, mint three is $2^2$ or 4 and so forth through seven.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 1 bad + | 2 good + | 4 good + | 8 good + | 16 bad + | 32 good + | 64 bad |
| -(1 good + | 2 good + | 4 good + | 8 good + | 16 good + | 32 good + | 64 good ) |

1 16 64

1+16+64 = 81**.** The 81 in this case stand for the total difference in weight by subtracting the weight of the honest coins from the total weight of honest and dishonest coins. These are our two weighings. The next step is to divide the number that resulted from the second weighing by the number of the first. The result is 27 or 81 divided by 3. The question now is to determine what possible combinations of base two numbers can result in a quotient of 27. For instance, 27 can result by dividing 54 by two. Is it possible to reach 54 by adding two numbers to the base two up to $2^6$ ? The answer is no. The closest numbers are $2^5$ or 32 plus $2^4$ or 16. This yields only 48.

The second possibility is to have 3 bad mints. Twenty-seven can result by dividing eighty-one by three. ***The three numbers, 1, 16, and 64, represent the numbers to the base 2 which have the digit 1 in the places of the bad mints! i***.**e**. 81 = $2^0 + 2^4 + 2^6$ In the base 2 number system this would be represented as 1000101 with the ones being the bad mints. In two weightings the three dishonest mints have been exposed!

This is a pretty nifty solution but one that is unfortunately flawed. Here is why:

Let us consider a case where there were four dishonest mints out of seven. They might sort out as following after one weighing:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 1 good + | 1 good + | 1 bad + | 1 bad + | 1 good + | 1 bad + | 1 bad |
| -(1 good+ | 1 good + | 1 good + | 1 good + | 1 good + | 1 good + | 1 good) |

1 + 1 + 1 + 1

The difference is four for weighing one. This is easy to see here but with the weight of a dishonest coin unknown, it is just a weight difference as in example one.

For weighing two:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 good + | 2 good + | 4 bad + | 8 bad + | 16 good + | 32 bad + | 64 bad |
| -(1 good + | 2 good + | 4 good + | 8 good + | 16 good + | 32 good + | 64 good) |

4 + 8 +32 +64

This problem works out to $2^2+2^3+2^5+2^6 = 108$. Dividing 108 by 4 yields the quotient 27. This is the same quotient as for three mints above. Since the weight of a dishonest coin is unknown, the results are ambiguous thus making this method invalid.

The actual solution involves using a similar method as above but designating the base number as the number of mints plus one. Astute students might note that this method would necessitate the use of huge numbers of coins if more than two mints is involved. That aside, the ambiguity resulting form two quotients of equal value is eliminated. The reason I am including this example is that it illustrates a case where a solution seems reasonable but can be proven false with just a little extra effort. That is the solution in the first case works for up to six mints but fails at seven. In mathematics and science it is a good policy to check thoroughly any proof or theory before trusting it completely.

**Einstein's equation says "this is the end" and physics says, "there is no end." … John Wheeler.**

I began this writing with a somewhat jaundiced view of common sense. The Aristotelian model of the universe, that is a common sense, observational construct, has proven inadequate in so many fundamental ways that to me common sense itself became inadequate for understanding. In a way I was misled by an invalid Aristotelian syllogism. To wit: Aristotelian common sense observations are all that are necessary to construct valid, working models of the universe. The universe is inadequately explained. Therefore common sense is an inadequate explanatory tool. Aristotle's observations and propositions were certainly lacking but perhaps common sense should be given a second chance. We live in a democracy, ironically a construct of Greek thought, and an informed citizenry exercising reason and commonly shared ideas i.e. common sense is necessary for its proper functioning. Arguments and propositions must be made transparent enough so that reasonable people can make reasonable, informed decisions based upon them. This does not mean that ideas must be "dumbed down" as some would argue. Quite the contrary! The giants of science and mathematics are remembered in large part because their ideas are transparent and comprehensible by any reasonably intelligent individual willing to make the effort to understand them. Archimedes, Galileo, Newton, Maxwell, Heisenberg, and certainly Einstein shook the universe with ideas that are models of clarity and insightful reasoning. A problem I perceive today is that so many branches of science and mathematics have become so specialized and esoteric that few outside the particular discipline understand them. The quantum theory, the Fermat proof, and the four-color theorem are only partially understood by the best scientists and mathematicians. Good teachers who are capable of making ideas transparent and understandable by the utilization of the natural light and reason common to all humans are needed as never before. I hope that the explanations and arguments I have advanced will help in this process. If common sense is dead, perhaps it needs to be resurrected albeit with the realization that very little can be accepted blindly at face value.

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TEACHER'S READING LIST

Burke, James, The Day the Universe Changed (Little, Brown, 1985)… *This book is an historical account of eight moments in history that radically changed man's perception of his universe. This book is highly readable and written by the author of Connections, the companion book to the popular 1980s TV series of the same name*

Davies, Paul, About Time (Simon and Schuster,1995)…*This book provides a highly readable account of time and Einstein's theories.*

Dennis P. Donovan, http://math.rice.edu/~ddonovan/Lessons/eratos.html …this web sight provides and interesting insight into Eratosthenes' monumental calculation of the Earth's circumference and a nice lesson plan that can be used by middle or high school students.

The Editors of Planet Dexter, This Book Really Sucks (Planet Dexter, 1999) …This book has a rubber cover made up of myriad suction-cups. This and the title make the book irresistible to my middle school scholars. The authors provide an irreverent, humorous, and entertaining examination of all things that "suck".

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Horgan, John, The Death of Proof (Scientific American, October 1993, pp. 92-103)… Mr. Horgan decries the increasing use of experimental mathematics and computers to prove theorems. He goes so far as to suggest that the reliance on these methods might be the future of mathematics thus jeopardizing formal mathematical proof as established by the ancient Greeks and practiced for over 2000 years.

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Loewen, James W., Lies My Teacher Told Me (Simon and Schuster, 1995)…*The author critiques twelve high school history books and delivers a scathing assessment of their shortcomings. The eye-opening account of Columbus and his accomplishments make the book worth reading.*

NOVA, Trebochet Construction (NOVA 1999) … French and American construction crews build two siege engines following medieval plans and launch 250 pound stones at a simulated castle wall. The trajectory of the stones in flight is evident and a reproach to Aristotelian logic.

Pappas, Theoni, The Joy of Mathematics (Wide World Publishing / Tetra, 1995)

## APPENDIX

The Pittsburgh Public Schools by whom I am employed has adopted Science Standards. The discussions and lessons in the paper, The Death of Common Sense, adhere to the following standards:

All students explain how scientific principles of physical phenomena have developed and relate them to real-world situations

All students demonstrate knowledge of basic concepts and principles of physical science.

All students explain the relationships among science, technology, and society.

All students construct and evaluate scientific and technological systems using models to explain and predict results.

All students develop and apply skills of observation, data collection, analysis, pattern recognition, prediction, and scientific reasoning in designing and conducting experiments and solving technological problems.