Functions in Motion

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Introduction

Students may gain a deeper understanding of mathematical concepts, if they can find and explore problems via hands-on experimentation. Learning can be especially great when students experience various ways of learning. The NCTM Standards for high school mathematics state that students should build on their prior knowledge and learn more varied and more sophisticated problem-solving techniques. They should increase their abilities to visualize, describe and analyze situations in mathematics terms. Problem solving gives students an opportunity to analyze and reason, which in the long run will prove to be a major part of success in mathematics (NCTM, 288).

The concepts of variables and functions are the building blocks of algebra. In a first year algebra course, the main emphasis is on functions. Functions allow students to describe a relationship and to predict future outcomes. For example, students can create a relation by recording and monitoring the growth of a plant over a period of time. In this case, the teacher is helping the students to build a foundation for the introduction of functions.

This unit will explore various functions in the field of motion. Constant motion and projectile motion are just two of several functions in the physical world.

Students learn to represent these functions symbolically, and graphically. Students collect data, analyze, and predict outcomes of motion problems. Students find their initial walking speed by taking the distance traveled over a period of time; students use the path of an object thrown in the air to investigate velocity, acceleration, distance and /or time.

The activities allow students to investigate problem situations that are related to places located around the world, The Golden Gate Bridge, The Empire State Building, etc. This alone will provide connection to the social sciences. To bring about active participation, this unit allows students to calculate and observe constant speed from a given distance over time (straight-line motion). Students explore projectile motion problems involving time, velocity and /or acceleration, in making conjectures about the behavior of the function. Also using the same concept, students can determine the time a particle arrives at a given distance, a particle's maximum height, the time for a particle to hit the ground, and the velocity and/or acceleration over a period of time.

Technology will play an important part of this unit as well. Multiple representations permit many problems to be solved using different methods. Students can view their results using tables and graphs that may bring about connections. Students can use their data to show relationships between the independent and dependent relationship of a function. The National Council of Teachers of Mathematics (NCTM) has provided standards that involve using technology for all grade-levels. Through the use of technology, students can obtain information from a whole new spectrum (visual representation). The NCTM report reveals that technologies are valuable educational tools that allow students to reach a higher level of mathematics ability. "Of all the influences that shape mathematics education, technology stands out as the one with the greatest potential for revolutionary impact" (Harvey, Waits, Demana, 75).

Unit Overview

The purpose of this unit is to serve as a resource for most courses at the high school level. Most courses require algebra as a pre-requisite. In the Pittsburgh Public Schools, most of the high school curricula are designed so that functions are explored, in subjects ranging from algebra through pre-calculus.

The main focus will be on application problems throughout the various lessons. These problems explore the linear and quadratic functions. In this unit, problems of various types that deal with motion, rate, speed, velocity, acceleration, height and distance will be investigated. Problem-solving activities will be provided, so students can participate through hands-on. Students will be allowed to experiment with different learning tools such as calculators, stop watches, rulers, etc.), in order to help them to build a deeper and richer understanding of mathematics. "Well-chosen problems can be particularly valuable in developing or deepening students' understanding of important mathematical ideas" (NCTM, 257).

Student involvement is necessary for the success of this unit. This unit has application problems, which connect to real world situations, and small group work activities. The problems are designed to bring about motivation, active participation, and hands-on experiences. This unit allows all students to get involved, and facilitate the transition from a traditional class format into one where discovery learning is taking place. This offers multiple representations of concepts and supports a tool-based approach. The kinds of problems given in this unit provide connections to other subject areas such as physics and social science.

This unit will provide an opportunity for students to become familiar with the use of the graphing calculator, TI83. The lessons provide an opportunity for students to graph functions, create tables, create regression lines, and use scatter plots. This tool-based approach allows multiple representations to be used in the classroom. This approach alone facilitates the transition from a traditional class format to discovery learning. As students work with tools over time, they may develop a clearer understanding of the concepts. Multiple representations of the concepts are needed so students can generalize and transfer their understanding.

For each of the various lessons, four to five days will be enough time for completing the assignments. The entire unit should take no more than two-three weeks to complete.

Lesson1- Constant Motion: Distance, Rate and Time

Objectives: Students will solve uniform-motion problems based on distance = rate x time. They will be able to calculate speed from distance and time measurements. Students will calculate time or distance using problems that are connected to real-world situations. Students can find the average speed walking by taking the average velocity = change in distance = $(x_2 – x_1) / (t_2 - t_1)$.

Rationale: The concept of functions is the focus of much of mathematics learning. Linear functions are the first which students explore. The following problems have been chosen, so students connect to the concept of function, and see the relationship of those functions to other topics such as physical science and social sciences. Using various representations, symbolically and graphically, students can become even more familiar with the linear function and its graph. Students need to develop an understanding of domain, range, slope of a line and y-intercept through hands-on activities. Connections are made easier for students when students can associate those connections to real –life events. Students will use formulas to create tables and graphs. This method will allow students to concentrate almost immediately on how the values of the function change. Also, the table approach keeps the concept of the solution in sight at all times. When students have access to tables, they can discuss the effect of a change in the variable values on the function containing that variable

Materials needed for the lesson: Stop watch, masking tape, measuring tape at least 50 feet or more, TI83 calculator, Excel (optional).

Instructional Methods: Once linear functions have been covered as a lesson objective, with concepts, such as line graph, slope, x and y intercepts, linear equation, slope-intercept form, students may participate in this lesson that involves equations and graphs of linear functions.

Students can be made aware of the simplest case of physical science involving straight- line motion. It is stated that the motion of a particle along a straight line, like that of a car moving along a flat, straight, narrow road is the simplest case of motion.

Change in distance = car (start)…………car(end)

x_1 x_2

The change is the car's position is called displacement, it is given by $x_2 - x_1$. Using the Greek letter Δ is a way to indicate the change in a quantity; this change is $\Delta x = x_2 - x_1$.

Velocity is created by the rate at which the position is changing. The average velocity of the particle is defined as the ratio of the displacementΔx to the time interval $\Delta t = t_2 - t_1$. Displacement and average velocity may be positive or negative. A positive value indicates motion in the positive x direction. The average velocity is the slope of the straight line connecting the points (t_1, x_1) and (t_2, x_2)

$V_{av} = \Delta \underline{x} = \underline{x_2 - x_1}$

$\Delta t \; t_2 - t_1$

The average speed of a particle is the ratio of the total distance traveled to the total time from start to finish. Since the total distance and total time are both always positive, the average speed is always positive.

Average speed = total distance = $\Delta \underline{s}$

total time Δt

Task 1: Students appreciate activities in which they are actively involved. Students will measure the time it takes to travel a known distance. In order to accomplish this task they must, first, measure the length of 50ft or more in the hallway, using masking tape for the starting and finishing lines.

The lines will be used for calculating velocity over a fixed distance. Next, find the time needed for each person to travel the fixed distance. Then, record the time and distance on a sheet of paper and create a data table. Students can now be introduced to the first type of uniform motion. Teachers can supply students with the given information about motion.

Uniform motion is motion at a constant speed in a straight line. Uniform motion can be described by a few simple equations. The distance (s) covered by a body moving with velocity (v) during a time t is given by s = Vt. Speed is the measure of motion. You can find it by dividing the distance covered by the time it takes to travel that distance.

Problem: Kendal's rate of speed can be found by taking the total distance traveled divided by the total time it takes to complete his trip. He traveled 30 ft in 8 sec. He must create a data table showing the different time intervals with the various distances. The graph he makes is the distance Vs time graph. He can analyze his data by making and examine scatter plots, modeling data, and using a linear regression line.

Average speed = 30 ft/8 sec = 3.75 ft/ sec

Table and graph:

Kendal's Walking Distance

Distance

45

37.5

30

22.5

15

7.5

2 4 6 8 10 12 Time

Table **Kendal's Distance Walking**

x	y
2	7.5
4	15
6	22.5
8	30
10	37.5
12	45

3.75 ft/ sec

The data table should contain the time it takes to travel each distance. The students can find their rates of speed by taking the distance divided by the time. Now, students can determine the distance over a given period of time (2min, 4min, 6min, 8min, 10min etc.) which can become their data. The graph each students is to make is a distance Vs time graph. Distance can be plotted vertically and time plotted horizontally. Students learn to analyze data by making and examining scatter plots, modeling data, and using linear regressions. Each student is to plot his/her own data. Discussion questions can be generated such as: Is the graph a straight line?

What does the slope represent? Compare and discuss your graph with other students. In which directions do the lines rise? Why? Based on their comparisons the students can then predict what the graph would look like for a person who is twice as fast or one-half as fast.

Supplement: Students can use their rate to determine how long it takes them to walk across various sites. Students can research other sites such as Fort Pitt Bridge, Yellowstone National Park, etc. to find their length. There are various selections to use so that student may become aware of other geographical locations. The Golden Gate Bride is just one of many. Students can find the time it will take to complete the trip if they were walking, biking or driving. Students can determine their rate biking over a period of time. Once students accomplish this goal, they may make a data table, create a graph, and a linear regression line. Students can compare their result walking, biking or driving.

The Golden Gate Bridge 1.2 miles long

Using the rate in the previous problem, it can be determined that if a student travels 3.75 ft/sec it will take that student 28.16 minutes to walk across the Golden Gate Bridge. To travel by car at a rate of 25mph it will take about 2.88 minutes (2minutes and 53 seconds). Last, students can determine the time it takes to travel by bike.

Students can determine how long it will take to walk across The Golden Gate Bridge.

$$\text{Time}_{(\text{walking})} = \frac{6336}{3.75 \text{ ft/sec}} = 1689.6 \text{ sec} = 28.16\text{min}$$

Next with this problem situation students can find how long it will take him to drive and bike across this famous site.

$$\text{Time}_{(\text{driving})} = \frac{1.2 \text{ miles long}}{25\text{mph}} = .048 = 2.88\text{min} \sim 2{:}53\text{sec}$$

$$\text{Time}_{(\text{biking})} = \frac{1.2 \text{ miles}}{5\text{mph}} = .24\text{hour} = 14.4 \text{ minutes}$$

Students can use other sites and events as well. Research can be done on other places of their choice. In this situation, the students decide to take a bike tour across Yellowstone National Park. Using the information given, students can create their own routes. Once they create their own routes, they can gather information on the total distance they will travel, the time it takes to travel to a certain destination, and their traveling speed. They can compare and discuss their information.

300ft

NW **N**
NE

W Gill Ave. E Gill Ave. N Willow St.

W. Broadway	E. Broadway
	E. Simpson Ave

W N Glenwood St S Willow St. **E**

Expected Outcome: This lesson should take at least 4 or 5 days to complete. As a result of this unit, students may be able to understand other characteristics of linear functions and how they can be applied to real-world situations. Students may become familiar with the graphing calculator via graphing, making tables, creating scatter plots, and making linear regressions.

Lesson2: Projectile Motion

Objective: Students will explore equations involving projectile motion in order to make conjectures about a quadratic function.

Rationale: When students are given a quadratic function $y = ax^2 + bx + c$, $a \neq 0$, the emphasis is placed on certain attributes of the graphs, vertex, and the various forms (shifts) involved. Students can learn to find the roots, the maximum value and minimum value, where the functions are increasing/decreasing. Various motion problems are chosen so students see relationships to the real world in the area of physical and social science. Students will investigate the path of a particle thrown in the air, then create the graph of a parabola. So that true connections are made of this function, students will explore various events, changes in time, height, and velocity and sites in the world in order to produce final outcomes that may differ.

Instructional model: Students will gather information about the different kinds of motion in order to define projectile motion. After the information is presented about the history of projectile motion, the teacher can lead a discussion about projectiles and how they connect to quadratic equation.

After completing tables and sketching graphs, the students can prepare a report to discuss their findings. They can include in their report, how changes in the one aspect can affect the motion of the object. The classroom teacher can set the stage for the activity by making it known that: When an object is thrown straight up, the relationship between the height of the object above the ground and the time after it was thrown, can be modeled by the quadratic function

$H = -1/2gt^2 + v_o t + h$, where (h) is the height, (t) represents the time after it was thrown, (g) represents the force of acceleration due to gravity, the linear coefficient, v_o represents the object's initial velocity and the constant term, h_o represents the object's initial height above the ground. Since it is difficult to analyze all of these variations at the same time, have the students explore factors involving different heights, different velocities, different planets- Venus, Earth, Mars, Jupiter, Neptune.

Students can be informed that acceleration is the rate of change of the instantaneous velocity. The average acceleration for a particular time interval is $\Delta t = t_2 - t_1$.

$A_{av} = \Delta V / \Delta T$

Instantaneous acceleration is the limit of the rate $\Delta V / \Delta T$ as ΔT approaches zero. The instantaneous acceleration at time t is the slope of the line tangent to the curve at the time:

$A = \lim \Delta \underline{V}$

$\Delta t \rightarrow 0 \ \Delta T$

Acceleration is the derivative of velocity with respect to time; dv/dt. Velocity is the derivative of the position x with respect to t, accelerations is the second derivative of x with respect to t, $d^2 x / dt^2$.

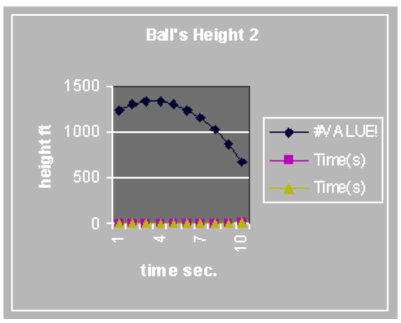
If the acceleration is zero, there is no change in velocity over time- velocity is constant. In this case, the curve of x verse t is a straight line. If acceleration is nonzero and constant, then velocity varies with time and the curve of x verse t.

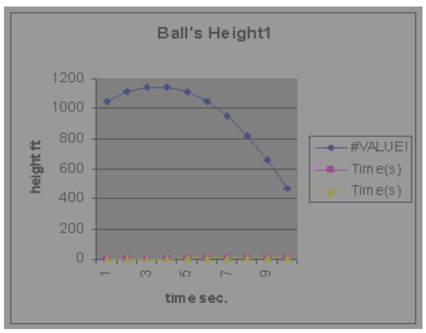
Part 1: The students can calculate, create a table, and record their findings for the given 5 objects thrown straight up into the air with an initial velocity of 80ft/sec but with different initial height. Different structures are used. Teachers can use the concept of projectile motion in connection to other subjects. History can be integrated into this lesson as well. Students can explore the variety in time and height of an object that is projectile from various structures such as:

The Chrysler Building, New York: 1,046
The Empire State Building, 1250ft(not including the mast)
The World Trade Center, New York, 1365ft
The Statue of Liberty 152 ft tall and stands on a 150-ft pedestal
Mount Rushmore (6,000ft), South Dakota
The Golden Gate Bridge (Main span 4,200ft high) or 12 miles long
The Great Wall of China 40 ft high
The Leaning Tower of Pisa

The Chrysler Building and the Empire State Building will be used to create tables and graphs. Students can analyze all of the variations that occur when a ball is thrown from the two sites. Students can compare data using both tables and graphs. Questions can be generated such as: How long does the ball stay in the air? When will the ball hit the ground? What is the maximum height of the particle? Determine the acceleration and velocity over a period of time.

The Chrysler building 1046 ft high **The Empire State Building 1250 ft**





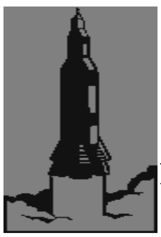
It is important for students to see how the different variations (height, velocity, and gravity) aeffect the outcome of the problem situation. When different structures are used, students can explore how the various heights can affect the location of the particle during various times. Also students can investigate how changing the velocity will affect the location of a particle at different times. Last, students can see that gravitation will also play a role in determining the location of a particle over a period of time.

Students are able to explore the various problem situations from various perspectives. Students may be able to make conjectures about quadratic functions by viewing the graphs or tables. Students can determine the particle maximum height, use the root-finder to see when the particle will hit the ground, predict if the ball is slowing down or speeding up, and /or its locations after a time period.

Students need to see how mathematics can be related to real-world events. To bring about motivation, problems are designed so students can explore sites around the world. By providing problems with real –world events, it may help students to become familiar with the world around them.

Part 2: The students should calculate, create a table, and record their findings for the given: 5 objects thrown straight into the air from The**Cathedral of Learning**, University of Pittsburgh. The objects have different initial velocity – 50 ft/sec, 70ft/sec, 80ft/sec, 90ft/sec and 100ft/sec.

Part 3: The students can calculate, create a table, and record their findings for the given problem: 5 object thrown straight-up into the air with the same initial velocity (50ft/ sec); the initial height will be that of a spacecraft, but are located on different planets- Venus, Earth, Mars, Jupiter, and Neptune.



Planet	**Acceleration due to Gravity (ft/sec^2)**
Venus	**29**
Earth	**32**
Mars	**12**
Jupiter	**81**
Neptune	**38**

Supplement 3: With a graphing calculator TI83, students can develop programs to determine the various heights of objects and the velocity of an object if the time is known and the height is known.

Once students have explored the various paths of an object using the symbolical and graphical methods, they can write their own programs. Students can write programs for determining the various heights, times, and acceleration of a particle projected. Students should have experience using the symbolical and graphical methods before they start writing programs. This may help them to explore problem situations from a much wider spectrum. This method will allow students to concentrate almost immediately on how values of the function change. Also, the table approach keeps the concept of the solution in sight at all times, when students have access to tables, they can discuss the effect of a change in the variable values on the function containing that variable. Here is an example of a program that was created by a student to find the height of a particle.

Program Height
Prompt V
Prompt S
Prompt T
($-16t^2 + vt + s$) yields H
Disph.

When students create their own programs, it allows them to participate in program design and development. This alone, promotes creativity and discovery learning. This is excellent because students can become inventors, using technology. "Today's mathematics technologies are generally characterized as tools and assistants… these technologies are useful only after a plan for using them has been developed by the user (Demana, Waits, Harvey, 95).

Expected Outcome: This lesson should take at least 4 or 5 days to complete. As a result of this lesson, students may be able to understand acceleration and velocity. Students should be able to identify with the various characteristics of quadratics by using the equations and graphs. Students should be able to make predictions about the different events that occur during different times.

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APPENDIX A: Content Standards

The Pittsburgh Public Schools have adopted Mathematics Standards that are used throughout the entire district. The Mathematics Standards describe what students should know and be able to do at four grade levels(third, fifth, eighth and eleventh). They reflect the increasing complexity and sophistication that students are expected to achieve as they progress through school. The lessons and tasks in the paper, Functions in Motion, adhere to the following standards:

. All students use numbers, number systems, and equivalent forms (including numbers, words, objects and graphics) to represent theoretical and practical situations.

. All students compute, measure and estimate to solve theoretical and practical problems, using appropriate tools, including modern technology such as calculators and computers.

. All students apply the concepts of patterns, functions, and relations to solve theoretical and practical problems.

. All students formulate and solve problems and communicate the mathematical processes used and the reasons for using them.

. All students understand and apply basic concepts of algebra, geometry, probability and statistics to solve theoretical and practical problems.

. All students evaluate, infer and draw appropriate conclusions from charts, tables and graphs, showing the relationships between data and real-world situations.

. All students make decisions and predictions based upon the collection, organization, analysis and interpretation of statistical data and the application of probability.

Appendix B: Glossary

A. Content Standards Specific information on what students are expected to know and be able to do for the nine academic goals. Content standards describe the knowledge and skill expected of students at important developmental stages. Standards are not curriculum, but guide the development of curriculum.

B. National Standards Statements of content and performance standards being developed by groups of experts, including teachers, in the art, English/language arts, civics, economics, history, social studies, science, geography, and world languages. All are following the model of the NCTM Curriculum and Evaluation Standards for School Mathematics (2000).