

How to Quantify the DNA Molecule

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Overview

This curricular unit will focus on size measurements of the DNA molecule, including actual lengths and masses of the base pairs, number of base pairs in DNA molecule, the number of base pairs per chromosome, and many more mathematical problems that arise when considering what is all of this genetic material that we are made up of? Further, when we examine our DNA, we see that there are huge quantities of it in our bodies, yet the size of them is minute. How do you deal mathematically with quantities so large, and also sizes that are so small? Technology will be a critical part of this unit; graphing calculators are a necessity to help the students with the computation involving scientific notation. This unit is intended for an eighth grade math/science curriculum. Topics discussed will be weights and measures, scientific notation, metric prefixes, and finally putting it all together with our genetic make up. This unit should take about 10 days.

Rationale

As a mathematics teacher in the Pittsburgh Public Schools, I created this unit with the current middle school curriculum, *Connected Mathematics*, in mind. The most effective teaching model for this unit would be the launch, explore, summarize lesson model that is used every day in *Connected Mathematics*. The launch of most of the lessons will be will be the introduction to DNA and genetics. Handouts will be available with this unit that contain all of the vital information and numbers necessary to complete the unit and should be photocopied for each student. The explore will allow the students to read through the materials and take out the information they need for each lesson. Finally, summarize will come at the end of each lesson when students share with the class their strategies for solving the problem. In the current curriculum, before even discussing the mathematics, a problem is introduced, and then you discuss what is necessary mathematically to solve it. I intend to use the same approach with this unit.

The idea for this unit arose when I attended the first Genetics and Genomes seminar. It has been awhile since I had studied genetics and DNA, and I was really stunned with the complexity of our genetic make up. All through that seminar, I was trying to quantify what all of this DNA stuff really was. How big is it? Does it exist in all of our cells? How much of it is there? How many A's, T's, C's and G's does it take to make us human? All of these questions led me to the idea to create a unit dealing with both really big and really small numbers. The current middle school science curriculum does not discuss genetics at all, so trying to go into any more detail than its size and basic function would be fruitless.

Objectives

The main objective for this unit is for students to become comfortable using scientific notation when dealing with both really big, and really small numbers. A good background with exponents is a plus, but not a necessity. Most of the students have probably already seen scientific notation when using the graphing calculators, they just did not know what the 7.5×10^6 meant. Graphing calculators are a necessity for completion of this unit, however, they should not be introduced before you have completed Labsheet 1.1, and Labsheet 1.2. It is critical to have the students understand the concept before they begin using the technology.

Strategies

One of the objectives of this unit is to increase the students interest in the topic of Genetics and to allow them to see how closely science is related to mathematics. In order to do this, you must incorporate all of the included activities. The first two labsheets should be completed as practice for the students in order to get them comfortable using scientific notation. Labsheet 1.3 should be completed with a graphing calculator, only after the students understand what these numbers mean. For some, more practice may be necessary to get them comfortable using scientific notation. Labsheet 1.6 is the last practice worksheet, it will familiarize the students with some metric prefixes and give them some practice making conversions between them.

The remaining labsheets should be used as application problems, along with their DNA information sheet. All of these incorporate problem solving skills that are critical to the future success of the student in their academic careers. -

Classroom Activities

Day One

The introduction to this unit should be a discussion to see how much the students already know about our genetic make up. Currently, it is not a topic of the middle school science curriculum, so a lot of what they know about it may come from the media, in which case, there are probably a lot of misconceptions about DNA. Some of the students may be under the impression that cloning is something that is very easily and very frequently done, when in fact it is a very lengthy and technical process. Why is it so difficult to do? Well, let's think about that in terms of the actual size of DNA. Since DNA is a very small molecule, would it be fairly easy, or fairly difficult to manipulate? What about money, wouldn't it be fairly expensive to find both the tools and the expertise to do something like clone a sheep? In the appendix you will find the DNA Information Sheet, which has all of the relevant numbers that you will need for this unit. Photocopy enough for all students and have them keep it in their notebooks for the remainder of the unit. Read through this handout with the class and naturally, more questions and discussion will arise. The first homework assignment should be to have them write down 5 questions they have about DNA.

Day Two

Now that we have discussed the basics about DNA and its uses, we will begin to discuss its size. To do that we will need to use scientific notation. Graphing calculators will be used eventually, but for the beginnings of this topic, use only calculators without scientific notation capabilities. Start by explaining to the students that this is a shortcut way of writing both really big, and really small numbers. Next, show the powers of ten, written in exponential form. For example:

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1,000$$

$$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$$

Now show some numbers multiplied by powers of 10. Start small, and work your way up to numbers large enough so that using scientific notation really is a shortcut to writing out the entire number. For example:

$$8 \times 10^2 = 8 \times 100 = 800 \text{ (8 is the base, 2 is the exponent)}$$

$$3 \times 10^4 = 3 \times 10,000 = 30,000 \text{ (3 is the base, 4 is the exponent)}$$

$$9 \times 10^7 = 9 \times 10,000,000 = 90,000,000 \text{ (9 is the base, 7 is the exponent)}$$

Do enough examples so that the students can see that it is indeed a simpler way of writing the numbers. Now you should look for a pattern in the examples you have just done. What was the power of ten for each one? How many zeros were at the end of each number? Now the rule can come in:

$N \times 10^x$ where N = number greater than 1, but less than 10, and x = exponent of 10

It is important to stress that the number must be between 0 and 1. Now do some more examples of taking a number and putting it into scientific notation. Example:

$$6,000 = 6 \times 10^3$$

Discuss how you went from 6,000 to just 6. You did that by moving the decimal three spaces to the left, or by dividing by 10 three times. Do some more examples with the class, again, do enough so that the students can see it as a shortcut. Labsheet 1.1 will give them some additional practice, however, you will need to discuss rounding the number to the nearest tenth or hundredth beforehand. You may want to do additional practice using simpler numbers first, depending on the level of your students.

Day 3

Now we will use scientific notation for really small numbers. You have already used it with really big numbers, so it should be fairly easy to transition your students to go the other way. To begin, remind your students that this is a shortcut, and it eventually it will make things easier. Negative exponents have not been introduced in math, so you will need to do that. Your students may not be ready for that,

instead we can say that positive exponents are used when you have a really big number, and negative exponents are used when you have a really small number. Again, show them the rule:

$N \times 10^x$ where N = number greater than 1, but less than 10, and x = exponent of 10

The only change is that now the x will be negative. Example:

$$0.00005 = 5 \times 10^{-5}$$

Here, 5 is the base, and -5 is the exponent. You can explain this as 5 being divided by 10 five times. Have them do this on a calculator to make verify that that is what is actually happening. How many places did you move the decimal so that you had a number between one and ten? Since you moved it to the right, the exponent must be negative. Show some more examples until they get the hang of it and do some really small numbers so that they can see that this is a shortcut. Labsheet 1.2 will give them additional practice.

Day 4

Once you have gotten them comfortable writing numbers in scientific notation, lets start multiplying them. This is a good time to introduce the EE button on the graphing calculator, or you can wait until you are done with Labsheet 1.3, and use the graphing calculators to check them. To multiply numbers in scientific notation, you first multiply the bases, then add the powers of ten. This will give you their product. Be careful, however, when the product of the two bases is greater than 10. For scientific notation, the base must be between 0 and 1, so you would have to move the decimal to the left and then account for that in the power of ten. An example is shown on labsheet 1.3.

To write numbers in scientific notation and have the answers in scientific notation on the graphing calculator, first change the mode to **Sci**. To write a number, first type in the base, then press the **EE** button and then the power of ten.

Day 5

Next we will use our DNA information sheet, along with what we have just learned about writing numbers in scientific notation to answer a few questions about DNA. Labsheets 1.4 and 1.5 each have 5 questions relating to DNA, its size and mass. At this point, more discussion can take place about the human genome project, genes and their functions, and units of measurement. Additional questions may be generated using the facts given, or through additional research. The Human Genome Project website has a lot of additional information and additional links that can be useful for gathering more information.

Day 6 and Day 7

The last mathematical topic we need to discuss for this unit is the use of metric prefixes. The context that your students will be most familiar with is in measuring lengths and distances in meters, centimeters and kilometers, so we will just expand on this knowledge. Start by taking a meter stick, and showing the basic unit of measurement as the meter. That is then divided up into decimeters. There are 10 decimeters in one meters, so you can say that a decimeter is one-tenth of a meter, or 0.1 meters. Continue this with centimeters being one-hundredth of a meter or 0.01 meters, and millimeters being one-thousandth of a meter or 0.001 meters. Continue this by saying that a millimeter is broken down into even smaller parts, and explain micrometers, nanometers, and picometers.

Use the same reasoning to explain the larger units, Decameter, Hectometer, Kilometer, Megameter, Gigameter, and Terameter. When you reach both the really big units and the really small units, some of your students should realize that this would be a good place to use scientific notation. Labsheet 1.6 gives them a table to use and 10 practice problems converting from one metric unit to the next. Before you give them Labsheet 1.6, you should start them off with some examples in class, and some guided practice. To tie this in with the scientific notation, in the space to the right of the table, have them write each of the prefix equivalents in scientific notation. You may want to use one day to discuss only the smaller metric prefixes, and a separate day to discuss the larger metric prefixes.

Day 8, Day 9 and Day 10

The remainder of the unit will be dedicated to incorporating the mathematics learned into the human genome. In the appendix you will find some multi-step problems involving the human genome that incorporate all of the mathematics introduced in this unit. These may be used as cooperative learning

exercises, independent practice, or as assessments. Graphing calculators are a must for these problems so that the students are not hung up on trying to do the calculations.

Annotated Bibliography for Teachers

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1. Davidson, Michael W. 2001. "Molecular Expressions Photo Gallery: The DNA Collection." Florida State University.

URL: <http://www.microscopy.fsu.edu/micro/gallery/dna/dna.html>

This website was used as a source for finding information on DNA size.

2. Human Genome Project Information Website. 2002.

URL: <http://www.ornl.gov/hgmis/fag/fags1.html#genetics>

URL: <http://www.ornl.gov/hgmis/elsi/forensics.html#2>

URL: <http://www.ornl.gov/hgmis/publicat/primer/prim1.html>

The problems on labsheet 1.7 and labsheet 1.8 were created from the information in these webpages. In addition, definitions and specific information about DNA shape and amounts contained your cells was obtained from this glossary to create the information sheet for the students.

3. Meeker-O'Connell. "How DNA Evidence Works." 2002.

URL: <http://www.howstuffworks.com/dna-evidence1.htm>

Information was taken from this website regarding coding and non-coding DNA regions.

4. Primer on Molecular Genetics. 2002.

URL: <http://www.ornl.gov/hgmis/publicat/primer/prim1.html>

This website will give more detailed information on the function of the DNA molecule.

5. Stewart, Robert. "A Few Words About DNA and Chromatin." PNNL-SA-30811

URL: <http://www.pnl.gov/berc/bg/dna.html>

More specific dimensions and weights of a DNA strand were given in this website and used for calculation purposes.

Reading List for Students

1. Human Genome Project Website Glossary. 2002

URL: <http://www.ornl.gov/hgmis/project/info.html>

This will be a useful website to use when first discussing DNA and its function.

2. Meeker-O'Connell. "How DNA Evidence Works." 2002.

URL: <http://www.howstuffworks.com/dna-evidence1.htm>

Use this website to answer questions about DNA testing in the courtroom and determining its reliability.

3. Park, John L. Metric Prefixes Website. 1998

URL: <http://dbhs.wvusd.k12.ca.us/Metric/Metric-Prefixes.html>

Provides a complete list of metric prefixes – both big and small.

Materials for Classroom Use

1. Logan, R.H. "What is Scientific Notation and How is it Used?" 2001.

URL: http://members.aol.com/profchm/sci_not/html

Provides detailed descriptions of scientific notation and how it should be used, along with additional practice problems.

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2. Park, John L. Metric Prefixes Website. 1998

URL: <http://dbhs.wvusd.k12.ca.us/Metric/Metric-Prefixes.html>

Provides a complete list of metric prefixes – both big and small.

3. Human Genome Project Website Glossary. 2002

URL: <http://www.ornl.gov/hgmis/project/info.html>

Provides a complete glossary and index of various DNA and human genome project terms and happenings.

Appendix I: Content Standards

Listed below are the mathematics standards used in the Pittsburgh Public Schools. I believe that with this unit, all of them are reached. In particular, standards 1, 2, 3 and 4 will be the focus of the unit. In completing the problems for the end of the unit, students will be expected to write complete explanations (in words) of the mathematics they are doing, and why they are doing it. It will be necessary for the students to extract the relevant information from their readings in order to answer the questions, which incorporates standards 6 and 7. All of this will be a challenge for the eighth graders. This material is very difficult, and putting it into a context they can understand is difficult.

1. All students use numbers, number systems, and equivalent forms (including numbers, words, objects, and graphics) to represent theoretical and practical situations.
2. All students compute, measure, and estimate to solve theoretical and practical problems using appropriate tools, including modern technology, such as calculators and computers.
3. All students apply the concepts of patterns, functions and relations to solve theoretical and practical problems.
4. All students formulate and solve problems and communicate the mathematical processes used and the reasons for using them.
5. All students understand and apply basic concepts of algebra, geometry, probability and statistics to solve theoretical and practical problems.
6. All students evaluate, infer and draw appropriate conclusions from charts, tables and graphs, showing the relationship between data and real world situations.
7. All students make decisions and predictions based upon the collection, organization, analysis and interpretation of statistical data and the application of probability.

Appendix II: Included Classroom Materials

In the appendix you will find all the necessary handouts that go along with this unit.

DNA Information Sheet -

1. Labsheet 1.1 – Writing numbers in scientific notation.
2. Labsheet 1.2 – Writing numbers in scientific notation.
3. Labsheet 1.3 – Multiplying numbers in scientific notation
4. Labsheet 1.4 – Application Problems
5. Labsheet 1.5 - Application Problems
6. Labsheet 1.6 – Metric Prefixes
7. Labsheet 1.7 – Size of the Human Genome
8. Labsheet 1.8 – Applications
9. Labsheet 1.9 – Applications

DNA Information Sheet

DNA is contained in nearly every cell in your body (white blood cells are the exceptions). It is in the shape of a double helix, imagine a ladder that is twisted and wound up to fit inside the nucleus of your cells. The rungs of this "ladder" are comprised of base pairs, pairs of the nitrogenous bases, Adenine, Thymine, Cytosine, and Guanine. Adenine can only pair with Thymine, and Cytosine can only pair with Guanine. Each base pair has an average mass of about 615 daltons (Da) (5). The entire sequence of DNA in one cell of your body is called the Human Genome. You have the exact same DNA in all of your cells; whether it is a skin cell, or a brain cell, all of the DNA is the same. A Human Genome consists of approximately 3 billion base pairs (4)! That is 3,000,000,000 pairs of Adenine, Thymine, Cytosine, and Guanine all squeezed into the nucleus of one cell! Also contained in the nucleus of your cells are your 24 chromosomes. Each chromosome has a single DNA molecule. If all the DNA strands in one cell were unwound and placed end to end, the total length would be approximately 5 feet in length (4). This is where the DNA ladders are; each chromosome has anywhere from 50 million to 250 million base pairs (2)! Genes are specific sequences of bases within our DNA that encode instruction on how to make proteins and they comprise only about 2% to 5% of the human genome. The entire human genome is estimated to contain 30,000 to 40,000 genes (4).

Labsheet 1.1 – Writing numbers in scientific notation.

Name: _____

Date: _____

To write a number in scientific notation, use the following form:

$$N \times 10^x$$

where N (also called the base) is a number between 1 and 10, and x is the power of ten.

For really big numbers, first move the decimal to the left as many places needed so that you now have a number between 1 and 10, that number is the N or the base. To keep it simple, round each number to the nearest tenth after you have turned it into a decimal. Next, count how many places you moved the decimal, that number is your x, or your power of 10. Now write it all out in the form

$$N \times 10^x.$$

Example: $1,500 = 1.5 \times 10^3$

$$3,528,680 = 3.5 \times 10^6$$

Try these for practice.

1. $3,600 =$ _____

2. $415 =$ _____

3. $3,400,000 =$ _____

4. $6,852,859,300 =$ _____

5. $5,802,368,258,258 =$ _____

6. $85,875,986,526 =$ _____

7. $975,875,896,321 =$ _____

8. 48,098,986 = _____

9. 201,777,425,587 = _____

10. 63,258,158,218,258 = _____

Labsheet 1.2 – Writing numbers in scientific notation.

Name: _____

Date: _____

To write a number in scientific notation, use the following form:

$$N \times 10^x$$

Where N (also called the base) is a number between 1 and 10, and x is the power of ten.

For really small numbers, first move the decimal to the right as many places needed so that you have eliminated all of the zeros immediately after the decimal and now have a number between 1 and 10, that number is the N or the base. To keep it simple, round each number to the nearest tenth after you have turned it into a decimal. Next, count how many places you moved the decimal, that number is your x, or your power of 10. **For really small numbers, the power of ten will be negative.** Now write it all out in the form $N \times 10^{-x}$.

Example: $0.015 = 1.5 \times 10^{-2}$

$$0.000036981 = 3.7 \times 10^{-5}$$

Try these for practice.

11. $0.0058 =$ _____

12. $0.000001875 =$ _____

13. $0.000036900 =$ _____

14. $0.00000000087515 =$ _____

15. $0.00368 =$ _____

16. $0.000009875 =$ _____

17. $0.55 =$ _____

18. $0.005698 =$ _____

19. $0.0003677 =$ _____

20. $0.26786 =$ _____

Labsheet 1.3 – Multiplying numbers in scientific notation

Name: _____

Date: _____

Numbers in scientific notation take the form:

$N \times 10^x$

Where N (also called the base) is a number between 1 and 10, and x is the power of ten.

To multiply numbers in scientific notation, use the same rules you would use for multiplying exponents. Multiply the bases – or the N – and add the exponents – the x.

Example: $1.5 \times 10^5 \bullet 3.0 \times 10^3$

$$(1.5 \bullet 3.0) \times 10^{(5+3)}$$

$$4.5 \times 10^8$$

$$2.0 \times 10^{-8} \bullet 8.1 \times 10^3$$

$$(2.0 \bullet 8.1) \times 10^{(-8+3)}$$

$$(16.2) \times 10^{-5}$$

Remember – the base must be between 1 and 10 –

You must move the decimal one more place to the left!

Since you moved it one place, add one to the exponent

$$1.62 \times 10^{(-5+1)}$$

$$1.62 \times 10^{-4}$$

Try these for practice.

1. $2.0 \times 10^5 \bullet 3.0 \times 10^7$

2. $1.5 \times 10^6 \bullet 4.0 \times 10^9$

3. $2.5 \times 10^5 \bullet 3.5 \times 10^{-8}$

4. $4.0 \times 10^{-12} \bullet 2.3 \times 10^4$

4. How many strands of DNA are in each cell of your body?

5. What is the approximate length of each DNA molecule?

Labsheet 1.5 - Application Problems

Name: _____

Date: _____

Use your DNA information sheet and your graphing calculator to complete the following problem.

1. If all the DNA molecules in one cell were placed end-to-end, approximately how long would it be?
2. How many base pairs are in the human genome?
3. What percent of the human genome is composed of genes?
4. Using your answers for #8 and #9, approximately how many of the base pairs in the human genome are genes?

5. Approximately how many base pairs would one gene contain?

Labsheet 1.6 – Metric Prefixes

Name: _____ Date: _____

Use the table below to answer the questions. Show your work!

Picometer	0.000000000001 meters
Nanometer	0.00000001 meters
Micrometer	0.000001 meters
Millimeter	0.001 meters
Centimeter	0.01 meters
Decimeter	0.1 meters
Meter	1 meter
Decameter	10 meters
Hectometer	100 meters
Kilometer	1000 meters
Megameter	1,000,000 meters
Gigameter	1,000,000,000 meters
Terameter	1,000,000,000,000 meters

1. How many meters are in 1 kilometer? _____
2. How many meters are in 17 hectometers? _____
3. How many centimeters are in 1 meter? _____
4. How many decimeters are in 54 meters? _____
5. How many micrometers are in 2 meters? _____
6. How many centimeters are in 1 kilometer? _____
7. How many millimeters are in 5 hectometers? _____
8. How many nanometers are in 1 meter? _____
9. How many decimeters are in 14 kilometers? _____

10. How many meters are in one terameter? _____

Labsheet 1.7 – Size of the Human Genome

Name: _____ Date: _____

Q. How big is the human genome?

“The human genome is made up of DNA, which has four different chemical building blocks. These are called bases and abbreviated A, T, C, and G. In the human genome, about 3 billion bases are arranged along the chromosomes in a particular order for each unique individual. To get an idea of the size of the human genome present in each of our cells, consider the following analogy: If the DNA sequence of the human genome were compiled in books, the equivalent of 200 volumes the size of a Manhattan telephone book (at 1000 pages each) would be needed to hold it all. It would take about 9.5 years to read out loud (without stopping) the 3 billion bases in a person's genome sequence. This is calculated on a reading rate of 10 bases per second, equaling 600 bases/minute, 36,000 bases/hour, 864,000 bases/day, 315,360,000 bases/year. Storing all this information is a great challenge to computer experts known as bioinformatics specialists. One million bases (called a megabase and abbreviated Mb) of DNA sequence data is roughly equivalent to 1 megabyte of computer data storage space. Since the human genome is 3 billion base pairs long, 3 gigabytes of computer data storage space are needed to store the entire genome. This includes nucleotide sequence data only and does not include data annotations and other information that can be associated with sequence data (2).”

1. Find two metric prefixes used in this reading and explain them as thoroughly as possible.
2. How many telephone book pages would it take to store the entire human genome? Write your answer in terms of pages, and in terms of pages and in kilo-pages.
3. How many hours would it take to read aloud a person's entire genome sequence?

4. Using scientific notation, approximately how many bases can be read per minute, per hour, per day, and per year, according to the information listed above?

Labsheet 1.8 – Applications

“How is DNA typing done?”

Only one-tenth of a single percent of DNA (about 3 million bases) differs from one person to the next. Scientists can use these variable regions to generate a DNA profile of an individual, using samples from blood, bone, hair, and other body tissues and products. In criminal cases, this generally involves obtaining samples from crime-scene evidence and a suspect, extracting the DNA, and analyzing it for the presence of a set of specific DNA regions (markers). Scientists find the markers in a DNA sample by designing small pieces of DNA (probes) that will each seek out and bind to a complementary DNA sequence in the sample. A series of probes bound to a DNA sample creates a distinctive pattern for an individual. Forensic scientists compare these DNA profiles to determine whether the suspect's sample matches the evidence sample. A marker by itself usually is not unique to an individual; if, however, two DNA samples are alike at four or five regions, odds are great that the samples are from the same person.

If the sample profiles don't match, the person did not contribute the DNA at the crime scene. If the patterns match, it means that the suspect may have contributed the evidence sample. While there is a chance that someone else has the same DNA profile for a particular probe set, the odds are exceedingly slim. The question is, How small do they have to be when conviction of the guilty or acquittal of the - innocent lies in the balance? Many judges consider this a matter for a jury to take into consideration along with other evidence in the case. Experts point out that using DNA forensic technology is far superior to eyewitness accounts, where the odds for correct identification are about 50:50 (2).”

1. What percent of DNA differs from person to person? Approximately, how many base pairs is that?
2. What percent of DNA is the same for each and every person? Approximately, how many base pairs is that?
3. How many regions should the DNA samples be alike in for the samples to match?

Labsheet 1.9 – Applications

Read the passage and answer the following questions on a separate sheet of paper using complete sentences.

“Super Bowl XXXIV footballs and 2000 Summer Olympic souvenirs

The NFL used DNA technology to tag all of the Super Bowl XXXIV balls, ensuring their authenticity for years to come and helping to combat the growing epidemic of sports memorabilia fraud. The footballs were marked with an invisible, yet permanent, strand of synthetic DNA. The DNA strand is unique and is verifiable any time in the future using a specially calibrated laser.

A section of human genetic code taken from several unnamed Australian athletes was added to ink used to mark all official goods — everything from caps to socks — from the 2000 Summer Olympic Games. The technology is used as a way to mark artwork or one-of-a-kind sports souvenirs (2).”

1. Do you think this is a good strategy to use to ensure the authenticity of sports memorabilia? Explain your reasoning.
2. Would this type of marking increase, or decrease the value of the items? Explain your reasoning.
3. Do you think this is a good use for DNA technology, or do you think it is a waste of time? Explain your reasoning.