Cosmetic Algebra and Geometric Stagecraft

Esther Liston

Overview

I teach eighth grade connected math in the vocal room in front of a piano. I return to my regular classroom in time for violin, viola, bass, and cello lessons. While I prepare materials for my sixth graders, there is a brass ensemble playing in my classroom. I teach at Rogers Creative and Performing Arts School. I left a job at a high school to teach in this middle school, because I fell in love with life back stage as I painted characters to life with a palate of Mehron and Bob Kelly make-up.

Math should captivate through applications interesting to us, rather than hold us captive by the limitations of a typical curriculum. Musical theater is so much more than the song, dance, and play. It is the production, direction, and planning for the financial demands of the next play. My kids are not normal, average kids, but our textbooks target the typical young teenager. I want to reach my students better by modifying the problems to touch them better. If I modify the problems my students encounter in sixth and eighth grade connected math and scholars Algebra I so that they are appropriate for students in CAPA, they will see why math is a subject worthy of their study.

Concepts

Students explore common factors and common multiples early in middle school. At the same time, they can look at distribution of costumes and props among a cast. They can also predict when two commercials may air simultaneously.

Students investigate the relationships of real numbers. They begin to write part to whole comparisons and link the piece of the whole to a part of a hundred. When they calculate percent of an audience, they can compare different groups especially when they are not audiences of exactly 100 people.

Students investigate exponential growth in eighth grade. By looking at how money can grow in a bank with interest, they can predict how costs might increase over time. By looking at costs associated with Broadway productions in the past, they can grow these costs into the present and compare them to 2003 theatrical productions. They can look at wages then and make a realistic comparison to wages for performers now.

Students play games of chance in order to investigate probability. Students who determine all possible outcomes for a game of chance can leap to determine all possible color outcomes for a set design. Similarly, students who determine whether a shape will tessellate as a kitchen tile may design a tessellation with lights on a set.

Students solve systems of equation by making charts and graphs and solving equations. These equations have meaning when they have to calculate how many performances occur before money is made. Students have to figure out how much money goes to costs, how much money comes in, and how much profit is left to pay the performers!

Objectives and related Content Standards

National Council of Teachers of Mathematics standard	Pennsylvania state standard	student classroom objective
NCTM 5 & 6 for grades 5 to 8	PA 2.1 numbers, number systems, and relationships types of numbers and equivalent forms (fractions, decimals, percents)	Students calculate least common multiple and greatest common factor for distribution of prizes among cast members or among an audience.
NCTM 7 for grades 5 to 8	PA 2.2 computation and estimation	Students calculate percent by comparing parts of an audience to a whole audience.
NCTM 13 for grades 5 to 8	PA 2.3 measurement and estimation	Students calculate area through auditorium seating.
NCTM 3 & 4	PA 2.4 mathematical reasoning and connections	Students calculate compound interest investigating compound interest by looking at production costs for Broadway shows.
NCTM 1 & 2	PA 2.5 mathematical problem solving and communication	Students solve problems involving splitting up an auditorium.
NCTM 10	PA 2.6 statistics and data anaylysis	The student predicts percent of population based on sample size.
NCTM 11	PA 2.7 probability and predictions	The student determines permutations and probability of event occurrence (2 possibilities for n events) by choosing from 2 film colors for each of 1, 2, 3, or n lights.
NCTM 9 for grades 5 to 8	PA 2.8 algebra and functions	Students determine linear growth for production costs and revenue in order to determine profit.
NCTM 9 for grades 9 to 12	PA 2.10 trigonometry	The student applies the Pythagorean theorem to staging.

Rationale "The pleasure we obtain from music comes from counting, but counting unconsciously. Music is nothing but unconscious arithmetic."Gottfried Wilhelm von Leibniz

"When children make connections between the real world and mathematical concepts, mathematics becomes relevant to them. As mathematics becomes relevant, students become more motivated to learn and more interested in the learning process." (Albert 526) I believe that my students will respond better to problems which involve staging and theater history rather than regular textbook problems. I currently teach three different math courses: connected math grade 6, connected math grade 8, and scholars Algebra I. I have just under 120 students from a variety of majors. Drama majors act on a stage. Stagecraft majors build. Costume majors sew and design costumes. Dance majors dance on the stage. Vocal majors sing on the stage. Media majors photograph and videotape and plan lighting for the stage. Instrumental majors play and are involved in the staging of shows. Art majors draw and paint sets. Creative writing majors write and have friends involved in staging the shows they may write! Our all school musical occurs at the end of March and truly involves everybody whether or not they actually appear on stage. Everybody in our school supports the theater and can relate to things involving musicals on stage.

I plan to meet my objectives through the modification of existing problems
and the addition of new problems (as described in the following table).

original existing problem	integrated theater problem
Prime Time page 41 # 15 The school cafeteria serves pizza every day and applesauce every eighth day. If pizza and applesauce are both on today's menu, how many days will it be before they are both on the menu again?	If a commercial for the CLO is on NBC every 84 minutes, and is on ABC every 108 minutes, then when does the commercial air on both NBC and ABC at the same time?
Shapes and Designs page 9 Investigation 1.1 The honeycomb demonstrates that regular hexagons fit together to cover, or tile, a surface. Are there other shapes that have this same property?	lighting design tessellations Which shapes can be used to create a pattern with light on the stage without gaps or overlaps?
Bits and Pieces 2 page 43 Investigation 4	area of seating Redraw the map of our auditorium so

Redraw the map of Tupelo township so that farmers can expand the land they own.	that some major groups get more seats.
Bits and Pieces 1 page 54 Investigation 5.1 Which basketball player makes the most free-throws? 17/25, 15/20, 7/10	percent of an audience If 25/32 people in an audience were thrilled, how many out of 100 would also enjoy the show?
original existing problem	integrated theater problem
Moving Straight Ahead page 24 Investigation 2.4 Determine how long a race should be for little brother Henri and big brother Emile if the younger one walks 1 m/s with a 45 m headstart and Henri walks 2.5 m/s.	linear revenues Determine when you will earn enough profit $2500 per show to cover your $45,000 up front cost and of $1,000 per show.
Shapes and Designs page 19 (for Investigation 2.1, 2.2) Determine what if any shape can be construced from side lengths of 5,5,3; 5,5,8,8; 5, 8 15; 5, 6, 10.	stagecraft Determine whether triangular or quadrilateral supports can be built from lumber of lengths 5,5,3; 5,5,8,8; 5, 8 15; 5, 6, 10.
Growing, Growing, Growing page 34 Investigation 3.2 Determine the value of Sam's coin $2,500 collection if its value increases 6% each year.	production cost comparisons If performers in a Broadway revue earned $250 a week in the 1920's, how does that salary compare with those in 2004?
What do You Expect? Page 7 Investigation 1.2 Determine how often you will match colors on a half-blue/half-yellow spinner.	light patterns using two color films Determine ways to replace two spotlights with a red or blue film.

Strategies

Students investigate distribution of prizes for audience members or costumes and props for an audience, calculating percent of an audience, and determining audience seating. Students will also compare productions costs from past decades to current costs for Broadway shows. Students will determine options for lighting by looking at patterns created by shapes as well as color. Students compare linear costs and solve systems involving mixing dyes for costumes or make-up. Students will look at problems in construction involving right triangle geometry. "The reform of mathematics education in the United States and Canada focuses on helping us move our mathematics programs toward a curriculum that - represents significant, powerful mathematics for all students; emphasizes topics that are relevant to students' present and future needs" (Lappan 133).

I can't sing and can't dance but have done it any way, and my biggest challenge is to convince people who can sing and can dance who "can't do math" to do it any way. "Our vision is a future where citizens do not live in fear of mathematics; where people use mathematics to solve problems in their daily lives as well as in their workplace, community, and society; and where mathematics will build on the technological advances we already enjoy. We need to prepare the students in today's classrooms to live and work in tomorrow's world" I teach in the *Fame* School, the one where all of the kids sparkle- "that's our job" (Burrill 189).

Classroom activities follow in lesson plan format.

© 2003 instructor Esther Liston Anticipated Date October, sixth grade
Subject 6th grade connected math topic: GCF, LCM for the number of costumes
Materials: chalk, paper, pencils
Expenditures for this and all of the following lessons include paper to make consumable packets and poster board to display example problems created by students (also consumable). Students need polystrips to build triangles, and we already have shape tiles at school.

standard PA 2.1 recognize situations in which problems can be solved by finding factors and multiples

my rationale for the problem In the book <u>Prime Time</u>, students review multiplication and division and see division through factors. The book stresses the ideas behind factor, divisor, multiple, product, primes.

Objectives Students calculate least common multiple and greatest common factor for distribution of prizes among cast members or among an audience.

Opening/motivation Many t.v. shows give away things to the audience.

Review prior knowledge/warm-up Remember how to make factor trees. How can you factor 12? How can you factor 75?

Reading Emphasis/ New Words greatest common factor and least common multiple

Guided Practice/ examples
I want to divide 12 hats and 75 costumes equally among as many actors as possible.
12 = 2•6 = 2•2•3
3 is the only (and the therefore the greatest) common factor
75=3•25 = 3•5•5

3 actors can equally share the hats and costumes.
3 goes into 12 four times, so each gets 4 hats.
3 goes into 75 twenty-five times, so each gets 25 costumes.

If every 12^{th} audience member gets a free Coke, and every 75^{th} audience member gets a free piece of pizza, then every $3 \bullet 4 \bullet 25 = 300^{th}$ audience member gets both prizes.

If 12/75 people in the audience are in middle-school, then 4/25 people are in middle-school.

I want to divide 84 props and 108 costumes equally among my acting class. How many students can equally share these things?

$84 = 12 \bullet 7 = 3 \bullet 4 \bullet 7 = 3 \bullet 2 \bullet 2 \bullet 7$

Notice $3 \bullet 2 \bullet 2 = 12$ is the Greatest Common Factor

$108 = 12 \bullet 9 = 2 \bullet 6 \bullet 3 \bullet 3 = 2 \bullet 3 \bullet 2 \bullet 3 \bullet 3$

12 students can share 84 hats if each gets 7.
12 students can share 108 costumes if each gets 9.

If a commercial for the CLO is on NBC every 84 minutes, and is on ABC every 108 minutes, then the commercial airs on both NBC and ABC every $12 \bullet 7 \bullet 9 = 756$ minutes. This is how many hours?
If 84/108 people who audition get in, then 7/9 people get in.

Closure Ask students to summarize how "matching" numbers can help find the GCF and LCM.
Independent practice/Homework
Students complete a page with similar problems. If student becomes stuck, she should do her examples again.

Assessment PA 2.1 Check homework for accuracy.
See Appendix 5 for stretching and shrinking postcards, also related to standard 2.1.

Student evaluation for this and all following lessons includes weekly work packets or assessment and questions on unit tests.

Anticipated Date Spring, 6^{th} grade
Subject 6^{th} grade connected math
topic: PA 2.3/PA 2.5 auditorium seating

Materials: chalk, paper, pencils, a worksheet with the irregular areas labeled and the equally divided map not labeled

standards: PA 2.1, 2.4, 2.5 look for patterns and describe how to continue the
PA 2.1, PA 2.4 use physical models and drawings to help reason about a situation
PA 2.5 invent, select, and justify the appropriate methods, materials, and strategies to solve problems
PA 2.4 Make conjectures based on logical reasoning.

My rationale for the problem

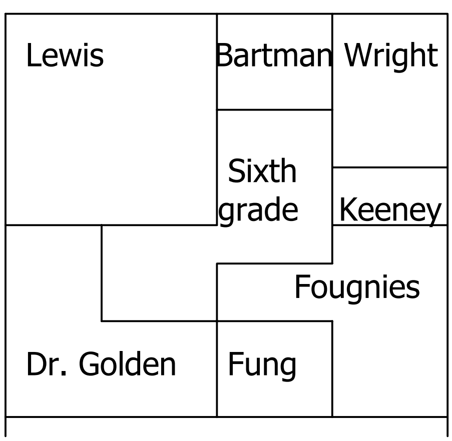
Students in our school sit with their majors during assemblies in our school auditorium. Bits and Pieces II exposes sixth grade students to fraction addition, subtraction, multiplication, and division. Students complete Investigation 4 in order to recognize fractional parts of 4, 16, and 32.

Objective: The student will determine what fraction of the auditorium belongs to each group by dividing an irregular map equally.

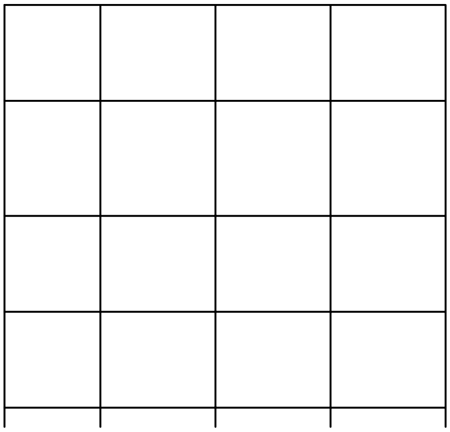
Opening/motivation
Have the irregular map on the chalk board.

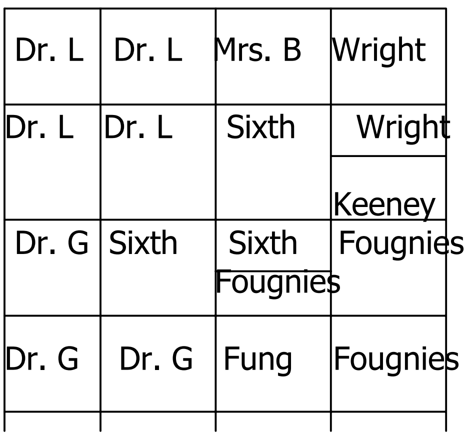
Review prior knowledge/warm-up
Ask students to list equivalent fractions for 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8.

Guided Practice/ example



Relabel each section with the corresponding name.





See APPENDIX 1 for the rest of the continuation of this problem.

Closure Ask students to label the map drawn on the chalkboard.
Independent practice/Homework
Ask students to complete the fraction section of the sheet.

Assessment PA 2.3/PA 2.5 Check student work for accuracy.

Materials: chalk, paper, pencils, copies of a chart with appropriate headings

standard GGG/KF PA 2.1 PPS 8-1 simplify numerical expressions involving exponents, scientific notation, and using order of operations
GGG/KF PA 2.1 PPS8-3 Simplify and expand algebraic expressions using exponential forms

my rationale for the problem
A theme in the growing, growing, growing book is exponential growth. Although the value of a baseball card collection is explored, it is more interesting to explore how the costs of productions on Broadway today can be compared to earlier productions. Students can use the compound interest formula to make predictions.

Objective: Students calculate compound interest investigating compound interest by looking at production costs for Broadway shows (PA 2.4).

Opening/motivation: The following table organizes calculations to figure out what production costs or salaries would be after 70 or more years given 6% rate of inflation.
What might gross 2 billion dollars today to compare to Christy Minstrels?
Who makes $9,000 a week in the theater? Who makes $26,000 on Broadway?
How much did Sebastian Bach get for his performance in *Jekyll and Hyde* in 2000?
What costs $3 million or $23 million to produce? These are the questions we get from our answers.

Review prior knowledge/warm-up
Students read the short story "John Jones' Dollar" (appearing in *Fantasia Mathematica* edited by Clifton Fadiman) before beginning *Growing, Growing, Growing* so that they see how $1.00 grows enormously even at 3% interest if compounded many, many times. Review what $(1.03)^{80}$ means and what $(1.06)^{80}$ means today.

Reading Emphasis/ New Words exponent is the "teeny-tiny" number that "floats"

Guided Practice/ example

kisl an ref.	P-principal, price, population	% Percent increase or decrease	N = time Year- start year	T = x(1+p)^n P(1$\pm$ %)^n= (1$\pm$%)^n P	total (total interest or loss)
	$1.00	3% = .03	1	$(1.03)^1$•$1 1.03• $1	1.03
20	Christy Minstrels grossed the sum of $317, 598	6% inflation?	2003-1850 153	$(1.06)^{153}$•$317,598	$2,364,150,633
	$1500 to appease the offended author $24,000 $35,000				
72	$90 a week salary inferior	6% inflation?	2005-1925 80 years	$(1.06)^{80}$•$90	$9,521
72	$250 a week salary in a Broadway revue	6% inflation?	2005-1925 80 years	$(1.06)^{80}$•$250	$26,449.00
84	$13,000 for first edition Follies production in 1907	6%?	2004-1907 97 years	$(1.06)^{97}$•$13,00 0 284.88	$3,703,499.437
	$289,000 Ziegfield production costs 1927	6%?	2004-1927 77 years	$(1.06)^{77}$•$289,0 00 81.62	$23,588,180
87	show cost $4000 or $5000				

Closure

Write a summary of how much a production originally cost, how 6% is relevant, and then how you make a prediction of future costs.

Independent practice/Homework Complete a similar grid at home.

Assessment PA 2.3/PA 2.5 Check to see that grid is completed. Read their summaries.

© 2003 instructor Esther Liston Anticipated Date September review, 8^{th} grade

Subject 8^{th} grade connected math topic: percentage of audience and polling to reteach long division

Materials: chalk, paper, pencils

standard PA2.2 move flexibly between fraction, decimal, and percent representations

my rationale for the problem

Sixth graders first see percent in Bits and Pieces I and have a chance to explore it again in *Bits and Pieces II*. Eighth grade students need to review percent. Context counts.

Objective: Students calculate percent by comparing parts of an audience to a whole audience.

Opening/motivation For example, if 25/32 audience members were thrilled with the show. What percent of 32 is 25? I don't want to say that 25 out of 32 people liked my show. I want to use a percent, especially if the percent is bigger than 25%!

Review prior knowledge/warm-up Remember that

$\frac{625}{8}$ means $8\overline{)625}$.

Reading Emphasis/ New Words per cent means per 100 After all, a cent is 1 out of a 100 pennies to make a dollar, and a century is 100 years.

Guided Practice/ example 25/32 = P/100 25 and 32 are relatively prime.

I cross multiply. 25•100 = 32•P

2,500 = 32P

so $\frac{2,500}{32} = \frac{32P}{32}$ $\frac{2500}{32} = \frac{2 \bullet 1250}{2 \bullet 16} = \frac{2 \bullet 2 \bullet 625}{2 \bullet 2 \bullet 8} = \frac{2 \bullet 2 \bullet 25 \bullet 25}{2 \bullet 2 \bullet 8} = \frac{625}{8}$

Since I now have a single-digit number, I can use long division.

Long division is great because it requires estimation, multiplication, subtraction, estimation, multiplication, subtraction until the division is done or you want it to be!

See appendix 2 for complete notes.

Closure Why is 78.125% the answer?

Independent practice/Homework

Ask students to write a similar problem and solve it. Ask them to complete a sheet with several long division problems.

Assessment 2.6 Check homework for accuracy.

 February, 8th grade

Subject 6th grade connected math topic: What Do You Expect? Pascal's triangle

Materials: chalk, paper, pencils, blank Pascal's triangle sheet (available at The Math Forum http://mathforum.org/workshops/usi/pascal/images/midd.comb1.gif)
See also Pascal's Triangle by Thomas M. Green and Charles L. Hamburg (Palo Alto: Dale Seymour Publications, 1986)
As students begin algebra and think of how to build circuits to support lights, an additional resource is "Electricians Need Algebra, Too" by Richard Hill in the September, 2002 edition of Mathematics Teacher pages 450 to 455.

standards: PA 2.8 Discover, describe and generalize patterns, including linear, exponential, and simple quadratic relationships.
PA 2.7 Determine the number of combinations or permutations for an event.
PA 2.7 Present the results of an argument using visual representations.
PA 2.7 Compare and contrast results from observations and mathematical models.
PA 2.7 Make valid inferences, predictions and arguments based on probability.

my rationale for the problem Although Pascal's triangle is not part of the curriculum, it should be. It fits well with 6th grade *How Likely Is It*?, 8th grade *What Do You Expect*? and Say it with Symbols, and the college prep mathematics introductory and Estimating Fish units.

Objectives The student determines permutations and probability of event occurrence (2 possibilities for n events) by choosing from 2 film colors for each of 1, 2, 3, or n lights.

Opening/motivation As an introduction to "Pascal's combinations," students should watch how the lighting changes from blue skies to reddish purple in the dream ballet sequence of *Oklahoma.*
If you have two spotlights and want to replace their films with red or blue, you have to determine the options. Both can be red. Both can be blue. Lastly, one can be blue and one can be red, but this can be done two ways: the one on the left can be red and the one on the right can be blue, or the one on the left can be blue, and the one on the right can be red.

Review prior knowledge/warm-up
Ask students to complete a blank Pascal's triangle with only a few hints.

It is also worth mentioning that this application for lighting design is also an application for geneticists as they study genetic outcomes including boy/girl combinations for people who have two children, positive/negative outcomes for two genes, etc. Similarly, lights or circuits could be on or off.

Reading Emphasis/ New Words

What is this called?

1

1 1

1 2 1

A. sum triangle B. Babbage's triangle
C. Pascal's triangle D. Fibonacci's triangle

Guided Practice/ example

Pascal's triangle provides an organized way of listing possibilities.
1B, 1R are the possible choices if one spotlight is out.
$1B^2$ (2 blues: BB), $2B^1 R^1$ (2 ways of getting 1 blue, 1 red: BR, RB), $1R^2$ (2 reds: RR) for 2 spotlights The sum of the exponents must be the same as the number of spotlights you wish to replace.
$1B^3$ (1 way for 3 blues: BBB), $3B^2 R^1$ (3 ways of getting 2 blues, 1 red: BBR, BRB, RBB), $3B^1 R^2$ (3 ways of getting 1 blues, 2 reds: BRR, RBR, RRB), $1R^3$ (1 way of getting 3 reds:RRR) Here the sum of the exponents, 3, 2+1, 1+2, 3, is three, for 3 spotlights. It is easier to list groupings of 1, 3, 3, and 1 rather than list all 8.

Similarly, the music of songs is frequently arranged in 4's. For example, ABAB stanzas A and A rhyme, and B and B rhyme. What are all of the rhyming patterns we can create for 4 stanzas with a choice of A or B rhymes?

Closure Review what the exponents mean.
Independent practice/Homework
Ask students to complete a blank Pascal's triangle and determine all of the boy-girl combinations a family with three children could have.
Assessment PA 2.7 Ask a three outcome question on their next quiz.
Check off all classwork in weekly packet.

 Anticipated Date
Subject 8th grade connected math/ Algebra topic: PA 2.8 systems
(Moving Straight Ahead beginning of the year/ Say it With Symbols end of the year)

Materials: chalk, paper, pencils

related standards PA 2.1 use the inverse relationships between operations to find missing values in equations
PA 2.8 create and interpret expressions, equations, or inequalities that model problem situations.
PA 2.8 Select and use a strategy to solve an equation or inequality, explain the solution, and check the solution for accuracy.
PA 2.8 Graph a linear function for a rule or table.
PA 2.8 Generate a table or graph from a function and use graphing calculators and computer spreadsheets to graph and analyze functions.
PA 2.11 Describe the concept of unit rate, ratio and slope in the context of rate of change.

Objectives
The student will determine how many performances with a given profit must be done in order to cover given costs through solving a system of linear equations.

Opening/motivation
In order to make money, producers must cover all expenses and costs.
For example, for production of a new show, there are fixed costs for advertising, and there are costs per performance (for electricity, heating, performer wages per performance). There are also per performance revenue. From this information, producers can calculate the number of performances needed before making money.

Review prior knowledge/warm-up
Remember Henri and Emile who raced each other in the beginning of the year.
Recall how to solve equations using cups and tiles.

Reading Emphasis/ New Words
break even problem
For example (Problem 4.2 in *Say it With Symbols*)

Guided Practice/ example
Let' s say that we have set costs of $100,000 for advertising, insurance, rent and a cost of $4,000 per show for (electricity, wages, heat). We also have a set revenue of $25,000 from the Hillman Foundation (or Heinz, or Grable Foundation, or

other arts supporter). We know that through ticket sales, we take in $7,000 per show.

$100 + 4x = 25 + 7x$

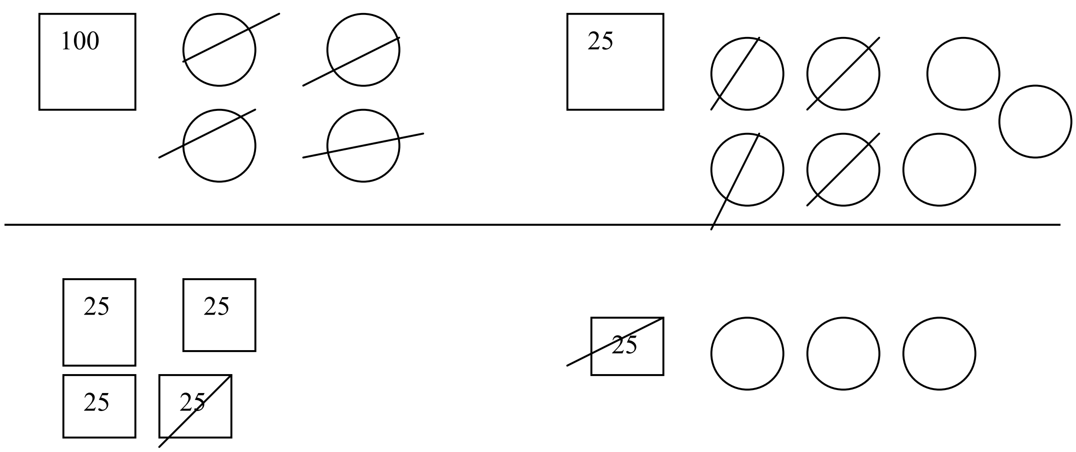
In other words, we have to come up with another $75,000 toward fixed costs, but we actually make $3,000 in profit per show. This algebraically is stated this way:

$100 + 4x =$	$25 + 7x$	
-25	-25	
$75 + 4x =$		$7x$
$-4x$		$-4x$
	$75 =$	$3x$
$75/3 =$	$25 =$	$1x$

We see that we must have 25 performances to come up with $75,000.

$75/3 = 25 = 1x$

Review "cups and tiles" otherwise known as Solving Linear Equations page 54



25 in each cup!

Closure

Independent practice/Homework

Rewrite today's example neatly and then solve a similar problem. Write down your homework in your assignment book.

See if you can make up your own problem and solve it.

Assessment Ask students to solve a similar problem as a graded warm-up the following day. Review rubric used on year-end problem.

 Anticipated Date Fall and Spring 8th grade
Subject 6th grade connected math topic: PA 2.8 linear systems

Materials: chalk, paper, pencils

standard: PA 2.8 E (advanced) define variables, write and solve simultaneous equations graphically or algebraically

Objectives PA 2.8 Given 800g of a dye solution that is 20% strength, the student will calculate how much must be added to make it a 50% strength (PA 2.5/2.8).

Opening/motivation
In order to dye the Sugar Plum Fairy's skirt the proper shade of purple, I need to create 800 g of dye that is 50% strength. I need to use leftover dye that is 20% strength and add some that is full (100% strength).
You might want to model this with different colored base ten blocks in transparent containers.

Review prior knowledge/warm-up
Ask how many students have applied pancake base.
Remember that Ben Nye (the make-up guy), Mehron, Bob Kelly, and other manufacturers of pancake base make-up for the stage mix different colors by using different amounts of pigment in their solution.

Remember to define variables, and recall that you can solve this problem with a "guess and check" table. L = leftover F = full

Reading Emphasis/ New Words
A mixture problem refers to mixing different solutions, usually defined by their percent component.

Guided Practice/ example See appendix 3 for details of the problem.
$F = 800 - L$ and $.2L + 1F = 400$
$F = 300$ (Notice again that 8 goes into 24 3 times).
Once again, we need 800 total, so we need 500 g of the Leftover dye.

Closure Ask students to summarize the steps of mixture problems. They should define their variables, write 2 equations for two variables, and then decide how to solve. They can substitute an expression in for one variable, and then substitute their solution for the other one.

Independent practice/Homework Recopy this example neatly into a "toolkit." Solve three similar problems. Write your own mixture problem.

A**ssessment** PA 2.3/PA 2.5 Check solutions to homework problems. Now is also a good time to review rubric for year-end exam.

 Anticipated Date January, 6th grade

Subject 6th grade connected math topic: PA 2.9 tessellating light designs

Materials: chalk, paper, pencils

For example, a writer may face the question, "Will a hexagon create a good pattern from lights above to a floor below?"

(Hexagon D from Connected Math Shapes set may be used.)

standard PA 2.9 PPS SD/CS 6-2 identify, name, draw, and list all properties of squares, cubes, pyramids, parallelograms, quadrilaterals, trapezoids, polygons, and rectangles.

PA 2.9 PPS SD/CS 6-4 analyze objects to determine whether they illustrate tessellations, symmetry, congruence, similarity, and scale

my rationale for the problem

Lighting sometimes creates a pattern on the stage floor. In *Shapes and Designs*, sixth graders determine which shapes tessellate, or tile an entire region without any gaps or overlaps.

David Hairston, a teacher in the Pittsburgh Public Schools, created the SPEAK UP method for writing mathematical essays. The SPEAR method is a variation of this method. This helps students understand, "So math isn't just answers; math is everything you do to get the answers, too" (Brown-Herbst 455)

Students are encouraged to write explanatory essays using the SPEAR method.

S represents "state an introduction." Students restate the question as a statement.

P represents "provide a picture."

E represents "explain your work."

A represents "add in the calculations."

R represents "respond to the question" with your result and a reason you know you are right.

Objective: The student will determine whether a given polygon tessellates.

Opening/motivation: Ask if any of the students have seen Fosse. If someone has, ask if she noticed the pattern of lights displayed on the floor of the stage. When I sat up high in the balcony, these patterns amazed me.

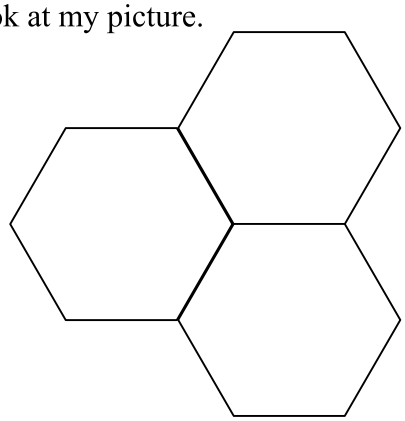
Review prior knowledge/warm-up: Remember that 1 triangle has 3 sides, a quadrilateral can be formed by 2 triangles with four sides, a pentagon can be formed by 3 triangles and 5 sides. Ask the students to explain that a shape with any amount of sides can be made by 2 fewer triangles than it has sides.

Reading Emphasis/ New Words
A tessellation is a covering of one or more than one shapes that does not have any gaps or overlaps.

Guided Practice/ example

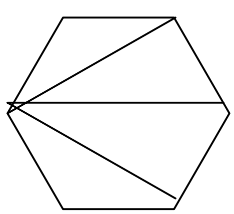
S I need to find out whether shape D, a hexagon, can be used as a tile so that there are no gaps or overlaps.

P Look at my picture.



E Three hexagons touch at once.

A I know that a hexagon is made from four triangles = 4 × 180° = 720°.



6 angles share 720°, so each angle is

120°
6 ⟌ 720°
6↓
12
12
00

120° goes into 360° 3 times as I can see from my picture.

Closure

R My result is that hexagons do tessellate which makes sense, since honeybees use this shape in their honeycombs. This means that I can use a hexagon to create a pattern of lights on a stage floor.

Independent practice/Homework

Assign the SPEAR essay for another shape.
Ask students to summarize what they did and write their homework in their assignment book.

Assessment PA 2.3/PA 2.5 Check to make sure that assigned essay is complete and accurate.

Anticipated Date
Subject 6th grade connected math
topic: triangle inequality

Materials: chalk, paper, pencils
standard PA 2.9Cp identify the set of three line segments that could form a triangle

my rationale for the problem

In order to determine whether are not leftover scraps of wood can be useful, the student can apply the triangle inequality. The triangle inequality can be modeled with scraps of measured paper. Although students first see this idea in late fall of sixth grade, the force of the triangle inequality drives the introduction to high school geometry.

Objectives The student will determine what, if any, shapes can be formed from given lengths.

Opening/motivation

State an introduction for your problem
Your friend who is a stagecraft major wants to know if the following lengths of lumber can be made into triangular supports or platforms for a set.
Can the following lengths make a triangle or quadrilateral? If so, what kind?

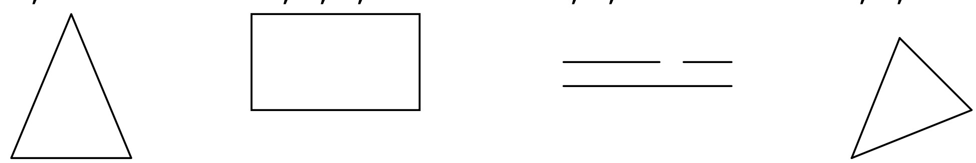
Review prior knowledge/warm-up

Review the different shapes that might be made with 3 or 4 sides. Check to make sure that square, rectangle, equilateral triangle, isosceles triangle, scalene triangle, trapezoid, parallelogram, just a quadrilateral, and nothing are included.

Guided Practice/ example

Provide a ***p***icture.

5, 5, 3   5, 5, 8, 8   5, 8, 15   5, 6, 10



You have to ***E***xplain what you calculated to your buddy.
5,5, and 3 are three lengths, so I can make a triangle as long as 5+3 is bigger than the third side = 5. 8 is bigger than 5, so I have a triangle. Since 5 and 5 are the same lengths, it is an isosceles triangle.

5,5,8, and 8 are four lengths. If the three smallest sides have a sum larger than the biggest side, the result is a quadrilateral. Since there are two pairs of equal lengths, the quadrilateral can be either a rectangle or a parallelogram. If the 5 and 8 meet at right angles, it is a rectangle. Otherwise, the sides are tilted and become a p-gram.

5, 8, and 15 do not meet. The 5 and 8 fall flat with a two inch gap. 5 + 8 = 13 which is not bigger than 15, so these are scraps.

5, 6, and 10 are different lengths. 5 + 6 is 11 which is bigger than 10, so these lengths form a scalene triangle.

The ***R***esult is my friend has 2 usable triangles, and one usable quadrilateral.

Closure
I ***R***eflect on my work. As long as the shorter side lengths add up to be bigger than the longest side, we can make usable supports and platforms.
Independent practice/Homework
Ask students to write a similar essay for other given lengths.

Assessment PA 2.9 Check homework essays for accuracy.

 Anticipated Date October, 8^{th} grade
Subject 8^{th} grade connected math/Algebra topic: PA 2.10 Pythagorean theorem

Materials: chalk, paper, pencils

standards: PA 2.2 Estimate the value of irrational numbers
PA 2.4 construct, use, and explain algorithmic procedures for computing and estimating with whole numbers, fractions, decimals, and integers
PA 2.10 compute measures of sides and angles using proportions, the Pythagorean theorem, and right triangle relationships

my rationale for the problem
Students can predict how long something must be or how high something might reach using right triangle relationships. If the student can master the relationships created by $a^2 + b^2 = c^2$, she can enter high school geometry and the S.A.T.'s feeling powerful. The problems listed below are based on the College Preparatory Mathematics unit "Birthday Party Pinata." The numbers used in the problems are the same as the book, but I shifted the context so that students only have to imagine a set.

Objective: The student applies the Pythagorean theorem to staging (PA 2.10).

Opening/motivation Ask the students to recall the ramps Mr. Shcheuring and the stagecraft majors built as part of the set for *The Wiz*.
The Pythagorean theorem is used in construction to figure out how long a ramp should be, how high a ladder should be, or how long a base should be.

Review prior knowledge/warm-up
Review squares and that the side of a square is the "square root".

Reading Emphasis/ New Words root, rational, Pythagorean, hypotenuse, leg, right angle

Guided Practice/ example See appendix 4 for example problems.
If a ramp is 5 feet and comes 3 feet out, then it will reach ***a*** feet up the wall.
$a^2 + 3^2 = 5^2$ or $a^2 + 9 = 25$ or $a^2 = 16$, so $a = 4$.

Closure Ask students to summarize how the Pythagorean theorem works.
Independent practice/Homework Assign the problems from the book or adapted worksheet (see appendix).
Assessment PA 2.10 Give students a short quiz with 2 problems, one where c is missing and one where a or b is missing.

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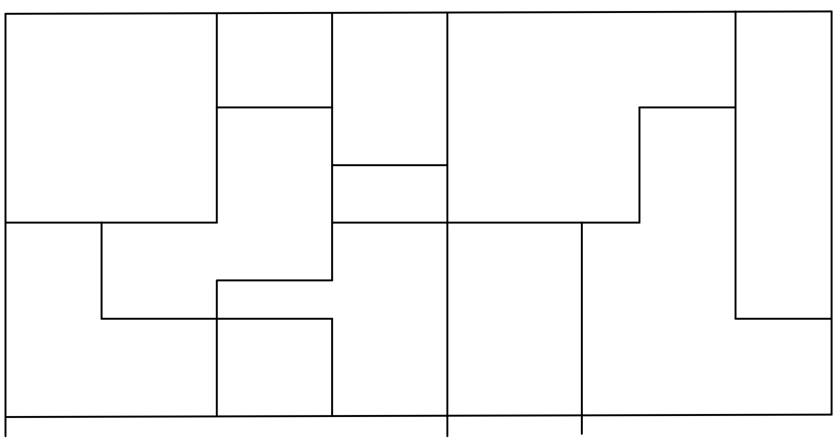
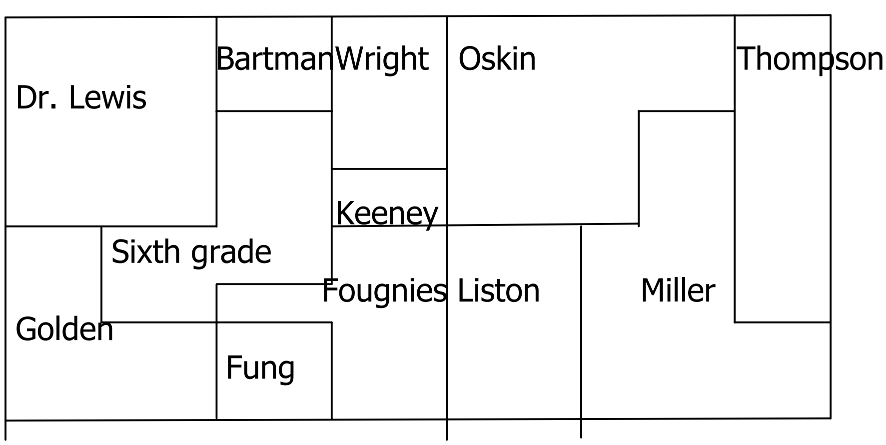
Appendix 1
Dr. Lewis has 4/16 or 8/32 or ¼ of the seats.
Dr. G has 3/16 or 6/32.
Mr. Fung has 1/16 or 2/32.
Ms. Fougnies has 2.5/16 or 5/32.
Ms. Keeney has ½ /16 or 1/32.
Ms. Wright has 1 ½ /16 or 3 / 32.
Sixth grade has 2 ½ /16 or 5/32.
Mrs. Bartman has 1/16 or 2/32.

8 + 6 + 2 + 5 + 1 + 3 + 5 + 2
14 + 7 + 4 + 7
21 + 11 = 32 spots.
Relabel each section with the corresponding name.

empty grids



labelled map



Appendix 1 continued
answer key

Dr. L	Dr. L	Mrs. B	Wright	Oskin	Oskin		Oskin	Thompson
Dr. L	Dr. L	6	Wright Keeney	Oskin	Oskin		Miller	Thompson
Golden	6	6 Fougnies	Fougnies	Liston	List	Mil	Miller	Thompson
Golden	Golden	Fung	Fougnies	Liston	List	Mil	Miller	Miller

Dr. L	Dr. L	Mrs. B	Wright
Dr. L	Dr. L	Sixth	Wright Keeney
Dr. G	Sixth	Sixth Fougnies	Fougnies
Dr. G	Dr. G	Fung	Fougnies

Appendix 2

```
           07
8        )625   8 does not go into 6
         - 56↓  because 7•8 is 56
            65     8 goes into 65?

           07 8
8        )625
         - 56↓  because 7•8 is 56
            65  8 goes into 65?
```

-64 because 8•8 is 64
1

07 8.1
8)625 .0
- 56↓ because 7•8 is 56
65 8 goes into 65?
-64 because 8•8 is 64
1 0 8 goes into 10 once I can bring down as many 0's as I want.
- 8
2

07 8.12
8)625 .00
- 56↓ because 7•8 is 56
65 8 goes into 65?
-64 because 8•8 is 64
1 0 8 goes into 10 once
- 8 8 times 1 is 8
2 0 8 goes into 20 twice
- 1 6 8 times 2 is 16
4

07 8.125
8)625 .000
- 56↓ because 7•8 is 56
66 8 goes into 65?
-64 because 8•8 is 64
1 0 8 goes into 10 once
- 8
2 0 8 goes into 20 twice
- 1 6 8 times 2 is 16
40 I bring down another 0, an 8 goes into 40 5 times.

78.125% is the answer.

Appendix 3

Together I want 800 g, so L + F = 800 g.
I see that if I add 100g of leftover, I still need 700 full strength.
If I use 200 g, we still need 600g of full strength.
If I use 300 g, then we still need 500 g of full strength, so another way of writing the expression L + F = 800 g is F = 800 – 300 g.
We also know that each dye has a value, and that our goal is to make 50% of 800g be dye.
.2L + 1F = .5(800)   or .2 L + 1 F = 400g (or F = 400 – .2L)
We now have a system of equations:
F = 800 – L and .2 L + 1F = 400

We can substitute the first equation into the second equation.
.2 L + 1(800 – L) = 400

.2 L + 800 – 1 L = 400
combine like terms
-.8L + 800 = 400
isolate the variable through addition property
-.8L + 800 = 400
-800 -800
-.8L + 0 = -400
isolate the variable through multiplication property

$$\frac{-.8L}{-.8} + 0 = \frac{-400}{-.8}$$

Remind students, how many times does 8 go into 40?

Appendix 3 continued

L = 500 Substitute this solution back into the first equation which tells us we need 800 g total, so we need 300g of Full strength.

Students may graph the equations F = 400 – .2L and F = 800 – L on a calculator to see where they intersect.

What if we had used the equation solved for L = 800 – F.
Plug this equation into .2L + F = 400

$.2(800 - F) + F = 400$

$160 - .2F + F = 400$ combine like terms

$160 + .8F = 400$ now isolate F through addition property

$\underline{-160 \quad\quad -160}$

$0 + \frac{.8F}{.8} = \frac{240}{.8}$ isolate F through multiplication property

F = 300 (Notice again that 8 goes into 24 3 times).
Once again, we need 800 total, so we need 500 g of the Leftover dye.

Early in the year, students are encouraged to make a list and check table.
All of the possibilities by 100's are below. This is a good way of checking.

leftover 20% L	full strength F	20% of L	100% of F	.2L + F check the sum
100	700	20	700	720 way high
200	600	40	600	640 high
300	500	60	500	560 high
400	400	80	400	480 closer
500	300	100	300	400 exact
600	200	120	200	320 too low
700	100	140	100	240 way low

Appendix 4

If I have 1-foot, 2-feet, 3-feet, 4-feet, and 5-feet lengths of wood to build props, I first need to figure out the combinations of wood lengths I can use.
I can have 1' with 1', 1' with 2', 1' with 3', 1' with 4', or 1' with 5'.
Similarly, I can have 2' with 1', 2' with 2', 2' with 3', 2' with 4', or 2' with 5'.
3' with 1', 3' with 2', 3' with 3', 3' with 4', or 3' with 5'.
4' with 1', 4' with 2', 4' with 3', 4' with 4', or 4' with 5'.
5' with 1', 5' with 2', 5' with 3', 5' with 4', or 5' with 5'.

In BP-2, I introduce the right triangle with a story that someone walks 3 miles south and four miles east or 4 miles west and 3 miles north. Similarly, I build a ramp 3 feet high and 4 feet out, then the ramp itself is 5 feet. Then I have students give a story out loud for the remaining pictures with given lengths.

In BP 4, I ask students to state that they have to find the length of the ramp, because they know how far out the base is an how high it reaches. In c, they know how long the ramp is, how high it reaches, but they have to find out how far the base extends. In b, students state they need to find how high the ramp will reach if it extends 24 feet from the base and is itself 25 feet long.

In BP 13, students look at line segments on graphs. They determine how far up the line segment rises and how far out it runs.
BP 1, 2, 3, 4, 11, 12, 32
13

24 A similar "Romeo and Julie" problem appears in the Core Plus curriculum. Part of your set is a beautiful castle, 50 feet high, and yea, you actually built a mote with real water 10 feet across. The prince is to rescue the princess from her castle by tossing her a rope. How long must the rope be in order for the actress to catch the rope without falling from the set?

26 A ten foot ladder is leaning against the set. The foot of the ladder is 3.5 feet away from the base of the wall.
Draw a diagram showing this.
How high on the wall does the ladder touch?
Write an equation and solve it.

27 Part of your set included a pole anchored 5 feet below the stage surface. A stagehand carelessly backed into the pole, so that the pole cracked 7 feet up from the floor. It hit the stage 24 feet from the base. Draw a diagram. How long should the replacement pole be so that it can be anchored?

You are in Los Angeles and have to order the replacement, but they only told you it cracked 7 feet up and hit 24 feet out.
Write an equation. Solve it.
Show all of your thinking.

32 You need a brace for a set whose wall is 8 feet high and has a bottom anchor extending 9 feet behind it. Think of a Christmas tree eight feet high on the set of the Nutcracker with an anchor laying 9 feet behind it.

You have a 21 m long piece of lumber, and it must go up 18 m. What is the longest its base can extend out?
Scaffolding on my set is created by a series of rectangles with a support on the diagonal. This support piece is 7 feet long and the length is 4 feet. What is the width?

If there is time, students will explore30-60-90 degree triangles. Given lights tilted back at a 30° angle and their length, the students calculate how long their base and height must be (PA 2.10).

Appendix 5

© 2003 instructor Esther Liston Anticipated Date Sept. review, 8th grade
Subject 6th grade connected math topic: postcard stretching

Materials: chalk, paper, pencils

Objectives If given the dimensions of a postcard advertisement and the width of a billboard, the students calculate what length the billboard must be.

Opening/motivation
Students compete to have the best poster design for the Nutcracker. They submit entries on an 8 ½" by 11" piece of paper. If the poster is 17" wide, how high should it be to easily transfer the design?

Review prior knowledge/warm-up
Remember the stretching and shrinking book and the comparing and scaling book from 7th grade. Remember to compare things that are the same- small side to small side, longer side to longer side.

Reading Emphasis/ New Words
The ratio of similitude tells us how much we multiply by to get from one numerator to the next numerator, etc.

Guided Practice/ example

8 ½	width	17"
11	length	x"

Since	8 ½ * 2 =	17"
we figure	11 * 2 =	22"

Closure Ask students to summarize that we do not use addition in these problems.
Independent practice/Homework
Ask students to complete similar problems for homework.

Assessment Check homework for accuracy.