

**An Analysis of the Development of Data and Chance in Everyday Mathematics
Plus An Enrichment and Acceleration Probability Unit**

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Overview

Over a period of years, the National Council of Teachers of Mathematics has developed and revised content standards. New textbooks and programs have been developed to reflect those standards. More generally, there has been a move within education to develop standards based systems. The hallmarks of standards based systems are content and performance based standards along with assessments aligned to the standards. The impetus for the development of standards based systems was the poor showing of American students on the international TIMSS tests and a subsequent analysis of texts and teaching methods across high and low scoring countries.

While there is great interest in and commitment to developing standards based systems, in actuality, there are many impediments. Organizations such as the NCTM will develop a set of standards, but individual states will then develop similar but not identical standards. Tests that students are required to take also vary from national norm referenced tests such as the Iowa Test of Basic Skills, to state mandated tests such as PSSA in Pennsylvania, to tests such as the New Standards Reference Test that are related to standards developed by the NCTM. The state tests are generally not related to particular programs or texts, and the New Standards Reference Tests require application and problem solving, but similar types of test questions are not included in texts such as *Everyday Mathematics* that are considered standards based. In other parts of the world there is much more likely to be integration of standards, texts, and tests since such systems are developed by a country wide agency that develops programs and materials for the entire country. The standards and materials and tests are then actually used by the entire country. It is interesting to note that, in the United States, economists have argued for competition in education spawning the charter school movement, which results in even more decentralization. Yet, countries in Europe and Asia that have high levels of achievement also have highly centralized educational systems.

Ideally, decisions about education should be based on research, but the variables affecting educational achievement are so complex that it is often hard to sort out the important variables. For example, is the success of Japanese students due to texts, methods, societal values, cram schools, or interactions among these variables? How can experiments be designed to hold these various variables constant?

The hope was that with the development of the NCTM Standards, and standard based curriculums, mathematics achievement would rise in the United States. The new curriculums emphasize problem solving, reasoning, and connections between topics. In addition topics such as geometry, data and chance, and algebraic reasoning are included in early grades. There was little inclusion of those topics at the elementary level in traditional texts.

In this paper, I will examine the development of the content areas of data and chance (K-5) in the standards based curriculum know as, *Everyday Mathematics*. The purpose of this paper will be to examine these topics as they are developed K-5. I will, first, examine the NCTM standards for these content areas. I will, then, show how these standards are developed grade by grade by the authors of *Everyday Mathematics*. Finally, I will present an enrichment and acceleration unit for probability that can be used in conjunction with *Everyday Mathematics* in a pullout program for the Gifted and Talented.

Rationale

Everyday Mathematics is a standards based curriculum developed, by a group at the University of Chicago, with backing from the National Science Foundation. *Everyday Mathematics* is a comprehensive mathematics program for grades K-6. It was designed to make mathematics meaningful, explore important mathematical ideas, and relate mathematics to everyday applications. Problems are to be solved in context. Students use manipulatives and teachers are expected to foster children's mathematical thinking. In addressing fifth grade students, the authors say, "The authors of *Everyday Mathematics* hope that you will appreciate the real beauty and widespread usefulness of mathematics, and that you will have fun in the process. But most important, the authors want you to become better and better at using mathematics, so that you may better understand the world you live in."

Everyday Mathematics has been a somewhat controversial program. Some educators and parents believe that *Everyday Mathematics* does not provide enough practice in basic skills and confuses students with multiple explanations for basic computation skills. Researchers, on the other hand, point to increased achievement when compared with traditional programs (Fuson 2000; Noyce and Riordan 2001; Carroll and Fuson 2000). Most importantly, the increased achievement has been across socioeconomic groups. (Briars and Resnick, Fuson 2000)

Many teachers have been resistant to implementing *Everyday Mathematics* because of belief, or because their own mathematical backgrounds are insufficient. Parents of high achieving children sometimes complain that there isn't enough opportunity for enrichment and acceleration.

The spiral design of the curriculum, in *Everyday Mathematics*, is considered to be a strength of the program by the developers and a weak point by the critics. The definition of spiral needs to be clarified. Researchers, who in light of TIMSS test results, studied curriculum and textbooks across countries, criticized American texts for spiraling. That definition of spiraling means that the same content is repeated year after year without any additional depth. In *Everyday Mathematics* topics are repeated within grade levels and across grade levels, but with increasing depth and sophistication. In the preface to the *Kindergarten Program Guide and Activity Booklet*, the authors give a rationale for the spiral design: “People rarely learn something the first time...each strand is designed as a spiral that begins at a simple level and then grows with each child’s experience...The spiral design ensures that an activity is not finished the first time it is encountered, but recurs periodically for further consideration and exploration.” (4)

Fuson (2000) shows positive results with *Everyday Mathematics* in comparison to traditional American students, but weaker results on some items in comparison with Japanese students. In regard to basic skills, *Everyday Mathematics* students were able to perform as well as traditional students, but they didn’t perform as well as might be hoped. On the other hand, she concluded that *Everyday Mathematics* students did learn more about topics not included in traditional texts. She felt that indicated students could learn more advanced topics at an earlier age. She offers a caveat in regard to the spiral nature of the curriculum. Fuson indicates that teachers reported difficulty with the approach, and they often supplemented the materials. She also reported that in *Everyday Mathematics*, students were expected to spend more time discussing their thinking, and the teachers were expected to cover more material. Fuson feels the tradeoffs between the breadth of the content and depth that is needed for students to discuss their thinking in *Everyday Mathematics* should be examined in future research.

Carroll analyzes *Everyday Mathematics* in light of the third TIMSS. He indicates that textbooks are the major reason for the poor showing of US students in eighth and twelfth grades. One of the shortcomings of the texts according to Carroll is that there are too many topics and that creates a lack of focus within the textbooks. One could question whether the spiraling curriculum of *Everyday Mathematics* actually helps the textbooks degree of focus. In addition a lot is covered in a single year.

In 2001 NCTM convened a dialogue on Asian mathematics achievement. Thomas J Coney, the editor concluded that “societal conditions” were very important to Asian achievement. Many of the authors who took part in the dialogue mentioned societal conditions such as parents willing to send their children to cram schools. One author,

Swee Fong Ng , points to the improvement of Singapore students on the TIMSS test in the last ten years. She indicates that a high value is placed on education by both parents and students. She, also, thinks that increases in achievement are due to revisions in the curriculum made by the central ministry of education (which are readily accepted by the population). Great emphasis is placed on teacher training. New teachers are mentored and have reduced workloads.

Data and Chance in *Everyday Mathematics*

Exploring data is one of the ten major content strands in *Everyday Mathematics*, which first introduce students to data collection in Kindergarten. The *Everyday Mathematics K-3 Reference Manual* indicates the rationale for early data collection. They say that from the beginning of school children should have the opportunity to decide on methods of collecting, representing, and explaining their data. *Everyday Mathematics* presents many situations that require data collection and analysis but they also encourage teachers to personalize data collection based on personal classroom concerns.

In *Everyday Mathematics* the most common purpose for collecting data is to answer the question how many. Children are encouraged to record, organize and display data using tables, line plots and bar graphs. Analysis of the data focuses on landmarks such as maximum, minimum, range, mode, and median. The authors, caution since, "...the usual reason for analyzing data is to solve a problem, or make a decision, or make a prediction...students should discuss the data they have collected and reach some sort of conclusion"(*Everyday Mathematics K-3 Teacher's Reference Manual* 9).

The authors indicate that is important for students to study probability since, over the course of their lives, students will have to make many decisions based on the chance of an event. It is important for students to become familiar with the vocabulary of probability including the words sure, certain, probably, 50-50 chance, not likely, and impossible. Though children are exposed to randomness, through the use of spinners and dice, a formal definition of random is not given until fifth grade.

The authors would like children to come to the understanding that the greater the number of trials, the more likely the outcome will reflect theoretical probability. The idea of increasing sample size and expected value is illustrated by a block drawing experiment in third grade.

Everyday Mathematics provides opportunities for data collection through the use of routines, data days, and games. A key goal for children in understanding data

exploration is that they be able to understand it as, “ a sensible process.” That is can they ask sensible questions, make sensible graphs, and make sense of graphs (*Everyday Mathematics K-3 Teacher’s Reference Manual*. p.12)

NCTM in their Table of Standards and Expectations lists Data Analysis and Probability as a main heading. The overriding objective is that students be able formulate questions that can be answered by collecting data that the students then organize and display the data in such a way as to answer the posed question.

The NCTM expectation K-2 is that all students should formulate questions that:

- Pose questions and gather data about themselves and their surroundings;
- Sort and classify objects according to their attributes and organize data about objects
- Represent data using concrete objects, pictures and graphs.

Students should select and use appropriate statistical methods to analyze data that-

- Describe parts of the data and set of data as a whole to determine what the data show.

Students should develop and evaluate inferences and predictions that are based on data that:

- Discuss events related to students’ experiences as likely or unlikely.

Appendix: Table of Standards and Expectations

Kindergarten

In *Everyday Mathematics* in Kindergarten, students have the opportunity to examine data that relates to age, birthday, favorite color, pets, and method of traveling to school. They also have an activity where they throw a single dice and record the results.

Grade One

In the first grade journals of *Everyday Mathematics*, data days are introduced. In the sections called data days, students are asked to create graphs and compare quantities. Students begin to describe parts of the data set. The concepts of typical value and range are introduced. Students are asked to make simple bar graphs. The topics in first grade include class heights, class weights, and end of year weights.

Grade Two

In the second grade, student data days are continued with more elaborate topics. For example, in generations, students find the birth dates of several generations of

their families. For each generation, they find the middle value or median. They, then, subtract the differences between the medians. The age difference between generations of families is usually twenty-five years (*Everyday Mathematics Teacher's Manual and Lesson Guide Vol. A. p.111*). Students compare the average age difference of generations with age differences in their samples. In standing jump, students are introduced to more elaborate collection methods. Students are given roles. A "line judge" sees to it that the "jumpers" toes don't cross the line. "A marker" marks where the jumper lands, and a "measurer" measures the length of the jump. (*Everyday Mathematics Second Grade Teacher's Manual Vol. B. p.302*) In Jumping Starts, students are asked to interpret graphs in the journal. They are also asked to find the median value. At the very end of the second journal the students are introduced to the concept of mode. (*Second Grade Teachers Manual Vol.B. p. 505*)

In the K-2 curriculum of *Everyday Mathematics* it appears that students have been given opportunities to meet the standards as delineated by the NCTM including posing questions, sorting and classifying, and representing, and describing parts of the data set.

The specific NCTM standards for expectations in grades 3-5 are as follows:

- Design investigations to address a question and consider how data-collection methods affect the nature of the data set;
- Collect data using observations, surveys and experiments;
- Represent data using tables and graphs such as line plots, bar graphs, and line graphs;
- Recognize the differences in representing categorical and numerical data.

The specific standards for using statistical methods are as follows:

- Describe the shape and important features of a data set and compare related data sets with an emphasis on how the data are distributed;
- Use measures of center, focusing on the median, and understand what does and does not indicate about the data set;
- Compare different representations of the same data and evaluate how well each representation shows important aspects of the data..

Grade Three

In the third grade students continue gathering and displaying data. In the third lesson of the year the students collect data about students zip codes and telephone prefixes. Students are asked if there might be a relationship between the two. Possible relationships are not examined in formal ways. Students also examine the number of letters in first and last names to determine if, on average, first or last names are longer. The concepts of median, and range are used to examine the data. Students are asked to make a frequency distribution and use that to make a bar graph.

Throughout the year in the *Everyday Mathematics* program, students are given routines of collecting data on a regular basis. In the third grade, students collect data about sunrise and sunset, and graph the data by shading in blocks on a chart designed by the teacher. Later in the first journal, students make a line graph from the data that reflects the length of days.

In lesson 83, in *Journal B* in third grade, students are first, formally, introduced to the concept of mean. The children first make bar graphs of the number of family members, and of ostrich age clutches. They find the mean by using pennies to even the bars. In Lesson 100, students again work with the concept of mean. Working in groups of four, children find their average weights using both median and mean and they compare the differences. A suggested independent activity is finding the mean and median height and arm spans for adults. In the following lesson (101), students make a frequency distribution of the heights of the entire class and find the mean, median, and mode for the distribution.

A large portion of the last unit in *Journal A* of grade three is devoted to probability. The objectives related to probability in three lessons are as follows:

- To provide a variety of informal activities related to chance and probability.
- To examine and summarize the results of yearlong, data-collection projects about length of days and temperature ranges.
- To present you and the children with information about their progress on some of the concepts and skills that have been covered so far...
Manual B. p.48

The first lesson (107) in *Teacher Manual B* (498-499) on probability in Unit 11 focuses on the vocabulary of probability and the meaning of the vocabulary. The students are asked to make lists of certain and uncertain events. In Lessons 110, 111, and 113 (505- 509 and 512-513) children are asked to conduct probability experiments in which the experiments are fair that is there is an equally likely outcome for each event, and unfair that is there is not an equally likely outcome for

each event. Lesson 114, (515-516) involves sampling with replacement. The students are asked to describe the contents of a bag based on their sampling of the contents.

The last three lessons 114, 115, 116 in Unit 11 are based on chance and data. The lessons involve looking at previously collected data and the related graphs. In lesson 116, (520-521) students are asked to look for patterns within the length of day, and sunrise and sunset records, and determine relationships between the two charts. In Lessons 117 and 118, (522- 526) students are asked to take the data in national high and low temperatures tables and use it to make a frequency chart, graph data, and find the mean and median of the data.

Fourth Grade

In the first fourth grade journal, the first data lesson (15) (74-77) involves using $\frac{1}{2}$ ounce raisin boxes as a means of using estimation, recording and organizing data, and finding landmarks such as maximum, minimum, range, median, and mode. Students are each given a box of raisins to count. They organize the class data by first making a line plot and then use the line plot to make a bar graph. The next lesson (16) (78-81) emphasizes median. Students are asked to collect data about family size. They construct a line plot and use the line plot to find the median value. In the following lesson (17) (82- 85), students again construct a bar graph in order to find the median head size circumference of the class.

The last three lessons (62-64) in the *Fourth Grade Teacher Manual A* (283- 292) involve chance. Students are asked to analyze spinners to determine chance as a fraction. They are also asked to design spinners to certain specifications. In a coin dropping experiment, students construct a 10 by 10 grid and color it a specific way; 50 white, 35 green, 10 blue, 4 red, and 1 black. Before students toss a coin onto the grid 50 times, they are asked to compute the expected values. They, then, perform the experiment and record the data. They also compile the class data. Students then compare their individual drops with the expected value and then the class drops with the expected value. Students can convert the coin drop into a carnival game deciding how much to charge and payback amount based on expected values.

There are a series of lessons (94-97) *Fourth Grade Teacher Manual* (454-474) in which the students do research about mammals. Students are asked to use the skills they have developed in collecting, organizing, and analyzing data when researching mammal statistics and making comparisons among groups. They are asked to organize data within ranges such as short, medium, and tall. They are also asked to examine relationships between two sets of data such as height and weight.

Fifth Grade

In the fifth grade a lot of coverage is given to Data and Chance. In *Volume A* of the *Fifth Grade Manual*, students are asked to perform two experiments that relate to reaction time (73- 76). In the first experiment students, stand in a circle and at a signal start squeezing hands. Someone times how long it takes from the first squeeze to the last squeeze. The average reaction time is found by dividing the total time by the number of students in the circle. In the second experiment, students try to catch a Grab-It Gauge. Each person conducts the experiment ten times taking turns being the tester and contestant. It is not at all clear from reading the manual and looking at a small replica of the journal page exactly how this experiment works. Once again the students analyze the data by looking at the landmarks in the data.

Probability is again introduced in *Volume A* of the *Fifth Grade Manual* (Lesson 15 pages 77-80). Students are first introduced to a poster of a probability meter. A probability meter is a number line from 0 to 1 that is divided into 100 equal parts. Fractions, decimals, and percents all appear on the meter. Students are then asked to perform a thumbtack experiment. Students, working in partners throw ten thumbtacks out of a cup and record the number that landed face down. They repeat the activity ten times. After a hundred tosses they record the fractional part that landed face down on a post it note and they attach the post it note to the probability meter. The students then use the distribution of post it notes to develop a class average. The average number of tacks that land face down becomes the estimate of how many tacks will land face down in a toss of one hundred.

In the cumulative review in Lesson 26 (133), students are asked to create a bar graph using a set of landmarks in the data. I think this a very interesting assignment and mimics the type of questions asked in the New Standards tests.

In lessons 40 – 42 (197 – 212), students are taught how to make pie graphs. This lesson comes after lesson involving relationships between percents, fractions, and decimals. First, students physically make a circle and analyze aspects of the circle based on the percent of boys and girls. Then, students are asked to construct pie graphs by hand. The authors of *Everyday Mathematics* recognize the widespread use of calculators, expect students to use calculators, and indicate that because of the availability of calculators students do need to practice multiple digit multiplication. Yet, nowhere have I seen reference to the use of computers. Computers are widely used in classrooms and surely it is easier to construct pie graphs using computers.

All of Unit 5 in *Vol.A* is devoted to collecting and working with data. In the introduction to the unit the authors feel compelled to justify and explain the unit. They say that the unit builds on the previously developed concepts of collecting and analyzing data. In the fifth unit, students learn about conducting and designing surveys, appropriate sample size, and use of stem and leaf plots. They even indicate

that the unit covers a formidable amount of material, but that the unit had been field-tested and students responded well to it.

Unit 5 (Lesson 45 p 232) begins with the students playing a card game called twenty-one. Two students play this game a number of times. The first player reveals a card from the pile, and announces its value. The second player reveals a card and sums the two cards. Players keep revealing and cumulatively summing cards until a player wins by calling twenty-one or more. Students keep a tally of whether the first or second player wins, and find the percentage of wins for each player. In the lesson, students also collect data about number of states visited by students and make a stem and leaf plot for the class data.

Lessons 46 – 48, involve having students take measurements of hand, finger and arm spans. They use stem and leaf plots to analyze the data. As a form of practice students are given a math box in which they draw conclusions from both bar graphs and stem and leaf plots.

Lesson 49 (250) deals with sample size and good conclusions. Students analyze both player results and class results from the twenty-one, card game. The authors expect this exercise will help students draw the conclusion that large samples provide more “trustworthy” evidence. They also look at sample size when drawing candy from a bag to determine how many samples are needed to require an accurate understanding of the number of candies of various colors in the bag. The authors don’t indicate that, in real life, researchers because of time and cost must try to have an optimal sample that is one that is big enough to be reliable without being too burdensome. I like the Marilyn Burns, activity (“The Popcorn Lesson” pages 149-158) better, wherein students sample with replacement from a paper bag to determine how many draws are needed to get a least one of each of six items. Each group of students performs multiply samples, and devise a frequency table for the class data. In this experiment students can use landmarks such as mode, median, and mean to see that it might be best to have X number of samples, but that that number of samples might not be sufficient in all cases, and therefore the median might result in error some percentage of the time while the maximum class trial might be needed only once in awhile.

In Lesson 51 (258), students are introduced to survey data. Students go to other classes in the school to conduct a survey on questions of interest such as how do you travel to school. Students collect, organize, graph data, and draw conclusions.

Lesson 52 is an assessment for Unit 5. In it, students are asked questions about bar graphs, pie graphs, and stem and leaf plots. The first volume of the *Fifth Grade Journal* devotes a great deal of time to data collection.

In lessons 86, 88, and 89 and 90 of *Volume B*, the authors present a series of ambitious experiments that combine science and mathematics as they collect and graph data. Students work with rate and interpret line graphs. In Lesson 89, students are presented with mystery graphs and are asked to match the graphs with descriptions of the graphs. In Lesson 90 students do an experiment with pendulum swings and length of string, weight of pendulum, and size of arc. They graph the results.

In Lesson 101 (534), a very interesting lesson is presented where students collect data from a videotape to determine how many cars have headlights on before and after sunset. After collecting the data, they make a line graph to show the results of the data collection. In Lesson 112, students again use a videotape to collect data of a variety of people walking between two set points. They then graph the data and solve rate problems. Both of the videotape lessons are ambitious and interesting, but it is not clear that the lessons are often used. A poll of my students in the gifted program did not reveal anyone who had actually seen the videotapes. These lessons would be excellent to use in the gifted program.

In Lesson 120–123, students collect data about the effect of exercise on heart rate. They make a class profile by making side-by-side line plots. In Lesson 123, students again collect data from videotape though again it is not clear if these lessons are actually used.

A large percentage of the fifth grade curriculum is devoted to data collection. The data collection integrates science and mathematics in a very sophisticated manner. It is not clear if some of the lessons are widely used. It might be beneficial to have these lessons as a supplement for a gifted program.

A comparison between *Everyday Mathematics* and *Addison-Wesley*, the former text used by The Pittsburgh Public Schools, shows a huge difference in the presentation of data in the fourth and fifth grades. In fourth grade *Addison Wesley* has a small unit on data. The focus is graphs and students are asked to answer questions about the graphs rather than collecting data themselves. In the fifth grade the data unit is a bit larger, but again students are asked questions about graphs. In one lesson students are asked to enter given data into a template of a graph. Certainly it would seem that students would be more interested in making graphs from data they have collected. That has been my experience in the gifted program.

Conclusions About Data and Chance in *Everyday Mathematics*

In the standards based curriculum known as *Everyday Mathematics*, exploring data is one of ten major content strands. Data collection is part of the NCTM standards and the standards of the state of Pennsylvania. In *Everyday Mathematics* data is collected

by the students, based on everyday questions such as how do we get to school and what is the average class height, that are presumably of interest to the students. Students make graphs based on the data they have collected. Students look for landmarks in the data such as measures of central tendency. It has been my observation that students who attend the gifted program have an excellent understanding of the concepts of mode, median, and mean and how to compute them though they sometimes need clarification of terms.

Probability is covered year by year in *Everyday Mathematics*, but not as extensively as data collection and analysis. There is an attempt have the students understand basic concepts. Students conduct some experiments with coins and spinners, and objects whose expected value cannot be known without conducting an experiment to obtain an experimental probability.

I do not think the spiraling nature of the curriculum of *Everyday Mathematics* causes difficulty with mastery of content in regard to data or chance. The authors clearly develop data collection and analysis in a very sophisticated manner. In the fifth grade, the concepts may be too sophisticated and some of the lessons might not be actually used in classrooms

If data and chance are deemed to be important topics for inclusion in elementary school then *Everyday Mathematics* does an excellent job of presenting the topics. Certainly the “hands on approach” of collecting data directly and analyzing that data is more compelling than analyzing graphs prepared by someone else.

Enrichment and Acceleration Unit on Probability for Fourth and Fifth Grade

This probability unit is an enrichment and acceleration unit that can be used in conjunction with *Everyday Mathematics*, in either a fourth or fifth grade class. The objectives of the unit are as follows:

Students will be able to determine the probability of an event.

Students will be able to compare results of experiments to the expected value.

Students will be able to determine if a game is fair or unfair.

Students will design a game of chance.

Activities

1. Launch Simulated Horse Race

The teacher simulates a horse race (*Projects to Enrich School Mathematics*, 72) by rolling and summing two die. Each student is given a number from 1 to 12. Students are asked to identify good and bad numbers. The students keep track of the race by entering the winning number for each roll until one number wins the race by getting nine wins.

a. Analysis of simulated horse race

Students are then asked to analyze the probabilities by filling in a chart (*Middle School Mathematics Project Probability*, 41) wherein they find all combinations of sums when two dice are rolled. Students use the chart to find the probability for each horse number, and for even and odd numbers (eighteen out of thirty-six for both even and odd.).

2. Experiment with odd and even numbers

Students then perform an experiment by rolling and summing two dice twenty-four times. Students record whether sum is odd or even. They total the odd and even sums. A line plot is made and analyzed for the class data.

a. Analysis of other two dice games

Students fill in a six by six array of all possible products when two dice are rolled and the values are multiplied together. They analyze the table by finding the probability of products of three and products of four. To test the expected values students roll two dice and multiply the values together and record whether the results are a multiple of three, four, or both.

b. Pascal's triangle

Students are shown the algorithm for finding the rows of Pascal's triangle. As the students fill in rows of an unfinished Pascal's triangle, (*Pattern's in Pascal's Triangle*) they are asked to identify patterns such as the rows being symmetrical. After students fill in fifteen rows, the teacher demonstrates how Pascal's triangle is related to probability by using the third row to demonstrate all possible ways three independent births may occur. The students are then asked to take the fourth row and list the outcomes of heads and tails when four coins are tossed. As a way of reinforcing times tables and divisibility rules students are given three Pascal's triangle with fifteen completed rows and asked to find the multiples of 2, 3, and 5. They all make different patterns of triangles.

c. More experiments comparing a single experiment with the expected value.

Students are given a wide choice of activities from the series, “What Are My chances *A and B*.” These include such activities as flipping a penny 50 times and tallying heads and tails, and rolling a dodecahedron die 120 times and tallying the numbers that come up. Results of several students can be combined in order to see that as the number of trials increase, the results become more like the expected values.

d. Computer probability experiments

Using the Web site (<http://www.mathsonline.co.uk/nonmembers/resource/prob/>), students can select from a variety of experiments. They can, for example, select the number of coins and the number of trials, which enables them to view how the number of trials effects the distribution of the results.

4. Fair and Unfair Games

a. Rock, Paper, Scissors

In order to examine fair and unfair games students are asked to play the game Rock, Paper Scissors (*Investigations in Data, Number and Space Between Never And Always*, 60). The game calls for three players. Each player responds to the call, rock, paper scissors by either making a fist (rock), putting a hand out flat (paper), putting the forefinger and middle finger in a scissors like way. If all the hands are the same, A gets a point, if all the hands are different B gets a point, if there are two of one and one of another C gets a point. Students play thirty rounds and record the points. After the game is finished students make an organized list of all the possible points. It is a very unfair game since the probability for A getting a point is $3/36$, for B the probability is $6/36$, and for C the probability is $27/36$.

b. Race to the top- looking at distributions of a fair game

In pairs of two, students play a game called race to the top. Students use a spinner that is divided equally into two parts. Students take turns spinning ten times. The player whose color or number comes up gets an X on the score sheet even if it wasn't their turn. Players play the game for six rounds. After six rounds, each player would expect to win three rounds each. That may or may not happen. In order to examine class results, students make a line plot. The line plot will probably have a normal distribution with mode number of wins being three.

5. A student developed game of chance

First students discuss the differences between games of chance and games of skill. They make a list of games that are dependent on skill, on chance, and on both skill and chance (*Investigations in Number, Data, and Space Between Never and Always*, 101). Students working in pairs, or individually, design their own games of chance. The games can be fair, or unfair. Students determine the probability of winning the game. They write up the rules and provide a computer generated graphic, advertisement for the game. Students can invite students from other classes to play the games, and record the results. Students keep track of the results and compare the results with the expected values.

General Conclusions

I have always covered the topics of data and chance for fourth and fifth graders at the Pittsburgh Gifted Center. I was anxious to see how the topics were covered in the home school program by *Everyday Mathematics*. I wanted to see if I was repeating too much. Since probability is not covered as extensively as collecting and analyzing data. I believe that the probability unit I have developed fits very well as enrichment and acceleration unit. In the regard to data analysis, I think that I can do less reviewing of the topic. In the 2002/2003 school year, I had the students write up a data collection research project according to the guidelines Presented by the American Statistical Association for their 4-6 research contest. I think that students have had enough background in data collection and analysis in *Everyday Mathematics* to perform such a task, though writing a statistical research report was novel and a bit challenging.

I did not come across any research that directly examined student achievement in the areas of data and chance by *Everyday Mathematics* students. One researcher indicated that students did better on topics not covered very much in traditional texts. One can assume that that related to data and chance though it was not specifically stated. I did not examine *Everyday Mathematics* in regard to basic skill achievement. The research of one author indicated that while *Everyday Mathematics* students did no worse than traditional students in basic skills, they also did no better. I do think that *Everyday Mathematics* does an excellent job of developing data and chance. I think many of the activities are very interesting and engaging though some in the fifth grade may be a bit too sophisticated.

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Students can control the number of coins and the number of trails. They can watch
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