An Interdisciplinary Unit on Cryptography

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Overview

My goal is to create an interdisciplinary unit on cryptography that can be used as a supplement to the high school curriculum. History, science, and mathematics lessons will be taught as part of the same overall unit on cryptography. The unit will cover more general aspects of cryptography and how it has been used as a means of secret communication throughout history. The bulk of the unit, however, will focus on the enigma code-breaking machines that were used by the Germans during World War II. My research will show how mathematics played a key role in the outcome of the war, and continues to play a key role in modern technology.

While part of the paper will focus on my research, I would like much of the paper to consist of lessons that could facilitate the students' understanding of the material. My general philosophy on education is that students learn best by conducting their own research and learning the material on their own. I would like to provide educators with lessons that will provide the students with hands-on experience with cryptography. They will write papers, decode encrypted messages, and actually work with models of the Enigma machines that are available online. I would then like to extend this knowledge to ways in which cryptography is applied today. This is a field that is constantly changing, and certainly not only a lesson in the mathematics of the past. Cryptography is used in much of the students' daily lives. They will explore the use of cryptography in Internet commerce and emailing.

Combining history, science, and mathematics into one unit with many different teachers is a difficult task. In most schools, no guarantee exists that the students in one math class will even be in the same history class. In addition, each teacher is normally expected to follow a very rigid curriculum. Therefore, this unit will not be practical under most circumstances. However, I feel that this unit may be possible as a one or two-week supplement to a small honors class curriculum. I am not suggesting that only honors students will be capable of grasping the ideas that I will be presenting. For practical purposes, though, a small group of students who share the same teachers is ideal for a cross-curricular unit such as this one.

The mathematics involved in cryptography can be extremely sophisticated. Much of the material that I encountered in my research was well beyond my understanding, and certainly beyond the abilities of most high school students. However, I feel that high school students at all ability-levels will be able to understand the cryptography basics that are presented in this unit. I believe that they will be able to appreciate the beauty of mathematics and how it has been used throughout history to solve practical problems. Even more importantly, I believe that the lessons will be fun, engaging, and perhaps give the students a more positive outlook on mathematics.

I have outlined in detail lesson plans that will cover 1 to 2 weeks. While I have included fairly rigid lesson plans, teachers should feel free to modify these lessons to meet the needs and time constraints of their students. It is also worth noting that it is not necessarily required that the mathematics portion of the unit be taught in conjunction with the history lessons, or that the history lessons necessarily need to be taught. While I feel that they complement each other nicely, the mathematics of cryptography can easily stand on its own.

Rationale

Students often see mathematics as a tedious, isolated discipline only to be enjoyed by students who have been classified early on as "math people." In high school, mathematics becomes more complex, and students often come to view it as a series of formulas that can be applied to various contrived problem situations. These situations theoretically have applications in science, but the students rarely see these applications. Instead, the curriculum largely focuses on a narrow range of skills rather than applying problem-solving to more complex situations. Students generally learn a specific skill and model this skill after examples that have been presented in the previous pages of the text. While I see the benefit to learning rote math skills and formulas, this is often the only context in which students see math. They often develop negative attitudes about math at an early age because they fail to see its relationship to other disciplines. Richard Lesh, director of Purdue University's School Mathematics and Science Center, noted "Calculation and rule-following only make up a small part of the big picture in terms of what students really need to know (in math), but because they are the easiest skills to measure, that's what schools emphasize. And children who don't grasp it right away or don't enjoy doing it often end up fearful of all math" (Lesh,1999). I believe that this unit will automatically interest many students because it involves secret codes and espionage. Knowledge of cryptography can also scaffold upon their existing knowledge. Nearly all students have some experience with games and puzzles that involve putting clues together to solve a problem. Many have even tried to develop their own secret ways to communicate with their friends without the teacher or other students intercepting their messages.

A common concern among mathematics teachers is that their students do not retain information from one year to the next. For that matter, students often fail to retain information from one topic to the next. It is not uncommon for students to take a test on a

topic and then forget it after another topic is learned. Facts and pieces of information are quickly discarded with each new chapter. How many feet are in a yard? How many inches in a foot? How many days in a year? The students realize that at one time they knew this information, but it is no longer at their disposal. Key facts that "should" be memorized rarely are - at least not in the long term. This applies to algorithms as well as basic facts. High school students often have trouble performing long division, multiplication, and subtraction. Once a topic is aced (or at least covered in the text), it's full steam ahead and no looking back. While teachers are quick to blame the students, I believe that a large part of the problem lies in the way that we expect the students to learn. When students memorize facts that are not related to other information, or to their existing knowledge base, the information is rarely converted to their long-term memory. It was once thought that the human brain was similar to a computer in that it could store large amounts of information that could be recalled at appropriate times. It was believed that students could bring back information at will, regardless of how it connected with anything else. Over the years, this theory on the brain has changed. It is now believed that the brain operates more like a rain forest: it works best when a lot of things are going on that can be absorbed at once. It functions best with a whole lot of stimuli from different places, and everything connects together in various ways, building on prior knowledge. (Summer, 1999, p.8). By relating subjects to each other, we offer many more opportunities for the students to connect new information to existing knowledge. I believe that educators should modify their teaching methods so that new information relates to as many different things as possible. It should relate to other disciplines and connect with as many life experiences of the student as possible. Disciplines should as often as possible be intertwined so that it is difficult to clearly distinguish the math from the science, technology, history, and even literature. Life operates in this way - so should education. Interdisciplinary lessons not only allow the students to see mathematics as it relates to other disciplines, the lessons that I have incorporated allow the students to connect the mathematics to their own experiences. This helps to create a more inclusive environment where students work together and share their own research. This will hopefully bring more energy and enthusiasm to the classroom.

Because life rarely functions in neatly compartmentalized subjects, I feel that teaching mathematics in this way alienates many of our students. Algebra is an important tool that can be used to solve problems - not simply problems about trains traveling in different directions, or brainteasers that involve one number that is two times another number, but 5 less than another number. It exists to solve real, practical problems. It is difficult to show the relevance of mathematics to every single student, but we should attempt to relate the material to the experiences of our students in as many ways as possible. I am not suggesting that cryptography will relate or appeal to all students, but the lessons in this unit are designed so that the students can creatively look at problems and bring their creativity to the classroom. Instead of telling the students about this topic in history, I feel that they should research the material and experience it on their own. In "The End of Civilization," Neil Postman reminds us that boredom and rote memorization do not necessarily have to be part of school's daily life: "Generally, young people have too much curiosity about the world and far too much vitality to be attracted to an idea that reduces them to a single dimension." (Postman, 1995).

Teacher and student frustration are common features of today's education that revolves around standardized tests. Teachers are encouraged to administer a daily regiment of multiple choice questions. Monthly open-ended questions are administered as "dress-rehearsals" for the big tests that are administered every year. These tests are certainly a reality for teachers, students, and administrators. However, I believe that the constant barrage of facts and information does very little for the true education of the student. In fact, I feel that these measures are more detrimental in the end. A primary goal of education should be to produce students who enjoy learning and wish to continue learning throughout their lives. Students should be encouraged to find and pursue paths that are interesting to them, while at the same time making sure that they have the necessary skills. Broad, interconnected problem situations not only spark interest, but they are also more realistic in terms of the way that knowledge is applied in the world. In her article "Why Standardized Tests Are Bad," Terry Meier wholeheartedly agrees with providing unique opportunities for students to learn:

It is tragic that at the time when many developmental psychologists stress a broad and complex conception of intelligence and ability, and when one needs multiple talents to function effectively in the world, we have come to define excellence in our schools within the narrow parameters of what can be measured by standardized tests. (Meier, 2000, p.65)

This unit will certainly not serve as a solution to the standardized testing problem or any other crisis that faces our schools today. In general, though, I feel that an interdisciplinary curriculum will lead to higher achievement in mathematics. Regardless of the school's curriculum, these lessons can serve as a good supplement, and provide an alternative (at least for a week or two) to the high-stakes educational environment that we live in.

Part of my overall teaching philosophy is that learning should be student-centered. The role of the student should be to discover the information, while the teacher simply acts as a facilitator. The lessons generally involve the student researching information, solving puzzles, working in teams to present information to the class, and learning about cryptography using a hand-on approach. I have not included a teacher's guide, notes, or any kind of instructors manual because I believe the student should embrace their role as teacher. In using this lesson, the teacher should not be intimidated by the idea that the students may eventually know more about the topic than the teacher does. The teacher should learn the material along with the students and help them to locate resources. However, the teacher should not feel the need to present the material as notes, or to come across as an expert in the field of cryptography. The teacher only needs a very superficial understanding of the material and can learn in the process.

In this unit students will have the opportunity to reason through the mathematics and discover the rules of cryptography on their own. I believe that this will lead them to think creatively about problems that are now facing scientists who work with cryptography today. They will be asked to discuss solutions to these current problems; to

write about them and apply what they have learned to these new situations. Cryptography hinges on combinations, a topic that appears as part of the high school curriculum. Students will also graph and observe data trends in forming conclusions. While some of the mathematics will be difficult, the general ideas will be within their grasp and they will learn more with this hands-on approach. Real-life math often consists of problems that may take days, or even years, to solve. I feel that it is valuable to present students with examples of these problems that have historically taken teams of mathematicians a great deal of time to solve. Working collaboratively as a team to come up with tools for solving problems is not only a valuable learning tool, but often more realistic in terms of the way mathematics problems are solved in the real world. Professor Lesh also adds that it may lead to better results: "If we ask for an answer that isn't necessarily just a number, and combine that with the use of technology in an interesting way, kids will often invent better ideas than anyone has ever tried to teach them. It's another way to enlist their thinking in the service of doing something mathematical -- and everybody wins." (Purdue News, January 1999).

This cryptography unit will provide students with the unique opportunity to see how math and science came together to literally change the outcome of history. While science and mathematics have always played an integral part in the engineering of weapons systems, they played a unique role in wartime intelligence efforts during this time. I would like the students to learn the scientific and mathematical impact that codes and code-breaking had on the war. During their history classes, the students will learn the historical framework of Hitler's rise to power in Nazi Germany. Instead of studying history as an isolated subject, I would like to combine the history of World War II with the mathematical advances in cryptography that were developed in the years leading up to this time. Math and science are often viewed as interrelated subjects, but students rarely study the historical significance of advances in mathematics. While studying WWII, students can learn about the role that cryptography played in transmitting and decoding secret messages. They can learn the developments of this technology (from the telegraph to the telephone) and how this eventually led to the Enigma machines. The history of cryptography is not intended to replace any pieces of the history curriculum. I would only like this to supplement the current European History curriculum to include sections on cryptography so that the students can see the relationship between math and history during this period.

As the science part of the unit, students will study the Enigma code-breaking machines. They will study the mechanics of the machine and how it was used to transmit messages. The can examine some of the basic engineering principles of the machine. Online models of the machine can be used to actually simulate the Enigma machines that were used during the war. This portion of the unit will be covered as an extra project that spans only a few class periods.

I will design the project so that it focuses on many of the mathematics and communications standards that are required by Pittsburgh Public Schools. The students will learn to write mathematical conclusions to open-ended problems. They will communicate their findings with the rest of the class. Students will see advances in

technology, and use this technology to solve problems. This will include graphing calculators, the internet, and encryption devices. They will learn ways to apply their skills to unique problem situations. Even basic high-school math skills will be taught and reviewed as part of the unit. This includes, but is not limited to, combinations, factorials, graph distributions, addition, subtraction, and modular math. The students will set up various types of graphs and charts to represent their data. In fact, many of the skills that will be taught in these lessons overlap with the lessons that we focus upon in Standards Based Mathematics, the course that fulfills the mathematics portion of the graduation requirements.

Objectives

The main objective of this unit is to change the way students perceive mathematics. I would like to do this in several ways.

- I want to introduce the students to an extremely interesting mathematics topic that played an important role in history.
- I would like them to understand that many of the skills that they are acquiring in their mathematics classes were used in this historical context and continue to be used in this field today.
- The students will see how mathematics ties into other disciplines. Math is used in problem solving, and not simply a series of formulas and quick drill problems.
- Hopefully they will come to see mathematics as a vibrant, exciting field. They will see this in an interesting historical context and see how this evolved into internet applications.
- The students will see the kind of work that career mathematicians do. Very few people consider mathematics when they are thinking of career options. Perhaps this unit will change a few students' minds.

Strategies

My general philosophy on education is that students learn best by conducting their own research and discovering mathematics and history with the teacher acting as a facilitator. I have designed lessons that give students hands-on experience with code-breaking. Current technology and applications have also been incorporated into the lessons so that the students will see the relevance of the material. I believe that they should research this information on their own and bring this to classroom discussions and essays.

I will also relate the material as much as possible to previous mathematical concepts that the students have studied in the classroom. In this way the lessons relate to the mathematical standards to which they are held accountable by Pittsburgh Public Schools.

Classroom Activities

Mathematics Lessons

Day 1:

Objective: The students will use sections of the newspaper to graph the frequency with which letters appear in English. They will use this data to come up with conclusions about which letters and words would be easiest to use in code-breaking.

Materials Needed: Several newspapers that can be divided among class members, graph paper.
Basic knowledge of graphing and statistical analysis is required. The students should be comfortable setting up tables and graphing data.

Anticipatory Set: The warm-up activity should include a quick review of creating and analyzing bar graphs.

Lesson: The students will work collaboratively to plot the frequency with which letters of the alphabet appear in a random paragraph from either a textbook or the newspaper. They should be encouraged to come up with their own method for calculating and plotting the data. Most will set up a table that will include the letters of the alphabet in one column and "tick" markets that indicate the number of times that letter appears. The students will use one paragraph of data from the newspaper. The work can be split up among group members, but try to keep the number of letters below 500 so that the data collections does not become too tedious or time-consuming. After they have collected the data, the students should decide on a graphical method to present their data to the rest of the class. The students should calculate the percentage of times that each letter appears.
The teacher should generate a discussion about the data. Which letters appear most? Research suggests that the letter "E" appears 13 percent of the time - do your findings support this? Which appear least? Did all of the groups come up with the same results? Most importantly, how could this be useful in cryptography? What problems may arise and what other information may be necessary?

Conclusion: The students should write a short paragraph describing their results. They should focus on the mathematics, and how this information may prove to be useful in code-breaking.

Day 2

Objective: The students should work collaboratively to come up with their own methods for solving "ciphers" or coded messages in which every letter of every word is replaced by a different letter.

Anticipatory Set: The students should review their work from yesterday. What letter(s) of the alphabet appear most frequently? Least frequently? The teacher could also possibly do a quick cipher that is very easy and with enough clues that the students could solve it quickly.

Materials Needed: Approximately 5-10 old newspapers, preferably all from different dates.

Lesson: Ciphers appear very often in a puzzle section of nearly every newspaper. While these aren't nearly as difficult as the encrypted messages that were sent using the Enigma machines, the students will able to understand the basic premise of cryptography. Many early messages that were sent using telegraphs were also encoded more simply and using basic strategies such as those found in the newspaper. The students will work in groups of 3 or 4 to solve one of these ciphers. The students should keep a log of the methods that they are using to solve the ciphers. This may take some time, and if groups get stuck, they should be encouraged to talk with other groups and share strategies. Each group should have a different puzzle.

An important part of this lesson is the presentation. One person from each group should record their group's work on poster-paper so that it can be presented to the rest of the class. Each group can then show their work to the rest of the class, explaining how they solved their ciphers. The teacher (or different student) can keep a running log of all of the different strategies that the groups employed.

Conclusion The students should take note of all of the different strategies for solving ciphers. They should write a journal entry answering the following questions: What strategies from the other groups could have been used in their work? Is the list complete? Was working in small groups effective, or would there be a better way to solve the puzzles if the fate of the world depended on it? How could technology be employed to make their work easier? Try to brainstorm for new ideas.

Day 3-4

Objective: The students should compare the methods that they used for solving ciphers with those that were used by the cryptographers at Bletchley Park. (This will most likely take two class periods).

Materials Needed: Internet access and/or library research materials.

Anticipatory Set: Review various strategies for solving ciphers from the previous day's lesson.

Classwork: The students should use the internet to come up with at least 3 methods that the people at Bletchley Park used to crack the Enigma codes. The students will write a

summary of some of these methods used at Bletchley Park. The writing should be in their own words and can include examples. While this is a writing assignment, the students should focus on the mathematics involved.

Conclusion: Journal Entry: How were the methods that they used different from the ones that you used in your work from the previous day (solving cipher puzzles in the newspaper)? How were your methods the same?

Homework: (Extra Credit Project). NOVA Online web is a great source of information on code-breaking. Included on this site are several examples of ciphers that are similar to the ones faced by the code breakers at Bletchley Park. The teacher should print out one or two from this site and offer them as an extra credit assignment. The teacher should emphasize that the problems are very difficult, but it is interesting to see problems that are very similar to the ones that code breakers faced during the war.

Day 5 Combinations

Objective: The students will begin to learn combinations and factorial notation. This will be essential in understanding the true power of the Enigma machine, and the great task at hand in trying to crack the codes. The lesson will begin with very simple combination problems that the students can easily comprehend. These will involve this with which they have experience in their daily lives. Gradually it will progress to more difficult problems that involve more advanced code-breaking.

Anticipatory Set: The students should be presented with a couple of simple combination problems that will introduce them to basic combination theory and factorial notation.

Question 1: "If 4 students are going to line up in a row, how many different ways can they line up?"
The students should work together in coming up with a solution.
Question 2: "If the cafeteria has 5 main course, 3 desserts, 2 beverages, and 3 different vegetables, how many different possible lunches are there?"

Classwork:

The students will work on a fairly simple cipher called the "Caesar shift cipher", where each letter of the alphabet is substituted with a different letter of the alphabet. For example, "A" may be shifted to become "B", and "B" will become "C", "X" will become "Y" and so forth. This involves basic elementary math because the shift is simply adding 1 to the letter that is given. However, it is interesting to note that the students will become familiar (possibly for the first time) with the idea of modular math. Since there are only 26 letters in the alphabet, if a letter is represented by 27, it will be equal to 1 (mod26). (A picture and several examples of this cipher will be included so that the ideas will be clearer as the students are working on the problem.)
The key question for the students is the following: "If a code is presented in this form, with only one substitution allowed, how many different arrangements are there for the

letters?"

Conclusion: The students should present their work. While this may be difficult at first, the students should be reminded to think back to simpler problems and come up with a general formula. The students should present their findings.

Independent Practice: (Journal Entry) If an encryption device is using several wheels, how are the results changed?

Day 6:

Objective: The students should become familiar with the Enigma machine and how it was such a powerful tool for sending coded messages. They should understand the basic functions and how combinations were used.
Material Needed: Library/internet access.

Classwork: The students should work in pairs to research the Enigma. They should write a brief summary of the Enigma machine and how it was used. They should focus on the actual mechanics of the machine and how it operates, and how the various wheels are used to make a seemingly limitless number of possible code combinations.

Independent Practice: (homework). An excellent assignment for students was suggested by Simon Singh (Mathematics Teaching 183/Easter Conference Supplement/ June 2003) "One feature of the Enigma key setting is a set of 3 rotors, each of which can be placed in 26 orientations. How many settings is this? These rotors can be swapped around - how many ways are there to put three different objects in three positions? How many more permutations are there if you can choose three rotors from a selection of five and put them in three positions? The Germans, however, decided that no rotor could remain in the same position two days running, so how many permutations are available on any particular day?"

Conclusion: The students should consider their research in solving this problem.

Day 7: Use of the Online Enigma Machine

Objective: The students will gain experience working with an online replica of the Enigma machine.

Materials Needed: Internet Access

Classwork: The students should go to the NOVA website and use the online replica of the Enigma machine to send and decode messages. The students should be given a certain amount of freedom to simply work with the online replica to get an understanding of its functions.

Independent Practice: Write a brief summary of your findings.

Days 8-9: Extension of Cryptography Knowledge

Objective: I would like the students to research and write a 2-page paper concerning the relationship between cryptography and e-commerce.

Anticipatory Set: The students should think about and discuss the following questions:

"Can other people read the emails that you are sending?"
"How do people safely purchase items over the internet without having their credit card numbers stolen?"
"How is math/cryptography involved?"
"How does this relate to WWII and the work that was done at Bletchley Park?

Classwork: The students should work in pairs to research answers to these questions. This is the final activity of the unit. The teacher should make sure that the students are discovering the relationship between the work that they have done and current problems that are solved using cryptography.

English (and Art) Lessons

Day 1

Objective: The students should realize that not everybody was in support of U.S. involvement in World War 2. The students will see the role that propoganda played in rallying support for the war. In the end, the students will create a propoganda poster that could have been used during this time period. This will lead to discussions about the current U.S. involvement in the Iraq War. This lesson should take approximately 2 to 3 class periods.

Materials: Internet access, poster-board, construction paper, markers, and library materials such as books and magazines.

Anticipatory Set: Have the students brainstorm and free-write answers to the following questions: "Is the support of the citizenry necessary in wartime? How does the government enlist the support of its citizens in wartime? What can ordinary citizens do to support a war effort if they are not in the military?

Classwork: Have the students visit the "Poster Gallery" on DiscoverySchool.com to see examples of posters that the United States used to gain support for the war. The students may also look up examples of their own by searching the internet. The students should choose one and write a short paper on it. The student should focus on the following questions:

> What is the meaning of the poster?
> Are there any hidden messages?
> What are the intended effects of the poster?
> How does it relate to the situation of the United States in 1940?
> What does "isolation vs. intervention" mean and how does it relate to this poster?

Days 2-3

Objective: The students will use their artistic talents and creativity to design a propoganda poster of their own.

Anticipatory Set: Have the students try to put themselves back to the WWII era and discuss reasons for supporting and opposing WWII. Have the students choose a position and explain their reasoning. Have the students jot down ideas of things that they could have done to support their conviction.

Classwork: Create a propoganda poster. Have the students design a picture and come up with a slogan of their own. The work should be creatively done and clearly define their position. Once the poster is complete, the students should be prepared to present their work to the rest of the class. The teacher will eventually have a gallery of student artwork

that can be displayed in the classroom.

Credits: The main idea for the English lessons for Days 1,2, and 3 came from Tish Raff at Sequoyah Elementary School in Derwood, Maryland. I found the lesson at DiscoverySchools.com and modified it to support my unit.

Day 4

Objective: The students will learn and reinforce vocabulary words encountered during their study of WWII in their history class by creating a crossword puzzle.

Materials: Internet Access

Classwork: The students will make a crossword puzzle involving the vocabulary that was learned in their study of WWII history. The students will come up with short definitions for each word. They can either create the puzzle by hand, or go to puzzlemaker.com and create a version of their crossword puzzle online. The following vocabulary words can be used, but the teacher can modify this list to suit the needs of his or her class.

Vocabulary List: accord, treaty, agreement, dictator, fascist, kamikaze, radar, veteran, axis, neutrality, pact, Nazi, Hitler, Mussolini, Hirohito, Churchill, Truman, Eisenhower, Roosevelt, emaciated, deportation, rendezvous, clandestine, Gestapo, propoganda, anti-semitism, genocide, appeasement, blitzkrieg, Luftwaffe.

Independent Practice: As a homework assignment, the students should trade crosswords and try to complete a puzzle that was created by another student.

Day 5

Objective: The students will write a newspaper article about a WWII topic of their choice.

Materials: Newspapers

Classwork: The students should look at current newspaper articles to become familiar with their organization and content. They should begin the lesson by observing a newspaper article and taking notes, focusing on the following questions:

How is the article organized?
What information is presented in the first paragraph?
What information is in each of the following paragraphs?
What type of language is used? Is the article only fact-based, or are opinions presented?

The students will use these articles as a model in writing their own newspaper article. They can choose from one of the following major events: The D-Day invasion, Pearl

Harbor, the Holocaust, or the bombing of Hiroshima and Nagasaki. Of course, with the permission of the teacher, the student should be able to choose other topics.

History Lessons

World War II is a major topic that will obviously be covered in any high school European history class. The manner in which the material is to be covered will be left to the discretion of the history teacher. The only thing that would be helpful in achieving the goal of this interdisciplinary unit is the timing of the history lessons. It is important that the teachers communicate and plan their lessons together. The math, English, art and history lessons covering this one topic would be most effective if they were taught over the same period of time.

Annotated Bibliography/Resources

Lesh, Richard. Purdue News, January 1999

Postman, Neil. *The End of Education*.
New York: Alfred A. Knopf, 1995

Swope, Kathy. *Failing Our Kids: Why the Testing Craze Won't Fix Our Schools.*
Milwaukee: Rethinking Schools, 2000

Summer. "Crossing Discipline Lines"
Curriculum/Technology Quarterly, 8, A-D.

Appendix-Content Standards

I. Mathematical Standards

In graphing and representing data through charts, tables, and decoding encrypted messages, the following standards are met:

M1: All students use numbers, number systems, and equivalent forms (including numbers, words, objects and graphics) to represent theoretical and practical situations.
M2: All students compute, measure, and estimate to solve theoretical and practical problems, using appropriate tools, including modern technology such as calculators and computers.
M5: All students understand and apply basic concepts of algebra, geometry, probability and statistics to solve theoretical and practical problems.
M6: All students evaluate, infer, and draw appropriate conclusions from charts, tables and graphs, showing the relationships between data and real-world situations.
M7: All students make decisions and predictions based upon the collection, organization, analysis and interpretation of statistical data and the application of probability.

In presenting their mathematical findings and writing journal entries, the following mathematical standards are met:

M4: All students formulate and solve problems and communicate the mathematical processes used and the reasons for using them.

II. Communication Standards

In writing journal entries, presenting mathematical findings, and conducting research the following communication standards will be met.

C1: All students use effective research and information management skills, including locating primary and secondary sources of information with traditional and emerging library technologies.
C2: All students read and use a variety of methods to make sense of various kinds of complex texts.
C3: All students respond orally and in writing to information and ideas gained by reading narrative and informational texts and use the information and ideas to make decisions and solve problems.
C4: All students write for a variety of purposes, including to narrate, inform, and persuade, in all subject areas.
C6: All students exchange information orally, including understanding and giving spoken instructions asking and answering questions appropriately, and promoting effective group communications.
C8: All students compose and make oral presentations for each academic area of student that are designed to persuade, inform or describe.
C9: All students communicate appropriately in business, work, and other applied situations.