Fractals, Chaos and Brain Functioning

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Overview

My first introduction to fractals, unbeknownst to me, involved my early fascination with drawing leafless trees. As a child, I would incessantly draw the intricate, repetitive, but not identical patterning of a tree's trunk, boughs and branches. I recognized the necessity of making the branches curve, split and shrink in scale in a controlled but not entirely predictable fashion. In effect, this was an introduction to fractals with a sense of chaos. Fractals demonstrate a repetitive scaling that makes them identical on all scales, whereas chaos introduces a seemingly random aspect to the patterning. It wasn't until I took this class on fractals and chaos that I realized how pervasive fractal patterns are in nature, that I was seeking those patterns at an early age and that in the midst of that attraction to pattern was an innate sense that chaos provides a divergence from that pattern.

This unit is intended to introduce the pervasive concepts of fractals and chaos to high school physics students. The unit will provide the mathematical and conceptual ideas for a teacher to be able to present the unit. The level of the unit can be tailored to suit any level of physics class from general to A.P. The unit is meant to be included as a part of the modern physics curriculum but also can be included to supplement the mathematical concepts of a high school physics course. The information on brain functioning is meant to add relevancy to the topic. Brain functioning is meant to stimulate interest in the challenging concepts of chaos and fractals. Although this unit is intended for high school physics it is my hope that teachers in other areas who wish to explore brain functioning, chaos or fractals may find this unit engaging. This could potentially include Biology teachers who are introducing the issue of brain function. I can also imagine this unit being used for the mathematical content on fractals and chaos.

The unit could also be utilized by any teacher who wants to introduce the concept of chaos and finds the connection to brain functioning engaging.

Rationale

In my unit I will explain the nature of fractals in order to introduce the concepts of chaos. In order to explain fractals I will discuss mathematical concepts of infinity and explore the implications of the Cantor Set. I will introduce and explain the related but distinct concept of chaos. Chaos involves systems or equations that are extremely sensitive to initial conditions and that preclude a prediction of long term behavior as a result of the inability to perfectly define the initial conditions such as the weather. Chaotic systems often appear random and unpredictable but they are often defined by simple equations and demonstrate intermittent periodic behavior. Chaotic systems are known as non-linear equations meaning that a small change in the initial parameters does not result in a small change in the outcome, but instead is widely divergent and results in unpredictable outcomes. These systems can be very challenging to deal with mathematically, but in the last 30 years the pervasiveness and utility of such non-linear systems has become apparent. One area in which the usefulness of chaos theory as presented itself is in the study of brain functioning. It turns out that although chaotic systems are difficult to deal with that evolutionarily they may offer benefits that do not exist in linear systems. Brain functioning is an extremely complicated system that involves a concept of "strange attractors." This is the idea that although chaotic systems are unpredictable and that they are extremely sensitive to initial conditions that in fact in some chaotic systems there is some pattern and that system tends to fluctuate around a given state. This state or value is called a strange attractor and it is one of the compelling reasons to believe that brain functioning is a chaotic system. I shall look at this idea in depth.

I will also explore the functioning of the brain to expound on the brain as a chaotic system. This research ties into my research last year at the University of Pittsburgh developing a RF remotely powered vagus nerve stimulator for the treatment of epilepsy and severe depression. Our invention has a pending patent. The vagus nerve stimulator applies a programmed electrical signal to the vagus nerve, which sends a regular impulse to the brain to prevent the periodic brain patterns that result in epilepsy. It was found in experiments that this treatment also mysteriously helps to alleviate depression and was also approved for this treatment by the FDA. It is not understood why a regular pulse (once tuned to the proper frequency, pulse width, amplitude and current output) will result in reduction in epileptic seizures except that the signal somehow prevents periodic signals from establishing themselves in the brain. Our contribution to the VNS was to make it remotely powered by RF (Radio Frequency- the same waves as a transistor radio) which is much less intrusive and results in an implantable chip in the neck at the vagus nerve. We also demonstrated that we could administer different current outputs from an RF powered device.

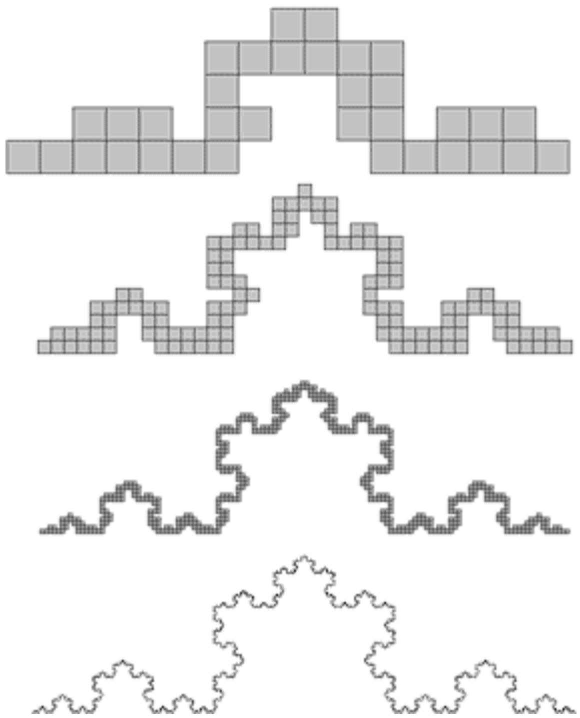
It is apparent from my reading that understanding the brain as a chaotic system helps to explain why supplying an electrical signal to the brain does prevent seizures. Seizures result from regular or periodic brain patterns. It seems very reasonable that applying an impulse to a chaotic brain system would stimulate the system to return to its attractor. The regular stimulation prevents the system from falling into a periodic rhythm. I will be continuing my research at the University of Pittsburgh Swanson Center for Invention and it is my hope that understanding the current brain research on chaotic functioning will enable me to explain the chaotic mechanism by which our RF neural stimulator operates.

Fractals

Fractals are repetitive patterns that display characteristics of scaling and self-similarity. Fractal scaling means that the same level of detail exists at all levels. Scaling is the property of being the same on all scales, from the macroscopic to the microscopic. Self-similarity means that the fractal is identical at all scales. Self-similarity means that shapes at one level resemble shapes at all other levels. Fractals introduce the intriguing concept of dimensionality. Normally we think of dimensions in terms of intergers. A one dimensional object is a line, a two dimensional object is an area and a three dimensional object is a volume. Fractals by definition, however, are objects that have a fractional dimension! What does this mean? Well, let's consider a coastline. The coastline is jagged. If we measure the length of the coast we would initially think that it is a line. However, on smaller and smaller scales the coast line is in fact, very jagged. The smaller the ruler the more length we are able to measure… for a fractal this can go on infinitely, but in real life there is a limit. In either case, the line becomes more than one dimensional… it starts to have characteristics similar to filling in area… and the dimension will be between 1 and 2 such as 1.2. This is a logarithmic scale

$$D = \lim_{h \to 0} \frac{\log N(h)}{\log(1/h)}$$

h = length of the ruler; N(h) = minimal # of rulers of length h required to cover object



An illustration from http://hypertextbook.com/chaos/ of the Koch coastline.

An example of this in biological terms is the lungs. "Self-similarity on every scale greatly increases surface area without increasing volume" (Ives,4)

A good mathematical illustration of this is a mathematical concept known as the Cantor Set. The Cantor Set is simply described but has some very strange mathematical properties related to length, infinity and dimensionality. The Cantor Set is defined by the line segment from 0 to 1 with the middle third repeatedly removed, infinitely. So let's consider what is left:

E_0 - whole set 0 1 [0,1]
E_1 (mid 1/3s removed) 0 1/3 2/3 1 [0,1/3] [2/3,1]
E_2 (mid 1/3s removed) 0 1/9 2/9 1/3 2/3 7/9 8/9 1 [0,1/9] [2/9,1/3] [2/3,7/9] [8/9,1]
… continued forever _ _ _ _ _ _ _ _
So, How "big" is the Cantor Set?
Infinity is a wild concept. There are different kinds of infinity, which is demonstrated by the Cantor Set.

Now let's consider the strange concept of infinity. Is there such a thing as infinitely small? Maybe not in reality because space may have a limit. Is there such a thing as infinitely large? Maybe not. The universe is probably infinite but maybe it isn't. This is true in the physical world, but in math world there are not limits and these terms have definable meanings. So what do we mean by infinity?

Oddly, mathematicians claim that there are an infinity of odd integers because there are the same number of odd integers as all integers. What do mathematicians mean by the same amount? An integer is z = {1,2,3… } An odd integer is given by the formula 2n -1. Any time you have an integer you can make it odd, z = {1,3,5…}. So equal infinities mean a 1:1 correspondence between z and odd z. Both are infinite of equal size because there is a 1:1 correspondence. This is called countable infinity. The mapping of a 1:1 correspondence is called cardinality. So are there bigger sets of infinity? Let's look at rational numbers, which include fractions, etc. p/q. How many rational numbers are there? More, less or the same? It can't be less because there are at least as many as the integers. However, it turns out, again oddly, that there are exactly the same number because again there is a one to one correspondence. The rational numbers, therefore, have the same cardinality as the integers and are said to be denumerable. Denumerable means countably infinite. An example of this is knows as the Hilbert Hotel. There are a denumerable number of rooms in an infinite hotel. All the rooms are full but you get another guest. By having everyone move one room none has to move an infinite distance and you can always add more. Now let's consider the irrational numbers. This is the rational numbers such as $\sqrt{2}$ which can not be written as a fraction. So the claim is that there is a collection of rational and irrational numbers that is BIGGER than the set of counting numbers! So, a 1:1 correspondence does not exist. This is called non-denumerable. The set [0,1] is non-denumerable or infinite. But there are the same number on the whole number lines as on [0,1]! Infinity is very strange and tricky and is very important for fractals.

Now let's reconsider the cantor set in terms of its dimensionality. Dimension means it's capacity and we will find that the dimension of an object can be a fraction and consequently resulted in the term fractal. The formula for the dimension is:

$$D = \lim_{h \to 0} \frac{\log N(h)}{\log(1/h)}$$

h = length of the ruler; N(h) = minimal # of rulers of length h required to cover object

C_0 = [0,1] $h = 1/3^0 = 1$ $N(h) = 1$ 2^0
C_1 = [0,1] $h = 1/3^1 = 1/3$ $N(h) = 2$ 2^1
C_2 = [0,1] $h = 1/3^2 = 1/9$ $N(h) = 4$ 2^2
C_3 = [0,1] $h = 1/3^3 = 1/27$ $N(h) = 8$ 2^3
C_n = [0,1] $h = 1/3^n = 1/3^n$ $N(h) = 2^n$ 2^n

$$D = \lim_{h \to 0} \frac{\log N(h)}{\log(1/h)} = \frac{\log 2^n}{\log 3^n} = \frac{n \log 2}{n \log 3} = \frac{\log 2}{\log 3} = 0.63892$$

This is a fractal!

"Fractals are important in chaos because they dramatically illustrate complex systems with definite properties." (Ives,2) Many of the systems in our body demonstrate fractal characteristics. Later we will see how brain functioning demonstrates fractal characteristics and is most likely chaotic. Now let's consider the related topic of chaos.

Chaos

Chaos is defined in systems that are incredibly sensitive to initial conditions. Linear systems can be represented by the equation, $x \rightarrow ax + b$. Unlike linear equations, in which changes in initial conditions have a proportional effect, in a chaotic system very small changes in initial conditions can, and often do result, in totally divergent outcomes. In other words, you can not predict from where two points start where they will end up in relation to each other. Chaotic systems are known as non-linear equations and great complexity can result from simple equations. Chaos was first discovered using weather modeling but it has recently been acknowledged that non-linear equations, although they are much more difficult if not impossible to solve, are in fact more prevalent than linear equations in nature.

An example of a non-linear equation is the "3 body problem." Physicists have tried to answer this problem about the solar system for hundreds of years. And although the solar system has been stable for 4.5 billion years, the 3 body problem and the proof of whether or not the solar system is stable is a chaotic system and remains unsolved. The equation $x^n + y^n = z^n$ can only be solved if n=2 which is known as the Pythagorean theorem. For values greater than 3 it is unsolvable because higher orders result in results that are geometric shapes. Gravity is another example of a non-linear system!

Chaotic systems are interesting for how simple they can be. For example: $X_{new} = rx_{old}$ $(1- x_{old})$;$0 <= x_{old} <= 1$ Known as the logistic equation is related to population equations and results in chaotic system. The outcome grows to a certain point, until r = 3.5964 at which point a bifurcation, or split results, and two solutions arise. Then another bifurcation occurs and each trajectory spits into two more possible solutions, resulting in four possibilities. This process continues at a more rapid rate, thus the chaotic system fluctuates between seemingly random outcomes and times of periodicity.

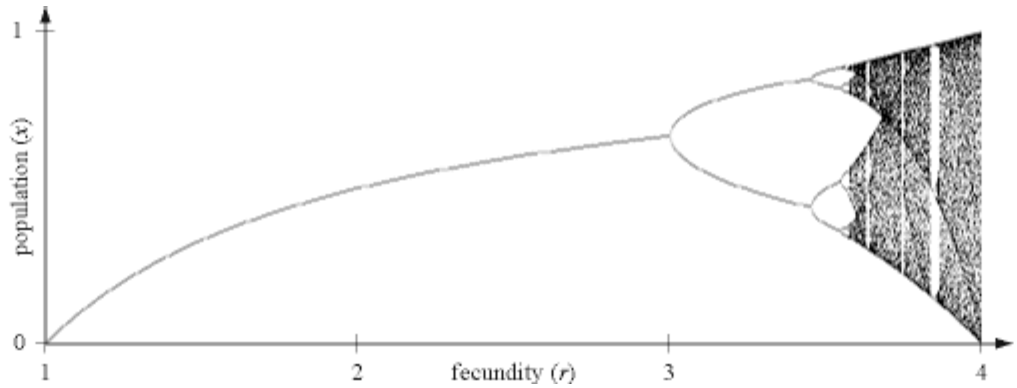


Illustration from http://hypertextbook.com/chaos/

It was believed that with the advent of powerful computers problems such as weather prediction would be completely solvable. However, the initial conditions can never be perfectly known, and in chaotic systems this prevents long term predictions. With time the initial error causes a divergence between the predicted and actual values and becomes unpredictable.

Chaotic systems often appear random. The difference between chaos and true randomness is that chaotic systems occasionally enter periodic states and that chaotic systems tend to stay around values known as strange attractors, whereas truly random processes show no preference for any values. Chaos and fractals are very related. These strange attractors are said to have a low dimensionality.

An example of a diagram of a chaotic attractor is the Lorenz attractor for atmospheric convection that deals with the weather.

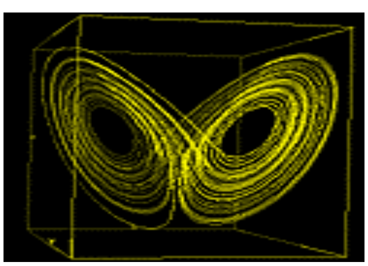


Illustration from http://hypertextbook.com/chaos/

Another example of a chaotic system is the equation $f(x) = x^2 + C$;where f(x) becomes x_0 the seed value.

x_0 ,

$x_1 = f(x_{0)}$
$x_2 = f(x_1)$
$x_3 = f(x_2)$
$x_n = f(x_{n-1})$
A fixed point is a point that doesn't change; $f(x_p) = x_p$
$x_p^{\,2} + C = x_p$ (quadratic solutions)
$x_p^{\,2} - x_p + C = 0$
$x_p = \frac{1 \pm \sqrt{1-4c}}{2}$ (if solutions exist we want real numbers, so (1 -4C) must be positive or zero. So c≤ ¼ is an important place! If c ›1/4 it won't generate a fixed point.

Using mathematica program or filling in values in a TI-84:
C > ¼ → ∞
C = ¼ → ½
C = -3/4 → -1/2, 3/2 (One is an attractor and one is a repeller)
-3/4 → -1/2 is the attractor and 3/2 is the repeller
< 3/2 → -1/2; more than 3/2 → + ∞

This simple chaotic equation illustrates a chaotic system and the presence of chaotic attractors and repellers.

Some examples of chaotic functioning in the human body include the beating of the heart and the functioning of the brain. Now let's look at how chaos is an evolutionarily beneficial and indispensable aspect of brain functioning.

Brain Functioning
The brain is a chaotic system and is only describable by non-linear mathematics. The brain as an internally generating information processing system is in a class of its own. Research over the past two decades has indicated that the brain is a chaotic system on the microscopic level of neurons and most likely is chaotic on the macroscopic level (Skarda and Feeman 280) As defined by chaos theory the small initial uncertainties are amplified over time which makes it impossible to predict long term outcomes, (Chaos and the New Science of the Brain, 281) "What we mean by a chaotic process is a strictly deterministic systematically generated process, which provides a seemingly irregular pattern that is not predictable at long times." (Disorder, 37) Brain functioning is nonlinear, and although it appears random it is not stochiastic (truly random). "The brain's unusual behavior has only been logically explained by non-linear mathematics" (Ives, 3)

Although it is difficult to definitively establish that any process is truly chaotic, there is good evidence to suggest that brain functioning is not only non-linear but that it is in fact chaotic on both the microscopic and macroscopic level.

Thinking is a creative process that involves the creation of strange attractors for different mental states. These attractors are associated with learning. It is the process of reconstructing these strange attractors that enables scientists to better understand the thinking process. Strange attractors exist in brain EEG's and are the result of an active process to create new brain patterns. Different states in the brain result in different strange attractors. "Brain activity is chaotic and unpredictable yet has a hidden order that, when plotted as a phase-space diagram, is attracted to a certain region. There are numerous fractal strange attractors in the brain that change as thinking processes vary. Paul Rapp, a neuroschientist for the Medical College of Pennsylvania comments on the discovery: 'For the first time we are able to see changes in the geometry of EEG activity that occur as the result of human cognitive activity'…These attractors in the brain are thought to play a key role that distinguishes the brain from all other computing machines: the ability to actively create." (Ives, 3) Chaotic attractors are present in the brain and have been demonstrated in research of the olfactory sense of rabbits. "These patterns of globally distributed activity, one for each discriminated odor, have been mathematically expressed as a collection of chaotic attractors. These are patterns that are sent out of the bulb to the cortex and that we suggest are behaviorally relevant for the correlations that are usually associated with learning and memory." (Skarda and Feeman, 278) The information that is collected must be graphed to reconstruct the attractors. Brains use chaotic dynamics and are nonlinear. New methods such as reconstructing attractors is the next step towards greater understanding of the brains processing.(Chaos and the New Science of the Brain, 281) The reconstruction of phase-space diagrams is possible because these diagrams exhibit a lower order fractal dimensionality, which means that they appear less variable than a random process.

History of Reductivist Linear Explanation of Brain Functioning

Prior to the presentation of the brain as a chaotic system, the prevelant explanation of brain functioning was a reductivist theory called the neuron doctrine which can be explained with linear mathematics. The theory of the brain as a chaotic system disputes the reductivist neuron doctrine, which states that "the physiological basis for behavior can be found at the level of individual neurons whose activity is triggered by a stimulus (Freeman, "Chaos in the CNS"). This explanation assumes that the brain can be explained by the properties of its parts and takes no active role in the thinking process, but only reacts to environmental stimuli and it is possible to describe this passive, mechanistic brain with linear mathematics. (Ives, 3) However, support of chaos in brain functioning leads to a rejection of the reductionist explanation because self-organization and chaotic dynamics are essential to brain function. (Skarda and Feeman, 275) The reductionist explanation purported a passive reaction to stimulus. It has been realized that brain functioning is active and selective and can not be accounted for by the reductivist model. (Skarda and Feeman, 276)

"We have found that brain function cannot be explained in terms of features of neurons taken individually or as part of a local network, nor is it adequately characterized as a passive reaction to stimuli. And while neural network theorists use nonlinear dynamics in modeling their networks and recognize that self-organization plays a role in the brain, they have yet to realize the radical implications of the concept of self-organization both with respect to their explanatory models and for the practice of neuroscience." (Skarda and Freeman, 277) Self-organization means internally generated. The self-organization in brain functioning can not be accounted for by the reductivist model.

Brain Functioning as Chaotic System

Chaos theory is essential to creative brain processes that result in learning. "A theory exists that learning takes place when a new stimulus leads to the emergence of an unpatterned, increasingly chaotic state in the brain. This activity causes a bifurcation that provides the substrate for a new nerve cell assembly and a new strange attractor" (Freeman 4). While more research needs to be done, it is clear that the brain's ability to generate new information internally is critical to our creative processes. It is likely that chaos is essential to that ability. (Ives,4) A bifurcation is a splitting of the possible outcomes in a chaotic system and is the mechanism by which different learning patterns are established.

Benefits of Chaotic System

There are many evolutionary benefits of a chaotic process for the brain. Chaos allows for the necessary consistent stimulation of all brain cells which was previously thought of as noise and it increases the speed of transmission. Neurons must be activated regularly or they die. Random firing of inactive neurons provides a suitable mechanism for maintaining neuron health. Therefore, "chaos is essential to normal brain functioning. What was previously dismissed as random noise is biologically relevant. Background noise in the brain is steady, stable, and controllable—but not absolutely so. Like the erratic beating of the heart, electromagnetic impulses in the brain are still chaotically random. Chaotic activity in the brain enables for rapid state transitions. Such transitions are essential for processing information." (Ives, 4) The importance of chaos has gradually been recognized. It was previously thought that chaos was undesirable. However, new data suggests the opposite view. Deterministic chaos is essential to normal brain functioning at many levels of activity. "What we previously dismissed as 'noise' in the system, something to be eliminated with filters… now appears to be the behaviorally relevant signal." (Skarda and Feeman, 279) Chaos is ubiquitous in neural functioning and provides many benefits. "We have suggested that the chaotic basal activity of the background state provides a suitable biological mechanism for this; moreover, one that is reliable because it is independent of stimulus input. The brain is built to ensure its own steady and controllable source of noise this is quite stable, but not absolutely so. We have also suggested that chaotic activity enables the rapid state transitions essential for information processing. Without this ability, the brain could not quickly concern itself with a new task. Thus we can thank chaos for the rapid transitions between perceptual states. Without it, perception would be agonizingly slow." (Skarda and Feeman, 280) There are many benefits to chaotic processes including the necessary regular stimulation of neurons and increase in the speed of transmission of brain signals, as the result of chaotic "noise" enabling and facilitating rapid state transitions because excitation processes are outbalanced by inhibition processes. Aside from the creative processes, the benefits of chaotic activity in the brain are similar to those of the heart in that chaos allows for rapid change of signals and prevents entrainment, the regular pattern of neurons in sync that results in a stroke.

Necessity of Chaos for Generation of Information

Chaos is necessary for the generation of information. It is through the use of chaos that brains are able to determine signal from noise. "Chaos is generated by a simple process, governed by a relatively small and controllable set of parameters. Noise is inevitable, but chaos is not." (<u>Disorder</u>, 45) Chaos helps brains to generate information. "Chaos has a role to play that sets brains apart from all other information processing systems. Chaos is not just an inevitable consequence of a highly interconnected complex system, it is essential for the creation of information. The brain, unlike machine systems, is selective, i.e. it does not process whatever information is received at the receptor level...receptor level activity only leads to the formation of a nerve cell assembly and bifurcation to the global activity pattern when the stimuli are reinforced and the animal is 'motivated'. This selection of the relevant information is not imposed on the system from the outside as is the case in a machine systems which use periodic or steady state dynamics and require filters designed by their creator to define in advance what is signal and what is noise. Brains have to accomplish this task themselves in the face of infinite environmental complexity." (Skarda and Feeman, 280) The essential mechanism of chaos that allows for signal selection is chaotic bifurcation. "A self-organized chaotic generator responds to environmental input by replacing it with an internally generated chaotic activity pattern. These self-organized chaotic activity patterns are transmitted further into the brain and provide the basis for future selectivity by (1) causing changes that mediate motivation, reinforcement and learning, and (2) modifying receptor input by causing direct environmental manipulation by the organism or by changing receptor positioning with respect to the world. The brain determines which input it will admit and what spatiotemporal form the resulting neural activity will assume. We suggest, therefore, that chaos is essential for input selection, processing and the creation of information in the brain." (Skarda and Feeman, 280) Motivation is a key aspect to creating the state in which the brain is prepared to receive or generate signals. "Motivation involves the creation of a self-organized internal state that destabilizes the system so that it becomes ready to respond to a specific class of stimulus input within a given sensory modality." (Skarda and Feeman, 279) Chaos is required for the brain to function properly.

Chaos Prevents Epileptic Seizure

Chaos is also invaluable as an evolutionary process because it prevents the synchronization of neuron signals that results in epilepsy. "Chaos is the mechanism whereby potentially fatal, and hence undesirable, periodic cortical behaviors are desynchronized." (Skarda and Feeman, 280) It turns out that non-linear processes are perfectly suited to brain functioning and that in fact too much order is the source of disease. Epilepsy is a perfect example of the need for chaos in brain functioning because epilepsy is the result of a periodic rhythm in the brain. The chaotic process surprisingly helps to sustain balance in the brain. "A linear process, given a slight nudge, tends to remain off track. A nonlinear process, given the same nudge, returns to its starting point. For this reason, nonlinear systems can withstand small jolts and operate over a broad range of environmental conditions. Systems disordered in the right way are highly adaptable and poised at a critical point, ready to react swiftly to any change (Ward 146). From seizures to leukemia, disease is finally being recognized for what it is: an acute attack of order." (Ives,5) When brain waves become periodic the result is a grand mal seizure. Epilepsy is the condition in which people experience regular seizures. Epilepsy is a clear indication that ordered brain functioning results in seizures and that it is probably a chaotic system. "Although an epileptic seizure occurs when spatiotemporal chaos in the brain fails, the seizure represents a mechanism for returning brain dynamics to a more normal (chaotic) state." (Sackellares et al, "Epilepsy – When Chaos Fails", Workshops on Chaos in Brain, 113) A seizure is a chaotic mechanism to return the system to its chaotic attractor.

Early Detection of Seizures

This is an explanation of our remotely powered RF Neural Stimulator and the early detection would allow the administration of an impulse only when needed instead of constantly, as is now the case. "Take for example an epileptic seizure. The amount of chaos in the epileptic's brain actually decreases as neurons at the seizure's focus begin to entice other neurons to fire in sync with them. This condition, called a dynamical entrainment, can start days before the attack….Sackallares, along with bioengineer Leonidas Lesemidis, have developed a technique that can identify seizures in the making ninety percent of the time with an average of seventy-five minutes notice. They do this by using dense mathematics to calculate a Lyapunov exponent- a measure of chaos in the brain. Such reliable advance warning of seizures could lead to the development of an implanted chip that would identify when seizure were about to occur and deliver an electrical impulse to return the brain to its normal chaotic state." (Ives,5) This is a possible explanation for how our RF Neural Stimulator actually works in the brain.

Remotely Powered RF (Radio Frequency) Vagus Nerve Stimulator

The RF (Radio Frequency) remotely powered Neural Stimulator that we developed and which is in the process of being patented operates on this premise. The device sends electrical impulses to the brain by stimulating the vagus nerve. These regular impulses, once modulated for pulse width, duration, and amplitude are constantly administered to the brain anywhere from every thirty seconds to every five minutes depending on the patient. This electrical stimulation activates the brain and helps to prevent the periodic, entrained signal that leads to an epileptic seizure, which is a means of returning the brain signals to a chaotic state. It is our hope this summer to pursue the possibility of utilizing the early detection of seizure onset to activate our RF Neural Stimulator so that the electrical impulse could be administered only as needed rather than at regular intervals. Understanding brain functioning will help in this endeavor.

It is generally accepted that the firing of neurons is chaotic and that brain functioning on the microscopic level of neurons is non-linear. There is evidence that neuron firing in brain functioning is chaotic. "This opens up the possibility that the timing of the switching between channel states arises from deterministic forces within the channel molecule rather than from its random thermal fluctuations." (Disorder, 97) It has been demonstrated that neuronal firing is chaotic. "The current knowledge about intrinsic membrane oscillations shows clearly that individual neurons may display different oscillatory states depending on initial conditions and generally behave as non-linear oscillators…the analysis of complex non-linear dynamics has shown some evidence that the underlying neuronal networks may possess multiple types of attractors including the possibility of exhibiting chaotic dynamics." (Silva et al, "Is Nonlinearity Evident in Time Series of Brain Electrical Activity", Workshops on Chaos in Brain, 69) The brain functions chaotically on the microscopic scale of neurons and appears to be chaotic on the macroscopic level of brain functioning.

Difficulty of Definitively Establishing Chaos

However, it is difficult to establish that a seemingly random process is chaotic, and there is some contention as to whether brain functioning is a true chaotic process. Brain functioning involves an incredibly complex system and it is difficult to establish that the system is not just non-linear but that it is chaotic. "[Schrieber] will show some cases of significant nonlinearity in time series none of which, however, shows deterministic chaos in any meaningful sense. The important question whether the observed irregularity is due to an intrinsic instability, the large number or neurons, or noise, may not be decidable based on time series data alone." (Thomas Schrieber, "Rhythms of the Brain: Between Randomness and Determinism",Workshops on Chaos in Brain, 69) This is indicative of the difficulty of definitively establishing chaotic processes. "Such systems that are deterministic, but whose output is so complex that it mimics random behavior, are now known by the jargon word 'chaos'. Chaos is a poor choice of words for this phenomena. In common usage chaos means disordered. Here, chaos means just the opposite, a highly ordered and deterministic system, but its output is so complex that it mimics random behavior" (Disorder, 95) As a result of the strict definition of chaos being a deterministic set of simple equations that generate seemingly random results, chaotic systems are very difficult to prove in reverse.

Fractal Dimension of Brain Function

Fractals dimensions and chaos, although somewhat contended on the macroscopic level of brain functioning are a useful tool for analyzing the brain's functioning. "We have shown that fractal dimension is a useful measure for the description of the brain complexity of brain dynamics in different tasks… fractal dimension is applicable for the distinction of brain dynamics, which result from different visual stimuli." (Preibl and Lutzenberger, Workshops on Chaos in Brain, 206) Ion channel kinetics, the structural mechanism of brain functioning is fractal because they demonstrate self-similarity and a scaling relationship. Self similarity means similar at different magnifications and the scaling relationship indicates that "the length depends on the resolution used to measure it." (Disorder, 88) In addition, the Lyapunoz exponent, the measure of the degree of chaos in the brain can be calculated. (Celletti et al, Workshops on Chaos in Brain, 213) Even though it is very difficult to prove that brain functioning is chaotic, there is scientific evidence to demonstrate that it is in fact chaotic. "There are experimental and theoretical findings supporting the idea that deterministic chaos can play an important role on different levels of neuronal systems." (Ladislav Audrey, "Analytical Proof of Chaos in Single Neurons and Consequences", Workshops on Chaos in Brain, 247). Brain functioning is most likely the result of a deterministic chaotic process. One way to demonstrate this is by showing that the fractal dimension of the phase space is a lower dimension than if the process was random. "One way to differentiate random data from deterministic, chaotic data is … each time series is used to generate an object. This is called an embedding. The objects here are constructed from plotting points whose x-coordinate is one value of the time series and whose y-coordinate is the next value of the time series. This is called a phase space. The topological properties of the object generated from a random process are different from that generated by a deterministic chaotic process… the object generated by the random process fills the 2-dimensional plot and its dimension is therefore equal to 2. The object generated by the deterministic chaotic process does not fill the 2-dimenstional plot. It is a 1-dimenstional object." (Disorder, 95) This lower order dimensionality is indicative of a less variable process and separates deterministic chaos from a totally random, higher dimensional phase space. "The number of independent variables needed to generate the time series is equal to the smallest integer greater than or equal to the dimensionality of the object in the phase space constructed from the time series." Whereas random processes fill n-dimensions, chaotic systems can be defined with less variables than random processes. Therefore, keen observation can separate chaotic process from random process because they are more ordered. The key is to be able to identify these lower dimensional systems. "The concept of chaos now opens the possibility that seemingly complex biological data may actually arise from much simpler, nonlinear mechanisms." (Disorder 96) It has been demonstrated that brain functioning is non-linear and compelling evidence that it is generated by deterministic chaos is gradually being accumulated.

Perception

Perception is a higher level of brain functioning and is very difficult to explain in physical terms. It has been demonstrated that brain functioning is chaotic, non-linear, active and selective and must be understood holistically as macroscopic ordering. There are many implications of the new chaotic model for understanding the nature of perceptual processing in the brain. "The conception that perception is caused by the stimulus or can be explained as the sum of responses to stimuli is not adequate. "The perceptual processing is not a passive process of reaction, like a reflex, in which whatever hits the receptors is registered inside the brain. Perception does not begin with causal impact on receptors; it begins within the organism with internally generated (self-organized) neural activity that, by re-afference, lays the ground for processing of future receptor input. In the absence of such activity, receptor stimulation does not lead to any observable changes in the cortex. It is the brain itself that creates the conditions for perceptual processing by generating activity patterns that determine what receptor activity will be accepted and processed. Perception is a self-organized dynamic process of interchange inaugurated by the brain in which the brain fails to respond to irrelevant input, opens itself to the input it accepts, reorganizes itself, and then reaches out to change its input. We suggest that the self-organized process that replaces environmental input with an internally generated, chaotic activity pattern is one the gives 'biological meaning' to the stimulus." (Skarda and Feeman, 279) The brain functions by motivation which enables it to select meaningful signals. Perception is an active process in which "perception does not just 'copy' objects, it creates their meaning for the organism: (the) function of the organism in receiving stimuli is, so to speak, to 'conceive' a certain form of excitation'[26]" (Skarda and Feeman, 279) Consequently, the brain must be activated in order to receive a signal and must be understood as an extremely complex nonlinear dynamical system which demonstrates self-organizing phenomena. (Skarda and Feeman, 281) Physiologists cannot make exact causal inferences from the level of individual neurons to that of mass neural actions, nor from the level of the activity of receptors to the internal dynamics. "The causal connection between past and future is cut" (Skarda and Feeman, 281) Perception is not passive, but instead is self-organized and cannot be understood from a reductionist point of view. The brain, in fact, creates the conditions for perception processing and this process requires deterministic chaos in order to function. It is difficult to definitively prove the existence of deterministic chaos, but it may be the existence of chaotic processes in your brain that are enabling you to read this, process it, remember it and form a perception of it. Without the evolutionarily beneficial processes of chaos in brain functioning consciousness might not even exist!

School Demographics

I teach at a comprehensive public high school in Pittsburgh, Pennsylvania. Peabody High School is located in the east end neighborhood of the city and serves six surrounding neighborhoods. The student body is predominantly African American (95% African American, 4% Caucasian, 1% other) with an enrollment of 660 students. Ninety per cent of the students receive free or reduced lunches. Information is based on the low socio-economic community in which the students live. Our school has been identified as an at risk high school. This means that our freshman class does not meet the state standards in reading and or mathematics. The benchmark year for testing in the state of Pennsylvania is during the junior year. The large majority of students have failed the PSSA tests in math, reading, and writing, and most scored below basic. At PHS tracking is implemented for all academic subjects; therefore, I teach two levels of physics. One course is designated for students who meet the state's gifted standards. The others are for regular education students. The students in my gifted class score significantly higher on the PSSA but some still failed. My honors students (3.0 GPA) in physics are above average for the school but likely failed the Pennsylvania System of State Assessment (PSSA) in math, reading and writing. My 9th grade general science students are representative of the school population.

Objectives

The students will get a basic understanding of chaos theory and fractals and be able to incorporate that knowledge into a greater understanding of physical science (S2). Students will improve their capacity for making discerning judgments about issues related to science and will be able to appreciate the scientific method as a valuable and useful approach to decision making. The students will be able to use their observations to evaluate the veracity of knowledge and be able to identify issues that are still unknown and form evaluative theories (S1, S5). The students will be able to understand the evolving nature of science and the method that scientist use to make truth claims (S7). The students will appreciate the role that the advances in technology have played in the advances in brain functioning and be able to access internet resources (S9). The students will acquire the terms and language to discuss their elevated ideas regarding fractal, chaos and brain functioning (C5). The students will be able to communicate their ideas, express their opinions, and defend the relative reliability of different theories. The students will engage in higher level class discussions (C6,C7,C8). The students will improve mathematical skills in understanding formulas, calculating results, representing their data graphically and comprehending the implication of mathematical expressions (M1, M2, M4, M5,M6).

Strategies

This unit is based on a constructivist, student-centered, hands-on approach in which students are encouraged to evaluate and create models that incorporate their understanding of fractals, chaos and brain functioning. The students are asked to make qualitative and quantitative observations whenever possible and perform labs. The goal is to develop enough scientific knowledge that the students are able to explain the main topics about what science currently knows about fractals, chaos and brain functioning and to be able to make a discerning evaluation of scientific claims.

The knowledge of chaos, brain functioning, and science in general, is evolving and the intent of this unit is to engage students in a scientific process to be able to appreciate how science knows what it claims to know. Partly, this will be achieved by guided questioning. It is my belief that instilling an enthusiasm for science through exciting hands on activities and relevant and challenging intellectual stimulation will produce more informed citizens who appreciate the utility of the scientific method. Is chaos a valid description of the processes of brain functioning? That is the question that we will attempt to solve. While doing this we will improve our mathematical skills that enable us to define chaotic systems and our intellectual capacities that enable us to evaluate these theories.

Classroom Activities

Lesson I- Fractals and the Cantor Set (3 class periods)

Infinity

A good illustration of this is a mathematical concept known as the Cantor Set.

The Cantor Set is simply described but has some very strange mathematical properties related to length, infinity and dimensionality. The Cantor Set is defined by the line segment from 0 to 1 with the middle third repeatedly removed, infinitely. So let's consider what is left:

E_0 - whole set 0 1 [0,1]

E_1 (mid 1/3s removed) 0 1/3 2/3 1 [0,1/3] [2/3,1]

E_2 (mid 1/3s removed) 0 1/9 2/9 1/3 2/3 7/9 8/9 1 [0,1/9] [2/9,1/3] [2/3,7/9] [8/9,1]

… continued forever _ _ _ _ _ _ _ _

So, How "big" is the Cantor Set?

Infinity is a wild concept. There are different kinds of infinity, which is demonstrated by the Cantor Set.

Now let's consider the strange concept of infinity. Is there such a thing as infinitely small? Maybe not in reality because space may have a limit. Is there such a thing as infinitely large? Maybe not. The universe is probably infinite but maybe it isn't. This is true in the physical world, but in the math world there are no limits and these terms have definable meanings. So what do we mean by infinity? Oddly, mathematicians claim that there are an infinity of odd integers because there are the same number of odd integers as all integers. What do mathematicians mean by the same amount? An integer is z = {1,2,3… } An odd integer is given by the formula 2n -1. Any time you have an integer you can make it odd, z = {1,3,5…}. So equal infinities mean a 1:1 correspondence between z and odd z. Both are infinite of equal size because there is a 1:1 correspondence. This is called countable infinity. The mapping of a 1:1 correspondence is called cardinality. So are there bigger sets of infinity? Let's look at rational numbers, which include fractions, etc. p/q. How many rational numbers are there? More, less of the same? It can't be less because there are at least as many as the integers. However, it turns out, again oddly, that there are exactly the same number because again there is a one to one correspondence. The rational numbers, therefore, have the same cardinality as the integers and are said to be denumerable. Denumerable means countably infinite. An example of this is knows as the Hilbert Hotel. There are a denumerable number of rooms in an infinite hotel. All the rooms are full but you get another guest. By having everyone move one room none has to move an infinite distance and you can always add more. Now let's consider the irrational numbers. This is the rational numbers such as $\sqrt{2}$ which can not be written as a fraction. So the claim is that there is a collection of rational and irrational numbers that is BIGGER than the set of counting numbers! So, a 1:1 correspondence does not exist. This is called non-denumerable. The set [0,1] is non-denumerable or infinite. But there are the same number on the whole number lines as on [0,1]! Infinity is very strange and tricky and is very important for fractals.

Lesson II- Dimensionality (1 class period)

Now let's reconsider the cantor set in terms of its dimensionality. Dimension means its capacity and we will find that the dimension of an object can be a fraction and consequently resulted in the term fractal. The formula for the dimension is:

$$D = \lim_{h \to 0} \frac{\log N(h)}{\log(1/h)}$$

h = length of the ruler; N(h) = minimal # of rulers of length h required to cover object

$C_0 =$ [0,1] $h=1/3^0 = 1$ N(h) = 1 2^0
$C_1 =$ [0,1] $h=1/3^1 = 1/3$ N(h) = 2 2^1
$C_2 =$ [0,1] $h=1/3^2 = 1/9$ N(h) = 4 2^2
$C_3 =$ [0,1] $h=1/3^3 = 1/27$ N(h) = 8 2^3
$C_n =$ [0,1] $h=1/3^n = 1/3^n$ N(h) = 2^n 2^n

$$D = \lim_{h \to 0} \frac{\log N(h)}{\log(1/h)} = \frac{\log 2^n}{\log 3^n} = \frac{n \log 2}{n \log 3} = \frac{\log 2}{\log 3} = 0.63892$$

This is a fractal!

Lesson III- Chaos (2 class periods)

Calculating a Chaotic Formula

Another example of a chaotic system is the equation $f(x) = x^2 + C$;where f(x) becomes x_0 the seed value.

x_0 ,
$x_1 = f(x_0)$
$x_2 = f(x_1)$
$x_3 = f(x_2)$
$x_n = f(x_{n-1})$

A fixed point is a point that doesn't change; $f(x_p) = x_p$
$x_p^2 + C = x_p$ (quadratic solutions)
$x_p^2 - x_p + C = 0$

(if solutions exist we want real numbers, so (1 -4C) must be positive

$x_p = \frac{1 \pm \sqrt{1-4c}}{2}$ or zero. So c≤ ¼ is an important place!
If c ›1/4 it won't generate a fixed point.

Using mathematica program or filling in values in a TI-84 calculator:
C > ¼ → ∞
C = ¼ → ½
C = -3/4 → -1/2, 3/2 (One is an attractor and one is a repeller)
-3/4 → -1/2 is the attractor and 3/2 is the repeller
< 3/2 → -1/2; more than 3/2 → + ∞

This simple chaotic equation illustrates a chaotic system and the presence of chaotic attractors and repellers.

If there is a desire to demonstrate another chaotic system, the logistic mapping equation F(x)=rx(1-x) can be used. There is a fixed point at 0 and x=1-1/r. This equation results in a downward parabola with two fixed points intersecting the line y=x. This indicates when the value for x will return itself! When r=4 there is an onset of chaotic behavior!

Lesson IV – The Chaos of Brain Functioning (3 class periods)

Describe linear processes (1/2 class period)

Describe non-linear processes (1/2 class period)

Explain how brain functioning is a non-linear process. What does it mean to be a deterministic chaotic process?
(1 class period)

Divide the class into two groups and debate the topic, Is brain functioning a chaotic process? (1 class period)

Annotated Bibliography:

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[Explains the history and concepts of fractals and chaos]

Illustrations

http//:hypertextbook.com/chaos/

Appendix-Content Standards

PENNSYLVANIA SCIENCE STANDARDS
S1. All students explain how scientific principles of chemical, physical, and biological phenomenon have developed and relate them to real-world situations.
S2. All students demonstrate knowledge of basic concepts and principles of physical, chemical, biological and earth sciences.
S5. All students construct and evaluate scientific and technological systems using models to explain or predict results.
S7. All students evaluate advantages, disadvantages and ethical implications associated with the impact of science and technology on current and future life.
S9. All students demonstrate basic computer literacy, including word processing, software applications, and the ability to access the global information infrastructure, using current technology.

PENNSYLVANIA MATH STANDARDS
M1. All students use numbers, number systems, and equivalent forms (including numbers, words, objects and graphics) to represent theoretical and practical situations.
M2. All students compute, measure, and estimate to solve theoretical and practical problems, using appropriate tools, including modern technology such as calculators and computers.
M4. All students formulate and solve problems and communicate the mathematical processes used and the reasons for using them.
M5. All students understand and apply basic concepts of algebra, geometry, probability and statistics to solve theoretical and practical problems.
M6. All students evaluate, infer, and draw appropriate conclusions from charts, tables and graphs, showing the relationships between data and real-world situation.

PENNSYLVANIA COMMUNICATION STANDARDS

C5 All students analyze and make critical judgments about all forms of communication, separating fact from opinion, recognizing propaganda, stereotypes, bias and recognizing inconsistencies and judging the validity of evidence.
C6 All students exchange information orally, including understanding and giving spoken instructions, asking and answering questions appropriately, and promoting effective group communications.
C7 All students listen to and understand complex oral messages and identify the purpose, structure, and use.
C8 All students compose and make oral presentations for each academic area of study that are designed to inform, persuade, and describe.