

Fractals and Chaos an Introductory Unit

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Overview

Students will begin the curriculum unit by compiling a working definition of the terms chaos and fractals. The instructor will guide the students through a brainstorming session that will be conducted within small groups. Students will be required to research mathematicians that have contributed to the subjects of fractals and chaos. This unit centers primarily around fractals with a small emphasis on chaos theory.

This curriculum unit is intended for Biology students, grades 9-12. Students will then proceed with their study of chaos and fractality with an introduction to the various terms that they will encounter. Various activities will be used throughout this curriculum unit in order to illustrate and create fractals. The Sierpinski triangle will be constructed using construction paper. Each small group will have to create a fractal, using the Sierpinski triangle. The instructor will set a goal for the number of iterations required by the students. Students will then be required to construct a fractal by folding paper in a prescribed manner. Each time the students fold their paper they will have to draw the resulting shape.

Students will then be introduced to the Mandelbrot set. The students will then be allowed to connect to the internet and go to various web sites which contain information on the Mandelbrot set. The Mandelbrot set can be accessed online and the resolution can be increased in order to see the degree of complexity and the patterns that emerge as you get closer to the image.

Fractality will then be extended to the recognition of fractals in nature. Fractals can be found within most systems. Fractals can be seen in biotic and abiotic factors. Many organisms such as fish exhibit fractality. In the case of

fish, fractality can be found within scales as a whole. If each scale is examined more closely, fractal patterns can be detected as well. Students will examine a few examples of fractality that are found within nature. The characteristics of fractality and chaos will be reviewed. The students will then be required to identify several examples of fractality that can be observed within living systems. Once again the working definitions of chaos and fractality will be revisited and revised. Students will wrap up their study of fractality researching fractal patterns that exist within the African culture. Students may pick any aspect of African culture in which to research. Several topics can be researched with the use of several web sites that are listed in the World Wide Web listing at the end of this curriculum unit.

Throughout this curriculum unit mathematics will be stressed which will include but not be limited to geometry, volumes, pattern recognition, logarithmic functions, geometric growth (as seen in bacterial growth), and a discussion on the concept of infinity. Hopefully through the use of this curriculum unit, students will be able to gain an appreciation for fractality and chaos. Students that are able to find understanding within an area, which at first glance seems abstract and impossible to fathom, will hopefully gain confidence to explore other areas of math and science.

Rationale

It is my intention that this unit will spark the interest of the student when it comes to biology and the study of fractals. It is no great secret that when the term mathematics is mentioned students tend to cringe. By using hands on activities students will be able to become active learners and apply the mathematics included within this unit. The activities within the unit are fun and interesting and they lend themselves to open-ended discussion, which is one of the cornerstones to inquiry-based teaching techniques. They also make good topics for essay writing. All educators know how important it is to get our students to become more engaged in the writing process. This unit will allow those students to do just that.

The adoption of math anchors in the district has challenged science teachers to include math related subjects in their discipline. This unit allows the biology teacher to draw from the study of fractals and chaos in order to introduce mathematical concepts such as pattern recognition and the application of formulas to systems that exist within nature. By recognizing patterns that are common within systems students will gain insight on how problem solving applications can be applied and utilized on other systems. Upper level cognitive reasoning will give students skills that are greatly needed in order to succeed in the academic arena.

Students will not only get to create formulas but they will get a chance to see them in action, by use of the activities such as the Sierpinski triangle and the paper folding exercise. Students' interests tend to be sparked when creativity goes along with certain activities. Students will be able to channel their creativity into the fractals they create and share them with fellow students.

Within the unit a timeline and a study of the history of fractality and chaos is included. This section is intended to give students an overview of how a concept comes into being and how it is developed throughout its life. I believe it is important for students to see how science works as a whole in order to make it more effective as a learning tool.

Students will also get a chance to examine a fish otolith for fractal patterns. This activity will help them gain skills on the microscope as well as dissection skills that have been left out of the curriculum. Students will gain an understanding of the structures found within their own bodies. Students will begin to recognize the intricacies that exist within our framework and hopefully gain a better appreciation for themselves as a working physical system.

Students will also gain skills using technology such as the computer. This unit will also teach students to research materials online as well as from library sources. The use of correct documentation and the ability to choose reliable websites will also be examined by the students using this unit.

Within the past few years there has been growing movement toward the adoption of inquiry based teaching techniques. This movement was sparked by the completion of the TIMSS study. The TIMSS study includes data from countries all around the world. It was found that students tend to grasp subject matter with greater ease when taught using inquiry-based techniques. As an attempt to stay current with contemporary educational practices, the Pittsburgh Board of Education has adopted an inquiry-based text for the district's Biology program.

Students include a wide range of learners from 9th to 12th grade. This curriculum unit can be taught to mainstream students as well as Scholars students. The district's Biology text, Biology a Human Approach, attempts to draw the student's imagination into the subject through discovery and open-ended questions. The text tends to center around the human body and the systems within it in order to apply those principles to systems outside the body. The current text does not include a section that deals with chaos and fractals. The idea of chaos and fractality lends itself to the biology curriculum while recognizing patterns and mathematics that exist within the natural world. Hopefully this curriculum unit

will get students started into thinking about this important area of science. I also hope this unit will give students an understanding of how different disciplines of science can intermingle. This curriculum unit will enable the teacher to incorporate mathematics that satisfies the standards and math anchors that the district strives to meet within the science curriculum.

What are fractals? What is chaos?

When beginning a discussion on fractals and chaos many questions come to mind. Can each of these subjects stand alone or is there a relationship between chaos and fractals. The latter of the two questions will provide a better basis for how these terms relate. In the book, Does God Play Dice, Ian Stewart comments that the study of chaos and fractals both deal with the structure of irregularity. Stewart goes on to say that geometric imagination is paramount when studying fractals and chaos (216). The difference between chaos and fractals is the fact that chaos depends on the dynamics more than the geometric properties of the system, while for fractals the opposite is true (Stewart 216).

Chaos and fractals go hand in hand; the study of one without the other would seem awkward at best. What thoughts are contemplated by a reader who for the first time encounters the term chaos? Does he or she automatically think back to Maxwell Smart and the famous sitcom *Get Smart*? Does the reader imagine disorder, confusion or even anarchy? These and hundreds of other potential questions may cross through the mind of the reader. Chaos describes a dynamic system that demonstrates sensitivity to initial conditions. As a result of sensitivity to initial conditions, the system may seem random although the system is defined (Wikipedia). Examples found that exhibit chaos could be the solar system, plate tectonics, and population growth. These systems along with countless others have been looked at with more scrutiny since the birth of the chaos theory.

Fractals as seen in many popular publications as well as on display hold deeper mathematical meaning. To the layman the fractal represents a beautiful landscape swirling with color but to the mathematician the fractal represents a set of points each a result of a previous function. Fractals can be seen in many natural systems. Fractals can also be constructed by the student in a controlled environment. The resulting fractals, constructed by students using a simple equation, can be surprisingly complex and interesting. According to Hakim, “fractals are geometric shapes with an infinite number of tinier and tinier repetitions that look the same as the overall shape” (81). Fractals are not only for mathematicians, fractals are enjoyed by many. The coming of the computer age allowed mathematicians to run lengthy programs in short amounts of time. The advent of the computer enabled scientists and mathematicians the luxury to study chaos and fractality in a whole new light. Hand computation was a thing of the

past and mistakes, on the behalf of the computer, are few and far between. Computer programs based on the concept of fractality are used to create computer scenes in movies. These programs can be used to create mountain ranges, grow trees and simulate battle scenes using animation (Hakim 81). Computer programs can also simulate plagues, earthquakes, and other weather phenomenon based on fractal patterns. Although these applications are not perfect due to the fact that fractals are sensitive to initial conditions they lend alternative explanations for these events. The applications of fractals to other facets of life are endless. Fractals can be seen in art, music, and poetry. For this unit the study of fractals in nature will be focused on most intently.

Fractals all have certain characteristics that help classify them as such. Each fractal has a quality called scaling behavior. Scaling behavior refers to the fact that all fractals have the same structure on all scales no matter how small one gets (Stewart 219). Each change in scale is attributed to an individual iteration. An iteration is the process of repeating a procedure over and over again while in turn creating more and more complexity (Stewart 18). Fractals also contain dimensionality which can be explained as the amount of space they take up. It is possible to have a dimensionality greater than one and less than two. Stewart refers to the dimensionality of a jagged line, "Jagged lines are better space fillers than a line which is dimension one but, not as space filling as a two-dimensional surface" (219).

Important contributors to the study of fractals and chaos

When discussing the topics of fractals and chaos many scientists come to mind. It goes unsaid that the process and evolution of mathematics are forever moving forward. All of the mathematicians that have contributed to modern thought most likely have not all been recognized. The works of many were probably never published or even viewed by others, this is truly a shame. As with all academic areas, many names stand out in the forefront, each person contributes a small piece to the overall puzzle. The process of discovery could probably be construed as fractal like in structure. The discovery of basic principles leads to more complex and in depth study as it proceeds. Although for the sake of time, five contributors to the study of fractality and chaos will be recognized for their work.

Georg Ferdinand Ludwig Phillip Cantor

Cantor was born March 3, 1845 in St. Petersburg Russia. Cantor died January 6, 1918 in Halle, Germany at the age of 73. He was born to Georg Waldemar Cantor and Maria Anna Bohm. His father was from Denmark and his mother was Russian, both parents were artistic and this had been shown in Cantor's upbringing. Georg Cantor was an outstanding violinist. The connection of

mathematics and music and their development, within the human mind, has been well documented (J.J.O'Connor, Cantor).

Cantor married on August 9th, 1874 to Vally Guttman, they had six children. Cantor's youngest son had died; this along with the death of his father seemed to cause problems within his life. Many of the relationships he had made within the mathematical world began to sour. Many speculate that Cantor's depression was caused by the deep mathematical problems and his obsession to prove them. Modern day understanding of depression suggests his depression was not caused by his bad relationships or mathematical worries. These aspects of Cantor's life were just magnified by his depression (J.J. O'Connor, Cantor).

Cantor worked extensively on set theory. The Cantor set is an interval of length 1, the middle third of the interval is then removed. The end sections of the interval are left and the process is repeated on these two sections. The Cantor set continues to progress making the interval tinier and tinier. These are all the properties required in order to be considered a fractal (Stewart 121).

Henri Poincare

Henri Poincare was born on April 29, 1854; he died on July 17, 1912. Poincare was called, by author E.T. Bell, "The Last Universalist, a man who made contributions in the fields of algebra, anatomy, topology, and physics" (Chaos). Poincare published many works; he strove to give the public a layman's approach to many scientific principles. Poincare first began working with chaos by entering a contest which the contestants had to show that the solar system was stable according to the laws of Newton. Poincare was not able to complete the task but his work was good enough to win the contest. The contest was judged by Weierstrass who was one of the lecturers that Cantor had studied while in school at the University of Berlin (J.J. O'Connor, Cantor). Poincare was a visionary and began looking at chaos and fractality much the way it is looked at today. He noticed that systems were sensitive to initial conditions Poincare states, "Small differences in initial conditions produce very great ones in the final phenomena", "A small error in the former will produce an enormous error in the latter" (Chaos).

Waclaw Sierpinski

Waclaw Sierpinski was born on March 14, 1882 and died on May 14, 1969 at the age of 87. Sierpinski taught most of his career at the University of Warsaw. Sierpinski's most important mathematical work was in the area of set theory, point set topology, and number theory (Riddle).

The Sierpinski Triangle or Sierpinski Gasket is probably his most famous fractal. The fractal is constructed by starting out with an equilateral triangle. The triangle is then divided into four smaller equilateral triangles. The center triangle is then removed. The remaining three triangles are then divided into four smaller equilateral triangles. The middle triangle is then discarded. This process is repeated and repeated. This procedure tends to mimic the Cantor set; the resulting surfaces make up the fractal.

Helge von Koch

In 1904 Helge Von Koch introduced what is known as the Koch snowflake. The snowflake starts as a triangle. Each side of the triangle is replaced by four segments the middle two which are forming a triangular bump. This is repeated again and again each time increasing by four thirds. The length of the Koch snowflake is infinite while its area is finite (Wikipedia).

Benoit Mandelbrot

The previous four mathematicians certainly laid groundwork for the present day understanding of fractals. Benoit Mandelbrot was largely responsible for the modern day interest in fractals (J.J.O'Connor, Mandelbrot).

Mandelbrot was born on November 20th 1924 in Warsaw Poland. All the articles, I was able to find, list him as alive although the most recent article was updated in 1999. Mandelbrot was introduced into mathematics by his uncles. Mandelbrot was mostly self-taught. The war as well as other factors made it nearly impossible for him to attend college. Mandelbrot continued to study and look at mathematics with a standard geometric approach. Mandelbrot eventually found himself in the United States as a fellow at IBM. IBM allowed Mandelbrot to choose the direction in which he wanted his work to take. This turned out to be a relationship that helped Mandelbrot to grow as a scholar. Mandelbrot worked with computer graphics and even developed some of the first computer programs that helped map the fractals he was working on (J.J. O'Connor, Mandelbrot).

Interestingly enough Mandelbrot was introduced by his uncle to a paper written by Gaston Julia on sets. At first Mandelbrot was not interested, but soon began working with the mathematics introduced by Julia. Gaston Julia was also the student of Henri Poincare (Stewart 234). Mandelbrot began to generate his fractals which can be seen on many different types of media.

Fractals in the human body

Fractals occur throughout natural systems. These can be discovered if a closer examination of a particular system is taken. Fractal patterns can be seen throughout the human body. Fractals are known to have large surface areas due to patterns existing within patterns. The human body and the organs and systems contained within it make use of increased surface area in order to make certain processes such as oxygen exchange more efficient. The lungs with their extensive branching into progressively smaller chambers allow for the maximum amount of oxygen to diffuse for every breath taken (Fractals). The air enters the lungs through the trachea. The trachea then branches into the bronchial tubes. The bronchial tubes then branch into smaller tubes called bronchioles. The bronchioles lead to the alveoli, which are surrounded by a fine capillary network in which the exchange of oxygen and carbon dioxide takes place (Mader 794).

Fractal properties are also evident throughout the circulatory system. The diameters of blood vessels follow a power law distribution. The power law distribution is a major characteristic of fractal systems (Fractals). The circulatory system consists of three types of vessels in which the blood travels the arteries, capillaries, and veins. The blood enters the aorta which is the largest artery in the body. As the blood continues to travel throughout the circulatory system, the diameters of the vessels become smaller and smaller. This decrease in diameter allows for the surface area to be increased thus allowing oxygen to diffuse to all points within the body (Miller 946). The study of medicine has recognized chaotic behavior in heartbeat irregularities. Normal heartbeat is referred to as regular sinus rhythm. When regular sinus rhythm is disrupted, a person will endure a heart attack in which the rhythm behaves in a random manner. The study of fractals and how they develop will hopefully allow scientists to understand the randomness of a heart attack (Fractals).

The digestive system shows fractal properties, the folded structures, within the system, increase the surface area for digestion and absorption. The small intestine is mainly responsible for the absorption of nutrients. The small intestine contains lumen which are folds inside its tubular structure. Each lumen has tiny microscopic extensions called villi. The villi are also subdivided into microvillus. Each villus contains a lacteal which is a structure that absorbs amino acids and sugars. All of these structures as well as the folding of the small intestine increase the surface area. In addition to these structures the other organs of the digestive system contain inner folding (Mader 781). In addition to the systems already mentioned within the human body, the nervous system also works by using extensive branching of nerve patterns in order to relay messages to and from the nerves and brain.

Fractal patterns in plants

Fractal patterns can be seen in branching patterns of leaves and trees. Japanese black pine will branch in a forked pattern at the terminal end of the branch. If the candle or new growth of the branch is sheared a candle will grow from each side of the base of the wound. By trimming the tree in this manner the fractal behavior of the tree can be manipulated into making the tree exhibit tighter growth. This principle is used by arborists in order to increase the attractiveness of ornamental trees. Deciduous trees exhibit similar fractal patterns in the structures of their leaves. The root structures of trees also display fractal properties. Each root tip will continue to split as it encounters small shards of grit within the soil. As the root tip splits it increases the surface area by which nutrients can be taken up by the tree. The bonsai enthusiast exploits this property by trimming the roots in order to increase surface area within a smaller container. The tree can then be potted while retaining the physical characteristics of a mature tree.

The dendritic nature of leaves allows nutrients to gain access to all points on the leaf surface. The vein patterns on leaves can take on many different patterns. In most leaves a midrib runs along the length of the leaf blade and veins emerge along its length. Some leaves have leaflets that emerge separately from the petiole. These leaflets can be in pairs or a group of leaflets may end in a terminal leaflet. The structure of the leaf is further divided into patterns of individual cells (Grimm 8). Similar scaling behavior is seen throughout many types of mosses and liverworts as well.

Fractals exist in the topography of surfaces

Fractals can be seen in the topography of coastlines as well as mountain ranges, river beds, and rock layers. As each part of the former examples is magnified the resulting structures are similar to the original viewpoint. Once again self similarity plays a role in the description of these systems. As the resolution gets greater the length continues to grow larger as well. The dimensionality becomes greater than one due to the twists and turns that are evident with each iteration (Stewart 215).

The structural topography of viral particles is host-specific. Each viral particle has a specific site it must bind to in order to infect its host. The structure of a viral particle consists of a tail piece that often binds to the surface of the host cell as in the case of the T4 bacteriophages. Bacteriophages are viral particles that infect bacteria. Bacteria that have been infected by viral particles can become extremely dangerous toxin producing organisms. The tailpiece often fits onto the host in a lock and key method. The surface of this site exhibits fractal properties (Stewart 222). Fractal properties are also seen in the capsid of the viral particle. The capsid is made up of protein subunits that are unique to individual viruses. The capsid also has binding sites that display fractal properties. These structures

can be seen in surface proteins of the influenza virus (Miller 483). Perhaps with a deeper understanding of fractal surfaces the keys to unlocking the mysteries of disease causing agents may become easier to discover.

Fractal properties of diatoms

Diatoms are members of the phylum Bacillariophyta. They are one of the most plentiful organisms on earth. Diatoms are constructed of silica and have many different patterns etched within their silica shells. Diatoms are autotrophs that contain chlorophyll that is used in photosynthesis. Diatoms are an important source of oxygen for many aquatic species of fish. Diatoms are comprised of two valves or halves. In diatoms, one half of the shell nests inside the other half. When diatoms reproduce asexually a side of the shell is given to the new diatom. The new diatom produces the half that is missing. The new resulting diatom is smaller than the original. Resulting generations get smaller and smaller until the asexual reproductive strategy is interrupted by a sexual phase of reproduction. The patterns seen on the diatom's shell display fractal properties that can be seen with a microscope. Diatoms are often harvested from the ocean and used as diatomaceous earth. Diatomaceous earth is used as a filtering agent due to its fine consistency (Mader 529).

Fractal Patterns are found in fish ears

Fractal patterns are evident in the ear plates of fish. Otoliths are calcium carbonate stones that are surrounded by nerve tissue in the head of a fish. The otolith continues to grow as the fish ages. The deposition of minerals to the otolith allows it to expand as the fish grows larger. Crystalline patterns can be seen under the microscope. The age of the fish can be estimated much like that of a tree. The only difference between the rings in trees and the rings in otoliths is that the rings in otoliths are deposited daily (Barkman 113).

Fractal patterns in crystal structures of minerals

Minerals each have characteristic crystal structures. The crystal structures can be seen by creating slides of a particular sample. A thin slice is taken off a sample the slice is then wet and pressed firmly onto a grinding wheel. Grits of varying coarseness are then poured onto the wheel while it is turning. The sample is then rinsed and the process is repeated until the sample becomes thin enough for light to pass through. The sample can then be examined and identified under the microscope based on the crystalline patterns that are observed. The crystal structures are also characterized by cleavage patterns that are a direct result of the shape of the bonds created within the atomic structure of the mineral. The shape

of crystals, are self similar, as the structure is broken down.

Fractal patterns are found within African culture

Dr. Ron Eglash is the author of the book African Fractals Modern Computing and Indigenous Design. Dr. Eglash's book focuses on the fractal designs that permeate African Culture. Dr. Eglash first noticed fractals in the architectural design of African huts. The thatched roofs of the huts were circular patterns inside larger circular patterns. Dr. Eglash first noticed the patterns when he was looking at aerial photographs of villages (African).

Dr. Eglash began looking at fractals in other areas of African culture. The planning of villages showed self similarity in the way the streets were organized. Streets turned into walks and then into foot paths. Dr. Eglash also noticed fractal patterns in religious practices and traditional artwork. Dr. Eglash noticed that wind screens made by certain tribes exhibited fractal patterns. The wind screens would become more intricate and dense at the top of the screen. The reason was that the wind is more powerful the higher up on the screen. The natives would spend more time on the top of the screen where the protection was more important. The natives not only created a screen that was practical but one that was frugal and attractive (African).

Fractals and cosmology

Many people believe that fractals extend throughout the Universe and matter as we know it. Many questions arise when talking about the Universe. How did the Universe form? How did the various parts of the Universe form? What does the Universe contain, and how are these parts inter-related? Stars and interstellar matter make up galaxies. "Interstellar matter is the dust between stars that provides new material for stars to be formed" (Dobson 528). Galaxies are substructures found within the Universe. D'Amico states, "It is estimated that the Universe contains 100 billion galaxies and that each contains an average of 100 billion stars" (6). Galaxies can be grouped into structures called clusters (Mitton 10).

The structure of matter does not only exhibit fractal properties as it gets larger. Matter exhibits properties that are fractal like, as the point of view becomes closer. Molecules contain atoms and atoms contain subatomic particles. Subatomic particles include neutrons, electrons, and protons. This is the smallest we can go due to the constraints of our technology. According to Rees, "We suspect that the underlying structures in nature may be superstrings or quantum foam on scales so tiny that they would require seventeen more zooms to reveal them" (6). This according to some constitutes a fractal. The defining of a fractal

must be able to be proven by mathematical formulas. The self similarity must be on infinite scales. The Universe does not follow these parameters. The structures in the Universe go as far as clusters. Larger structures have not been seen past these clusters. Some cosmologists contend that the Universe takes on a smoothing effect and that it is relatively the same throughout (Rees 60).

Fractals and patterning

Patterning is the ability to notice patterns within systems. The recognition of patterns is a basic skill that is needed in order to understand mathematics and the world around us. According to Barkman, “Numerical patterns have led to the discovery of pi and to Pascal’s triangle” (10). Fractals have been detected in music, art, and literature. The presence of fractals and self similarity is a recurring theme throughout our world and our cultures. The recognition of patterns is a basic characteristic that must exist in order for us to organize information within our brains. The organization and analysis of patterns is paramount to our ability to learn. The subject of fractals and chaos has been a recent topic for discussion throughout the academic world. The terminology may be new but the actual phenomenon has been around forever.

Through the use of hands on activities and computer software students will gain an understanding of how to identify chaos and fractals. The ability for students to identify these patterns will give them the ability to apply these skills to other situations. The use of fractals in mathematics and culture perhaps will spark creativity in the teaching of these subjects as well as the learning strategies adopted by educators and their students.

The application of mathematical skills such as the ability for students to construct functions and apply them will be strengthened through the study of chaos and fractals. Students while working together and constructing mathematical models will be able to examine the perspective of others. The great mathematicians of our past did not propel themselves to greatness; they rode the coat tails of those before them. Problem solving is a skill that is used by students every day. The student with the greatest ability to problem-solve will clearly become the most successful academically and personally.

Objectives

After completing this unit my students will have accomplished the following objectives.

- 1.) to have a basic understanding of fractals and chaotic systems
- 2.) to have an understanding of the history of the mathematics underlying fractals and chaotic systems

- 3.) to demonstrate how to construct fractals
- 4.) to explain the concept of self similarity
- 5.) to identify fractal patterns in nature
- 6.) to explain the term iteration
- 7.) to describe how resolution is increased and decreased
- 8.) to describe how fractal patterns can be found within culture
- 9.) to describe the viewpoints on fractality in the structure of the Universe
- 10.) to demonstrate the concept of dimensionality
- 11.) to calculate volumes and construct formulas to describe systems

Students will satisfy the standards and math anchors by modeling scientific concepts while integrating mathematics. Students will also be required to make detailed written descriptions of some of the concepts covered within this unit.

Strategies

Students will begin the unit by compiling a working definition of the terms chaos and fractals. The instructor will guide the students through a brainstorming session that is conducted within small groups. Brainstorming will be used throughout the curriculum unit. Brainstorming, guided discussion, and small group work is the cornerstone to inquiry-based courses. A constructivist approach will be taken to the unit. Student misconceptions will be identified and then revisited at the end of the unit. The students will then have an opportunity to review their knowledge base. Students will be required to conduct research on a mathematician. The research will then be put into paragraph form and placed on a time line for the entire class to examine. The use of concept maps will be used in order to develop relationships between the terms that will be introduced within the unit. The concept map will be posted and added to as the unit progresses. Students will be required to construct fractals. Hands on activities will help solidify the written definitions of fractality, iterations, and resolution. Students will get a chance to use the computer to identify fractal patterns and experiment with resolution of the Mandelbrot set. Students will be required to apply their knowledge of fractals and pattern recognition to finding fractals that exist in nature. Students will also be required to find fractality within a cultural setting. Mathematics will be used throughout the unit in order to satisfy the academic standards set forth by the state and local board of education. Students will be assessed in a number of different ways, based on the needs of the instructor. Students will be required to keep all of their work in a journal.

Classroom Activities

Day One

Procedure

Students will begin the lesson with a brainstorming session of the terms chaos and fractals. The teacher will guide but not add to the conclusions of the students. The instructor will also be used in an organizational capacity. Students will work in small groups of three or four. Each group will be required to come up with a list of ideas on the meaning of terms. Sometimes students need to be pulled into the discussion, the following list of questions can help start the activity.

- 1.) What comes to mind when you hear the term chaos?
- 2.) What other words does the term fractal remind you of?
- 3.) What types of patterns do you notice in everyday life?
- 4.) How can mathematics fit into pattern recognition?
- 5.) Does the term chaos necessarily mean disorder?

The instructor will take the lists of facts from the various groups and post them on the board. The lists will then be read and a master list will be constructed. The group will then begin to write operational definitions for the two terms. The definitions will be revisited throughout the unit.

Analysis

Have students write a few sentences on the following questions.

- 1.) What have I learned about the terms chaos and fractals?
- 2.) What purpose would the study of chaos or fractals serve?
- 3.) Is mathematics an invention of man or has it already existed?
- 4.) What would you like to learn about chaos or fractals?

Day Two

Procedure

Students will be taken to the library and given the name of a mathematician that has contributed to the study of chaos and fractals. The students will be required to write a brief paragraph on the person they were assigned. Each description must include a time period when the work was done and the concept that was either discovered or furthered by the individual. Relationships to other scholars may also be included in order to show the fact that science is a discipline of contribution and cooperation. All information must be properly documented.

Have students answer the following questions about the person they have researched.

- 1.) Has the discovery made a major contribution or a minor one?
- 2.) What do you suppose would have happened if the discovery was not made or shared?
- 3.) Does the discovery dovetail with other discoveries or does it stand alone?

To end the lesson students will place their mathematician's information on the time line.

Day Three

Procedure

The instructor will give notes on the various terms that will be used in the unit. Have students construct a concept map. Make sure to have the students place descriptors between the terms that they link. The following questions might be used to guide the students.

- 1.) Where are fractals found?
- 2.) Where would a person begin to look for chaos?
- 3.) Can the study of fractals be used for medical applications?
- 4.) Are fractals seen within nonliving systems?
- 5.) Are fractals found within cultures?

Have the students describe the thought process in the form of a paragraph. This exercise often leads students to the discovery of additional information.

Day Four

Procedure

Students will be required to construct a Sierpinski triangle. The instructor will describe the pattern of the Sierpinski triangle. The instructor will attempt to describe the process of dimensionality as it applies to the triangle.

Students will then construct their Sierpinski triangles according to the instructions of the teacher. Each group of students will consist of two or three students. Each group will be required to construct a triangle with a different number of iterations.

Have students compare triangles that are made. Have students describe how the dimensionality of the triangle changes as the iterations are increased,

Analysis

- 1.) How can the term dimensionality be related to a coastline?
- 2.) How far could you take the number of iterations until it became impossible to physically construct more?
- 3.) How would an increase in surface area be of importance in an organ system?
- 4.) Describe how the surface area of a box of tissues is increased by the way they are packaged.

Day Five

Procedure

Students will construct a fractal out of a strip of paper. Have the students begin with a piece of paper and fold it in half. The first fold is the first iteration. The paper should then be folded in half again. Each consecutive fold will be an iteration. Have the students draw each iteration as they are completed?

Analysis

- 1.) Do you see any patterns in the individual iterations?
- 2.) Can you predict what the next iteration will look like?
- 3.) Construct a function that would be used to describe the fractal geometry that is taking place.
- 4.) What could fractal geometry be used to create?

Day Six

Procedure

Students will be able to go onto the world wide web and examine the Mandelbrot set. Students will be able to increase the resolution of the Mandelbrot set.

Analysis

- 1.) How does the Mandelbrot set change with each increase in resolution?
- 2.) How far does the Mandelbrot set continue to show self similarity?

Day Seven

Students will examine the otoliths found in fish. Each student will get a fresh fish from which to extract the otolith from. The instructor should show the students

where the otolith can be located.

An introduction to fish anatomy may be covered. Have the students draw the otolith as they see it under the microscope. The students should draw the otolith at three different magnifications.

Analysis

- 1.) What was the difference in the three magnifications?
- 2.) How is the otolith fractal like?
- 3.) What other patterns in nature could you identify that are fractals?

Day Eight

Have students meet in the library. Each student will have to find a source of fractals within a culture. The African culture is rich in such examples. Other places to look would be in Indian and Aboriginal cultures. Background information can be given in the form of notes.

Each student should write a five paragraph essay on the information they locate. Essays will be accompanied by a visual of the fractal in question.

Day Nine

Have students revisit their definitions of chaos and fractals. Have students identify any misconceptions they might have had. Students should then construct a new concept map on the material they have studied. The new concept map will be compared to the old one in order to identify differences and similarities.

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Appendix A

Content Standards for the Pittsburgh Public Schools

Science and Technology

- 1.) All students explain how scientific principles of chemical, physical, and biological phenomena have developed and relate them to real-world situations.
- 2.) All students demonstrate knowledge of basic concepts and principles of physical, chemical, biological, and earth sciences.
- 4.) All students explain the relationships among science, technology and society.
- 5.) All students construct and evaluate scientific and technological systems using models to explain or predict results.
- 6.) All students develop and apply skills of observation, data collection, analysis, pattern recognition, prediction and scientific reasoning in designing and conducting experiments and solving technological problems.
- 9.) All students demonstrate basic computer literacy, including word processing, software applications, and the ability to access the global information infrastructure, using current technology.

Reading, Writing, Speaking and Listening

- 1.) All students use effective research and information management skills, including locating primary and secondary sources of information with traditional and emerging library technologies.
- 2.) All students read and use a variety of methods to make sense of various kinds of complex texts.
- 3.) All students respond orally and in writing to information and ideas gained by reading narrative and informational texts and the use of information and ideas to make decisions and solve problems.
- 6.) All students exchange information orally, including understanding and given spoken instructions, asking and answering questions appropriately, and promoting effective group communications.
- 8.) All students compose and make oral presentations for each academic area of study that are designed to persuade, inform, or describe.

Mathematics

- 1.) All students use numbers, number systems, and equivalent forms (including numbers, words, objects, and graphics) to represent theoretical and practical situations.
- 3.) All students apply the concepts of patterns, functions, and relations to solve theoretical and practical problems.
- 4.) All students formulate and solve problems and communicate the mathematical processes used and the reasons for using them.
- 5.) All students understand and apply the basic concepts of algebra, geometry, probability, and statistics to solve theoretical and practical problems