# FRACTALS AND NATURE – IT'S ELEMENTARY

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## OVERVIEW

This unit on Fractals will be used within a Geometry unit, to expand and enhance the concepts that are learned in Basic Geometry. Fractals are the study of the roughness of the world, its transformations and dynamical changes. Fractal Geometry studies the geometry in nature and the changes that take place all around us.

So what is a fractal? What does one look like? Who first coined the term? How can elementary students be taught to understand their significance?
The observation by Mandelbrot of the existence of a "Geometry of Nature" has led us to think in a new scientific way about the edges of clouds, the profiles of the tops of forests on the horizon, and the intricate moving arrangement of the feathers on the wings of a bird as it flies. Geometry is concerned with making our spatial intuitions objective. Classical geometry provides a first approximation to the structure of physical objects? Classical geometry is the language which we use to communicate the designs of technological products, and, very approximately, the forms of natural creations. So, fractal geometry is an extension of classical geometry. It can be used to make precise models of physical structures from ferns to galaxies. Fractal geometry is a new language to explain complex shapes. Applications of fractal geometry extend to biological modeling, physiological modeling, geography, coastlines, images, turbulence, feathers, computer graphics, and ocean spray. Applying it to computer graphics and, in particular, image compression for data transmission and reconstruction are exciting new developments.

Applications of fractal geometry to computer graphics have been investigated by a number of authors including Mandelbrot in 1982, and Oppenheimer in 1986. In all of these cases the focus has been on the modeling of natural objects and scenes. Both deterministic and random geometries have been utilized. The modeling of natural scenes is an important area of computer graphics. Photographs of natural scenes contain redundant information in the form of subtle patterns and variations.
Fractals have applications in medical research. All aspects of nature follow mathematical rules and involve some roughness and a lot of irregularity. For example, complex protein

surfaces fold up and wrinkle around towards three-dimensional space in a dimension that is around 2.4 Antibodies bind to a virus through their compatibility with the specific fractal dimension of the surface of the cell with which they intend to react. As a result, many of the current developments and findings in fractal geometry are in work with surfaces.
The dynamics of the AIDS virus in the human body has been modeled with fractal geometry. It provides the answer to the long-standing puzzle surrounding the unusually long incubation period of the AIDS virus. Many patients remain HIV positive for as long as a decade before the virus decides to kick in. The beginning of the full-blown disease does reveal itself in the body. As the immune system begins to fall apart, the AIDS virus begins to behave chaotically. Studies of the virus at this stage have revealed significant changes in the fractal structure. Fractal geometry unravels the structural differences that occur at the end of the incubation period of AIDS.

Fractal geometry is a relatively new mathematical language. Fractals are very familiar to us because we see them every day, everywhere we look. Therefore it is not surprising that fractal geometry is finding a host of applications in the study and the management of our environment. A prime example is acid rain. Earthquakes and epidemics in the human species show up with a clearly fractal signature. A key factor in understanding and describing nature's phenomena is self-similarity on all scales. Corrosion reveals the fractal nature of the process, suggesting ways and means to alleviate the problem.

Now used in the modeling of crowd flow is fractal geometry. Our behavior, in mass, has turned out to be fractal. A researcher's work has led to some unexpected revelations about human crowd behavior and how to manage crowds. Looking at our behavior in crowds, it resembles flocks of birds or shoals of fish as they move as one unit. Crowds entering, leaving, and moving around inside of a stadium reveal interesting behavioral patterns. And although we are individuals and can make decisions about which direction we should move, how far and how fast, in tight, busy, fluid situations, we actually behave like one organism. In a crowd, our focal point becomes the bubble of our own space and we can detect little beyond that. When crowd-flow computer simulations are run, they produce extraordinarily beautiful patterns. Because of their fractal nature, they have structure on all scales. Again, these patterns appear organic and are very beautiful. Flow simulations often resemble floral patterns. As a result of these studies, stadium design is used from the intricate modeling of ecosystems.

The night sky is one of the most beautiful fractals in nature. There is a star wherever you look in the night sky. Between any two stars there are always other stars in that space. Since luminosity decreases with the square of the distance, and so does apparent size, the total mount of light coming in from any direction in space should be the same. In 1909, Jac Fournier suggested a model to attack this problem of the distribution of matter in the universe. He suggested taking five points, arranged in a square with one in the middle. Replace each point with a called-down copy of the whole pattern. And keep going, replacing each point of this new diagram with a smaller duplicate of the whole shape, and so on. Although highly simplified, it showed that an infinite universe need not be equally bright in all directions, if it has this fractal structure.

Planets clump together to form solar systems, and the stars fit together to form clusters of stars. These clusters of stars come together to form galaxies, and these galaxies hang together to form clusters. These clusters of galaxies go on to make super-clusters. Some of these galactic clusters are composed of vast agglomerations of matter a billion light years across. By using fractals, researchers can now model the evolution and the structure – even the fate – of interstellar dust clouds.

Fractal geometry is being used in the initial detection of the presences of cancer cells in the body. The surface structures of cancer cells are crinkly and wrinkly. These convoluted structures display fractal properties which vary markedly during the different stages of the growth of the cancer cell. Using computers, mathematical pictures can be obtained, which reveal whether or not cells are going cancerous. The computer is able to measure the fractal structure of cells. If cells are too fractal, it is not good. There is something very wrong with those cells. The fractal dimension of cancerous material is higher than that of healthy cells. MRI breast imaging may improve diagnosis for the 4 million women at risk for whom mammography is not effective. Clinical application of MRI has been held back by difficulty in determining which masses are benign and which are malignant. Research has focused on developing robust fractal dimension estimates which will improve discrimination between benign and malignant breast masses in women.

There seems to be two characteristic features.

(1) The presence of complex geometrical structure and distributions of color and brightness at many scales: Natural boundaries and textures are not smoothed out under magnification. They preserve some degree of geometrical complexity.

(2)Natural scenes are organized in hierarchical structures. For example,

- A forest is made of trees
  - A tree is a collection of boughs and limbs along a trunk
    - On each branch there are clusters of leaves
      - A single leaf is filled with veins and covered with fine hairs

It appears many times in a natural scene that a recognizable entity is built up from numerous near repetitions of some smaller structure.

A fractal set usually contains infinitely many points whose organization is so complicated that it is not possible to describe the set by specifying directly where each point in it lies. So instead, the set may be defined by the relations between the pieces. It is like describing the solar system by quoting the law of gravitation and stating the initial conditions. Everything follows from that. It appears always better to describe in term of relationships.

Through interactive, engaging, and sequential activities on fractals, the student will develop a basic understanding of how fractals explain mathematical connections to nature. For some students, the knowledge will plant a seed for future investigations.

**RATIONALE**

I currently teach third and fourth grade mathematics at the Pittsburgh Gifted Center. We get a different group of students each day of the week. So I teach the same basic lesson, each day, for a week. I have them for a 2 ½ hour block of time, so I have the time to delve into a topic. Teaching fractals within a unit on Geometry would make a natural connection for the students. They could use their basic knowledge and understanding of geometry and apply it to a bigger picture. Since they are elementary students, the lessons and activities that I will involve them in will expose my student to the vocabulary of working with fractals, what the latest research says about them, and who the mathematicians are who are working with and studying fractals, and then do some hands-on activities creating fractals. Gifted students already have a strong grasp on the basics, so this unit will take them to higher understanding of geometry in their surroundings, and push them to think new thoughts about the order and chaos in nature.

Why study fractals? Well most of the math you study in school is old knowledge – organized thousands of years ago. Much of fractal geometry is relatively newer. It is currently being done. Modern day mathematicians are carrying out research right now. Engineers have begun designing and constructing fractals in order to solve and explain practical engineering problems. So, they are new, fresh ideas. Students can be taught to understand them and to identify them in their environment. Objects in nature look fractal in nature. There are new concepts connected to fractals. People use fractals to solve real world problems. Presenting the material on fractals using lecture, hands-on lessons, discovery, small group discussions, and having them be able to synthesize the information to novel situations, will address the different learning levels and styles of my students.

Did you ever notice bizarre drawings on chapter heading pages in "Jurassic Park" that look kind of like partial squares? They're labeled First Iteration, Second Iteration, etc., and they get more and more complex with each iteration. They are an example of fractal art.

Exploring patterns in fractal curves aids students in comprehending mathematical concepts, such as infinite curves. Mathematics is frequently described as the science and language of patterns. School mathematics has traditionally emphasized the creation of patterns based on simple rules and the classification of patterns by their fractures. The latest reforms in mathematics education recommend extending the study of patterns to include pattern finding. One such example from the NCTM (National Council of Teachers of Mathematics, 2000), says students should create and use tabular, symbolic, graphical, and verbal representations to analyze and understand patterns. In this article, they describe a technology-rich, pattern finding investigation focused on the area and perimeter of the Koch snowflake curve. As they work through this investigation, students discover that the Koch snowflake curve has two seemingly incompatible properties - the area within the snowflake curve is finite, the perimeter of the curve is finite.

The Koch snowflake is a fractal curve, and fractal curves possess a number of distinctive mathematical features. The most prominent feature is that it is self-similar. Self-similar means that at any magnification the curve has the same form. Construction of the Koch snowflake starts with an equilateral triangle. The middle third of each segment of this triangle is removed, and then an equilateral "bump-out" is inserted, making a six-pointed star with twelve segments. Then the middle third of each of these twelve segments is removed, and a smaller, equilateral bump-out is inserted. This creates a star with forty-eight segments, the next iteration, or repetition, of this process. Applied repeatedly, this process produces a sequence of curves composed of smaller and smaller segments. Since this process conceptually could be repeated an infinite number of times, the relating curve would be fractal.

Much of the research on fractal geometry is relatively new compared to basic geometry. Classic geometry was made and used to define the subject of geometry for millennia to come, laid down by Euclid of Alexandria around 300B.C. The Geometry that Euclid studied, concerning straight lines and circles, proved so successful in explaining the universe that scientists did not see their limitations, especially in explaining shapes in nature. A mountain may be triangular, but it is not a pyramid or a cone; a rock can be circular, but it is not a sphere; lightning does not travel in straight lines. Fractal geometry is an extension of basic geometry. It does not replace basic geometry, but it does enrich and deepen its meaning.

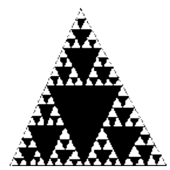
Many occurrences that happen in nature cannot be plotted to form a linear graph. Fractals try to explain the geometry, and the results are seldom an anticipated design, but a wild and crazy pattern of lines and colors. There lies the challenge of dealing with fractals.

The word “fractal” was coined in 1975 by Benoit Mandelbrot, born in 1924 in Poland. He used it to describe shapes which are detailed at all scales. Fractal Geometry is the study of the irregular shapes we find in nature. Generally speaking, fractals are characterized by infinite detail, infinite length, and the absence of smoothness.
When Mandelbrot was young, the cauliflower fascinated him. He observed that when you break off a branch from the cauliflower, the small piece looks just like the whole thing. He could continue to break off smaller and smaller branches of the cauliflower and that, up to a certain point, they continued to look like smaller and smaller versions of the whole vegetable.

These ideas started in the 19$^{th}$ century with discoveries by Karl Weierstrass (1815 – 1897), George Canton (1845 – 1918), and Henri Poincare (1845 – 1912). These ideas led towards the creation of a new kind of geometry. This geometry gave the power to describe aspects of the world inexpressible in the classic geometry of Euclid. Fractal Geometry is very much a geometry of the study of the practical, of the real nuts-and-bolts-world.

Self-similar at all levels, continuing to infinity, was revealed by Mandlebrot, as he studied computer printouts of different problems. In geometry, similar means something very specific. Geometric figures are *similar* if they have the same shape. Corresponding sides are in proportion (and the corresponding angles are also of equal measure), the figures are the same shape and are *similar*. Seen in a different way, for one figure to be similar to another, you must be able to magnify the length of the small figure by the scale factor, and it will become exactly the same size as the larger figure.

The Sierpinski Triangle is self-similar. First you start out with an equilateral triangle. Then look inside at all the equilateral triangles. Then there are infinitely many smaller and smaller triangles inside. All of these are similar to each other and to the original triangle, making it self-similar.



This brought Mandlebrot to grasp the concept and explain mathematically, the way things work in the real world. He sensed the underlying shapes of things in the world – the hidden order.

Warlaw Sierpinski entered the Department of Mathematics and Physics of the University of Warsaw in 1899. He was awarded a gold medal by the university for work in a competition on the theory of numbers. It was scientific work. It was printed in 1907 in the mathematical magazine "*The works of Mathematics and Physics*" published by Samuel Dickstein. Siepinski graduated in 1904 and worked for a while as a school teacher of mathematics and physics in a girls school in Warsaw. He attended lectures on mathematics, studying in addition astronomy and philosophy. He received his doctorate and was appointed to the University of Lvov in 1908. In 1907, Sierpinski first became interested in set theory. It happened when he came across a theorem which stated that points in the plane could be specified with a single coordinate. He began to study set theory and in 1909 he gave the first ever lecture course devoted entirely to set theory. Throughout his life, he received many honors and was awarded several honorary degrees.

Cantor, in 1883, was looking for meaning of continuity. It led him to the set, the Cantor Set, now named after him. If was one of the first fractals to be studied mathematically. Ten years previous, Cantor proved the rational numbers countable, i.e. they may be placed in one-one correspondence with the natural numbers. He also showed that the algebraic numbers, i.e. the numbers which are roots of polynomial equations with integer coefficients, were indeed countable. However his attempts to decide whether the real numbers were countable proved harder. Canton had proved that the real numbers were not countable by December 1873 and published this in a paper in 1874. It is in this paper in 1874. It is in this paper that the idea of a one-one correspondence appears for the first time, but it is only implicit in this piece of work.

I have chosen to present this material to my third and fourth grade students, in a very introductory, hands-on approach. They need to have a basic vocabulary and grasp of the terms to be used. They also need to measure, divide, draw, and calculate on their own, to see the fractal patterns develop.

## CONTENT STANDARDS

Having students think beyond the obvious while using their basic math skills, is the main goal of this unit. To expand their thinking, plant a seed of thought, and possibly to ignite a thought to ponder for further study at a later time are also objectives of presenting this unit. As they proceed through this unit they will:

1. Identify and use fractions to represent part of a set or region
2. Measure length in metric and customary units
3. Critique mathematical ideas
4. Use strategies to understand and solve non-routine problems
5. Use appropriate mathematical terms to explain and justify logical solutions to problems
6. Collect, organize, and display data
7. Interpret data from graphs
8. Predict and determine why some outcomes are certain, more likely, most likely, equally likely, or impossible
9. Recognize, describe, extend, create, and replicate a variety of patterns, including attribute, activity, number, and geometric patterns
10. Form rules to describe a specific pattern
11. Identify and construct 2D and 3D shapes
12. Use coordinates to identify points on a coordinate grid
13. Classify and compare properties of polygons
14. Understand mathematical concepts - patterns, ordering fractions, area, perimeter, similarity, measurement

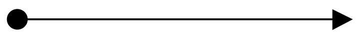
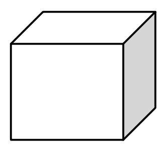
**Connecting Mathematics:**

- Connects math, art, writing, history, and careers
- Connects arithmetic and geometry

**STRATEGIES**

To give the students sufficient background knowledge from which to build from, we will use graphic aids to describe the basic geometric shapes. From there, they will go on to use a Venn Diagram to compare and clarify the various terms and to extend the vocabulary terms that will be used to explain fractals.

To get started with fractals, I will teach them a lesson on FRACTAL PROPERTIES:

- A **point** has no dimensions - no length, no width, no height
- A **line** has one dimension - length. It has no width and no height, but infinite length. 
- A **plane** has two dimensions - length and width, no depth. It is an absolutely flat tabletop extending out both ways to infinity. 
- **Space,** a huge empty box, has three dimensions, length, width, and depth, extending to infinity in all three directions. 
- Fractals are formed by infinitely many steps.

The next activity will be for students to study pictures and artwork of nature and fractal designs. A discussion of these pictures and what geometric shapes they can identify in the picture, will follow. After this activity, the students will be led to see that some shapes in nature cannot be so easily classified as one shape or another, nor as a combination of several different basic shapes. It is more complicated than that, yet there is order.

Then the students will work in small groups. Each group will be given a rock to measure. First they will have to use a 12 inch ruler to measure the distance around it and record their findings. Then they will be given a tape measure to repeat the process. Finally, they will be given a piece of string to wrap around the rock, then to measure it against a ruler. After charting their data, we will have a class discussion as to why the results vary. From there, we will talk about measuring the distance around an island, and how it can change, depending on the measuring tool. Incorporating this activity will be a discussion of measuring the coastline of Africa. Looking at a picture of the coast-line of Africa, you could measure it with rulers, and scale it down to get a reasonable measurement. What if you then measured it with a smaller ruler? Which measurement should give you a bigger measurement? Since the coastline is jagged, you could get into the nooks and crannies

better with the smaller ruler, so it would give you a larger measurement. Now, what if it were measured using even a smaller unit of measurement? You could really get into the teeniest of crannies there. So the measurement would be even larger. You could continue to measure it with shorter and shorter rulers, and the measurement would get longer and longer. You could even measure it with infinitesimally short rulers, and the coastline would be infinitely long. That is fractal!
Inform them that engineers have begun designing and constructing fractals in order to solve practical engineering problems. It is a field of mathematics that may interest them in the future.

We will then move on to defining fractals, and showing examples of some famous geometric fractals, starting with the Cantor Set. To familiarize the students with the name and his design, we will create a Cantor Set as a class. Then they will be given colored pencils, graph paper, and rulers to create a Cantor Set. This will require them to use their rulers accurately, be able to visualize how to divide lines into thirds, and how to follow directions to form the pattern. How many times they can iterate the pattern, will be up to their level of mathematical skills in dealing with fractions and measurement.

After students have an understanding of dividing lines into thirds, I will introduce the Sierpinski Triangle. On a large sheet of paper, I will have the students measure a 28 cm equilateral triangle on their paper, again reviewing the vocabulary of basic geometry, and using their measuring skills. Then, we will proceed by dividing each side into halves, marking the midpoints, then connecting them to form a triangle within the triangle. They will then shade out the triangle in the center. Then, they will draw another equilateral triangle with sides of four triangle lengths each. They will then connect the midpoints of the sides and shade the triangle that formed in the center. They will continue this procedure for as many steps as they can, based on their measuring abilities.

Outlined are the steps:

*Step one* will be to draw an equilateral triangle with sides of two triangle lengths each and then to connect the midpoints of each side, making four equilateral triangles.

Shade out the triangle in the center. Think of it as cutting a hole in the triangle.

*Step two* is to draw another equilateral triangle with sides of four triangle lengths each. Connect the midpoints of the sides and shade the triangle in the center as before. Notice the three small triangles that also need to be shaded out in each of the three triangle on each corner - three more holes.

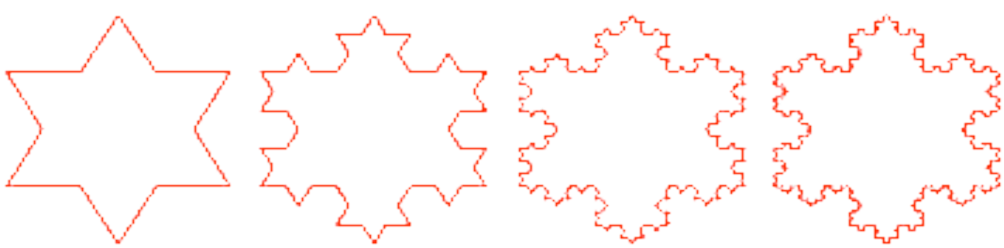
*Step three* will be to draw an equilateral ` eight triangle lengths each. Follow the same procedure as before, m he shading pattern. You

will have 1 large, 3 medium, and 9 small triangles shaded.

*Step four* you can be creative with your design. Do it on a poster board, following the pattern to complete the Sierpinski triangle. Use your artistic creativity and shade the triangles in interesting color patterns.

Moving along, the students will be introduced to the Koch snowflake. Niels Fabian Helge von Koch was born on January 25, 1870 in Stockholm, Sweden. Between the years 1893 and 1905 von Koch had several appointments as an assistant professor of mathematics. Von Koch was appointed to the chair of pure mathematics at the Kengliga Tekniska Hogskolan. In July 1911 von Koch succeeded Mittag Leffler as professor of mathematics at Stockholm University. Von Koch is famous for the Koch curve which appears in his paper *Une methode geometrique elementair pour l'etude de certaines question de la thheorie des courbes plane* published in 1906.
This curve snowflake is constructed by dividing a line into three equal parts and replacing the middle segment by the other two sides of an equilateral triangle constructed on the middle segment.

Repeat on each of the 4 segments. Repeat indefinitely. It gives a continuous curve which is of infinite length and nowhere differentiable. If one starts with an equilateral triangle and applies the construction, one gets the von Koch snowflake (sometimes called the von Koch star) as the limit of the construction.
The von Koch snowflake is a continuous curve which does not have a tangent at any point. Von Koch's 1906 paper mainly consists of a proof of these facts.



The students will be given a set of directions, rulers, large paper, sharpened colored pencils, with which to create this fractal. The more advanced students will be given the opportunity to produce more iterations, thus more sophisticated snowflakes. This will be an activity that can challenge them at various levels, differentiating their instruction. Every student is at a different learning levels and developmental stages, so this activity will challenge the students at higher levels, yet the other students will be able to complete a drawing with success, at their levels.

*Given these three different types of fractals*, students will then be given the chance to repeat the one of their choice, to improve on their design and final product.

As they finish, they will be given a mathematician to study further – either Mandelbrot, Cantor, Spierpinski, or Koch. They will be given a sheet of questions to answer about their mathematician, to guide their research. Then, in groups, they will combine their findings and create a product to display their findings.

As a review lesson on the terminology and the mathematicians we studied, we will play Jeopardy. I will plug in topics and terms from this unit and create a Jeopardy game that is on their ability levels. Then, as they play the game, I will observe to see what information and concepts they comprehended, and what information needs some more instruction.

After spending time on this unit, I hope my students will have a basic understanding of fractals and be excited to go out and learn more about them on their own! As they go about their daily routines, hopefully, they will see the world as a changing place, one full of possibilities, uncertainties, dynamic changes, yet order amongst the chaos.

## CLASSROOM ACTIVITIES

### Lesson One:

Basic Geometry

Objectives:

- The student will identify basic geometric shapes.
- The student will learn vocabulary terms dealing with fractals.
- The student will apply the terms to other situations.

Materials

Large Cards with geometric shapes

Large cards with definitions of geometric terms

Procedure

1. In small groups, have students brainstorm to list as many 2-D and 3-Dgeometric shapes as they can think of in ten minutes.
2. As a whole group, compare their lists and create a master list.
3. Go over each term, reviewing their attributes.
4. Using the terms polygons, quadrilaterals, and parallelograms, have the students create a three-circle Venn Diagram to see the similarities and differences in the terms.

**Appendix A Venn Diagram**

5. Go over the Venn Diagram and have the students explain the similarities and

differences in the terms.

6. As a review, tape large cards with the definitions on one side of the board and pictures of the shapes on the other side of the board, face down.

Match up the picture with its description by turning over one card from each side, at a time. Behind the cards, on the chalkboard, but up a rebus. As the students match up the cards, remove the cards from the board, and see if they can guess the rebus. If they cannot, they continue matching up the cards, and removing the cards, until they can solve the rebus.

**Appendix B REBUS PUZZLE**

## LESSON 2

Introducing fractal properties

Objectives:

- The student will become familiar with terminology dealing with fractals.
- The student will be able to apply the terms to novel situations.

Materials

Large paper and markers

Procedure

1. Have the students define and illustrate these terms on large sheets of paper:
   point, line, plane, space, iterate, infinite, fractal
2. After they complete them, have the students share in a small group, their definitions and illustrations, and discuss their findings.

## LESSON 3

Looking at fractal art and nature scenes

Objectives:

- The student will pick out basic geometric shapes in artwork and nature scenes.
- The student will discover that not all objects can be classified using basic geometric shapes.

Procedure

1. Hold up various fractal art pieces from books and art calendars. Discuss the picture - colors, designs, appeal.
2. Ask students to identify any basic geometric shapes that they see in the pictures.
3. Hold up the nature pictures. Again discuss the color, design, and appeal of the pictures. Then have them point out geometric shapes in nature.
4. Lead them to the discovery that not all things in nature can be simply classified into the basic geometric shapes.

## Lesson 4

Measuring the distance around rocks

Objective

- The student will measure objects using different measuring tools.
- The student will compare the results of using different measuring tools.

Materials

6 rocks, 12 inch rulers, yard sticks, tape measures, string, thread

**Appendix C - Worksheet**

Procedure

1.  On their record sheets, have the students describe their rocks, including geometric terms in their description.
2.  Have students measure the distance around their rocks using a yard stick, recording their measurement.
3.  Repeat using a 12 inch ruler, and then a measuring tape.
4.  Using the string, have them wrap it around the rock, mark the string, then measure the length of the string using one of the other measuring tools and record.
5.  Repeat step 5, using thread.
6.  Have students analyze their data by answering the questions on their worksheet.
7.  As a group, discuss their findings.

## Lesson 5
Defining and illustrating fractals

Objectives
- The student will define a fractal.
- The student will create a Cantor Set.
- The student will divide lines  into thirds.

Materials
inch rulers, yard sticks, large sheets of paper, calculators, markers
Procedure
1. Positioning the paper horizontally, have students draw a large line across the top of their papers.
2. Under that line, have the students draw another line, very lightly.  Then they are to highlight the first third of the line, and the last third of the line, leaving the middle third alone.
3. Then draw another line below that, dividing the first and last third, into thirds, leaving the middle third alone. They can use their calculators, to find the thirds, measuring to the nearest 16th of an inch.
4. They repeat this procedure, until they cannot draw any smaller thirds.
5. Give them time to practice this procedure a couple more times, letting them pick their best work to share with the rest of the class.
6.  Tell them that is called a Cantor Set, named after the mathematician who first designed it.
7.  Review the term ITERATE and how it applies to this activity.

## Lesson 6
The Sierpinski Triangle

Objectives
- The student will apply their understanding of dividing lines in thirds to create a Sierprinski Triangle.
- The student will use measuring tools to measure and divide lines into thirds.

Materials

Large paper, pencils, rulers, calculators, colored pencils
Procedure
1. Have the students measure a 30 centimeter equilateral triangle on their paper.
2. Then they are to divide each side into halves, marking the midpoints, then connecting them to form a triangle within the triangle.
3. Next, shade out the triangle in the center.
4. Draw another equilateral triangle with sides of four triangle lengths each. Connect the midpoints of the sides and shade the triangle that formed in the center.
5. Continue this procedure for as many steps as they can, based on their calculating and measuring abilities.
**APPENDIX D - Steps to the Sierpinski Triangle**

## Lesson 7
Koch Snowflake

Objectives
- The student will apply their understanding of dividing lines in thirds to create a Koch Snowflake.
- The student will use measuring tools to measure and divide lines into thirds.

Materials
Large paper, pencils, rulers, calculators, colored pencils

Procedure
1. The student will be given a set of directions, rulers, large paper, sharpened colored pencils, with which to create this fractal. The more advanced students will be given the opportunity to produce more iterations, thus more sophisticated snowflakes. This activity can challenge them at various levels, differentiating their instruction.
2. This curve snowflake is constructed by dividing a line into three equal parts and replacing the middle segment by the other two sides of an equilateral triangle constructed on the middle segment.
2. Repeat each of the 4 segments. Repeat indefinitely. It gives a continuous curve which is of infinite length and nowhere differentiable. If one starts with an equilateral triangle and applies the construction, one gets the von Koch snowflake (sometimes called the von Koch star) as the limit of the construction.

## Lesson 8

Research Project on Cantor, Koch, Sierpinski, or Mandelbrot

Objectives
- The student will use a variety of resources to research one of the four mathematicians
- The student will present their findings in a brochure, a report, poster,or a power point presentation.

Materials
Research books, internet access

Pencils, pens, markers, tag board, glue, scissors

Procedure
1. Give each student a choice of the four Mathematicians they would like to research.
2. Have them follow a research outline guide, to find specified facts about their Mathematician.
(APPENDIX E - RESEARCH REPORT GUIDE)
3. Have students use their guide sheets to create their presentation.

**Lesson 9**

Review lesson

Objectives
- The student will apply and share the knowledge they have learned to other situations
- The student will work cooperatively in a small group

Materials
List of questions/answers

Procedure
1. Divide the class into three groups
2. Set up the game board
3. Have students pick the category and the clues
4. See which team can earn the most points

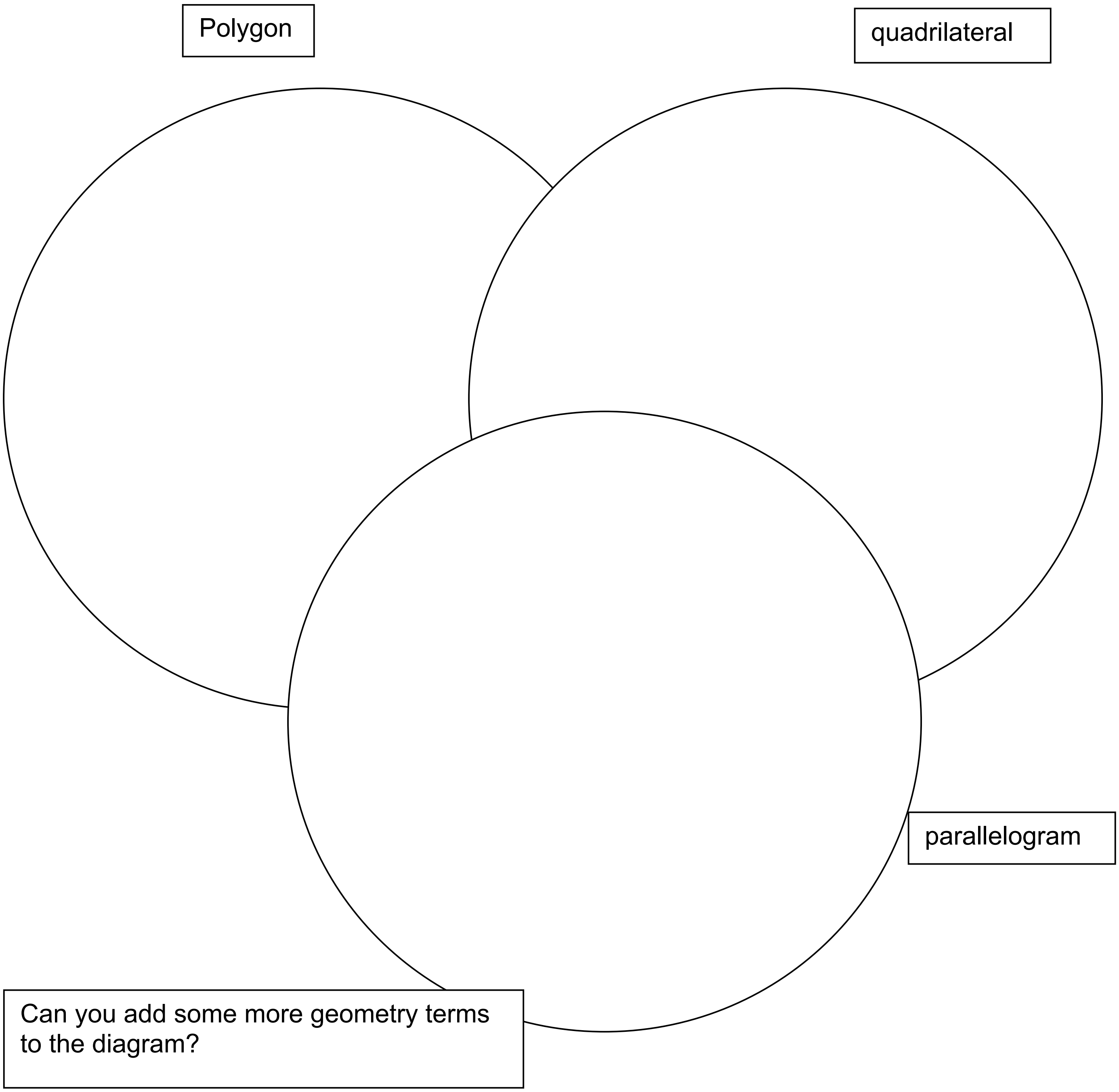
(APPENDIX F - QUESTIONS/ANSWERS)

## APPENDIX A

# VENN DIAGRAM

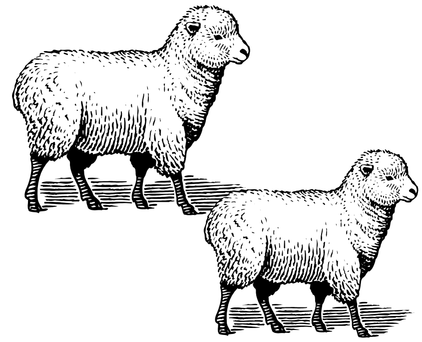
| | |
|---|---|
| Triangle | hexagon |
| Rectangle | octagon |
| Square | pentagon |
| Rhombus | trapezoid |

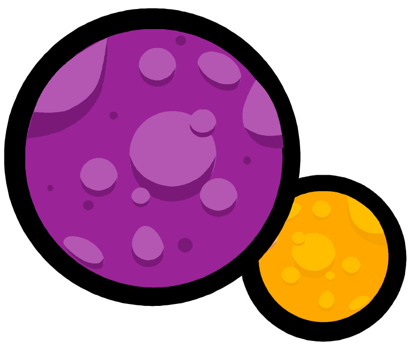
D*irections:* Using the terms in the box, put them in the correct circles.




**APPENDIX B**

{Enlarge and place on chalkboard}





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***Answer:*** *I use circles to compare.*

APPENDIX C

Name_____________________________ Date __________________________________

MEASURING ROCKS

Rock #_______

Describe your rock. Use geometric terms.

Directions: Measure the distance around your rock (to the nearest quarter inch) using the following tools:

Yard stick ______________________

12" ruler ______________________

Tape measure ____________________

String ____________________

Thread ____________________

A.. Are all your measurements the same, or are they different?

B. What are your observations?

## APPENDIX D - Steps to the Sierpinski Triangle

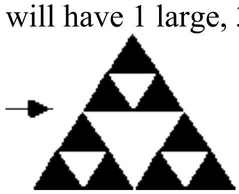
Outlined are the steps:

**Step one** will be to draw an equilateral triangle with sides of two triangle lengths each. Then to connect the midpoints of each side, making four equilateral triangles.

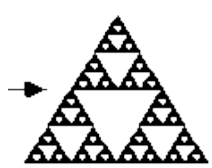
Shade out the triangle in the center. Think of it as cutting a hole in the triangle.

**Step two** is to draw another equilateral triangle with sides of four triangle lengths each. Connect the midpoints of the sides and shade the triangle in the center as before. Notice the three small triangles that also need to be shaded out in each of the three triangle on each corner - three more holes.

**Step three** will be to draw an equilateral triangle with sides of eight triangle lengths each. Follow the same procedure as before, making sure to follow the shading pattern. You will have 1 large, 3 medium, and 9 small triangles shaded.



**Step four** you can be creative with your design. Do it on a poster board, following the pattern to complete the Sierpinski triangle. Use your artistic creativity and shade the triangles in interesting color patterns.



## APPENDIX E

NAME: ______________________________________________ Date _________________________

### RESEARCH REPORT GUIDE

DIRECTIONS: Using the following guide, research a Mathematician and use this information to create a presentation in the form of a report, brochure, poster, or a power point presentation.

1. Mathematician's full name
2. Born
3. Died
4. Family members
5. Childhood home (country)
6. Education
7. Work Experiences
8. Jobs/positions held
9. Important contribution to the field of mathematics
10. Other interesting facts

## APPENDIX F

*Review Activity*

## QUESTIONS: ***SINGLE CHALLENGE***

Category #1 SHAPE UP {Identify the 2-dimensional shapes}

Clue #1 A polygon with 5 sides
answer: What is a pentagon?

Clue #2 Any polygon with 4 sides
answer: What is a quadrilateral?

Clue #3 A triangle with 3 equal sides
answer: What is an equilateral triangle?

Clue #4 A polygon with 8 sides and 8 angles
answer: What is an octagon?

Clue #5 A four-sided figure with one set of parallel sides
answer: What is a trapezoid?

Category #2 GEOMETRIC CALCULATORS {Use geometry to solve the problems}

Clue #1 The degrees in a right angle
answer: What is 90 degrees?

Clue #2 Sides of a square with a perimeter of 32 inches
answer: What is 8 inches?

Clue #3 Sides of an equilateral triangle with a perimeter of 27 centimeters
answer: What is 9 centimeters?

Clue #4 Degrees in a circle
answer: What is 360 degrees?

Clue #5 Total degrees of all angles in a square
answer: What is 360 degrees?

Category #3 THIRDS - {Divide each by 3}

Clue #1 21
answer: What is 7?

Clue #2 3 x 3 x 3
answer: 9?

Clue #3 36 inches
answer: What is 12 inches?

Clue #4 45
answer: What is 15?

Clue #5 330
answer: What is 110?

Category #4 PERIMETER {Calculate the perimeter}

Clue #1 A seven inch triangle
answer: What is 21 inches?

Clue #2 A five inch square
answer: What is 21 inches?

Clue #3 A hexagon with 4 inch sides
answer: What is 24 inches?

Clue #4 A 3 x 4 inch rectangle
answer: What is 14 inches?

Clue #5 A 9 x 7 inch rectangle
answer: What is 32 inches?

Category #5 NUMBER EQUATIONS {Identify each initial}

Clue #1 A polygon with 5 sides
answer: What is a pentagon?

Clue #2 Any polygon with 4 sides
answer: What is a quadrilateral?

Clue #3 A triangle with 3 equal sides
answer: What is an equilateral triangle?

Clue #4 A polygon with 8 sides and 8 angles
answer: What is an octagon?

Clue #5 A four-sided figure with one set of parallel sides
answer: What is a trapezoid?

Category #6: MATHEMATICIANS {Identify the shape/term associated with each mathematician}

Clue #1 Euclid
answer: What is classic geometry?

Clue #2 Sierpinski
answer: What is a triangle?

Clue #3 Koch
answer: What is a snowflake?

Clue #4 Cantor
answer: What is the Cantor Set?

Clue #5 Mandelbrot
answer: What is a fractal?

## APPENDIX F (CONTINUED)

### *DOUBLE CHALLENGE*

Category #1 PATTERNS {What comes next?}

Clue #1 23, 25, 27
answer: What is 29?
Clue #2 45, 54, 63
answer: What is 72?
Clue #3 33, 29, 25
answer: What is 21?
Clue #4 AZ, BY, CX
answer: What is DW?
Clue #5 16, 25, 36
answer: What is 49?

Category #2 STARTS WITH "M" {Words that begin with the letter "M"}

Clue #1 Abbreviated MM
answer: What is a millimeter?
Clue #2 100 centimeters in one
answer: What is a meter?
Clue #3 5280 in one
answer: What is a mile?
Clue #4 Middle number in a set of data
answer: What is the median?
Clue #5 Mathematician who worked with fractals
answer: Who is Mandlebrot?

Category #3 MORE, LESS, OR EQUAL – {Answer more, less, or equal}

Clue #1 Sides on hexagon: sides on an octagon
answer: What is less than?
Clue #2 14-5: 2 x 2 x 2
answer: What is greater than?
Clue #3 9 x 4 :7 x 5
answer: What is greater than?
Clue #4 5 x 5 : 3 x 3 x 3
answer: What is less than?
Clue #5 3/4 : 75%
answer: What is equal?

Category #4 FRACTIONS {Add or subtract the fractions}

Clue #1 1/2 + 1/2
answer: What is one?
Clue #2 1/6 + 4/6
answer: What is 5/6?
Clue #3 7/8 - 2/8
answer: What is 5/8?
Clue #4 1/2 + 1/4
answer: What is 3/4?
Clue #5 1/2 + 1/4 + 1/2
answer: What is one?

Category #5 MYSTERY NUMBERS {Name the 2 numbers that fit both equations}

Clue #1 Their sum is 4, their product is 4
answer: What is 2 and 2?
Clue #2 Their product is 36, their sum is 13
answer: What is 9 and 4?
Clue #3 Their sum is 10, their difference is 4
answer: What is 7 and 3?
Clue #4 Their quotient is 3, their difference is 4
answer: What is 9 and 3?
Clue #5 Their difference is 2, their product is 48
answer: What is 8 and 6?

Category #6: FRACTALS {Terms that pertain to fractals}

Clue #1 Goes on and on forever
answer: What is infinity?
Clue #2 To repeat a pattern
answer: What is iterate?
Clue #3 Same as itself, only smaller or larger
answer: What is self-similar?
Clue #4 Koch devised this curve
answer: What is the Snowflake curve?
Clue #5 Mathematician who coined the term "Fractal"
answer: Who is Mandelbrot?

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URL http://math.rice.edu/-lanius/fractals/self.html