

Fractal Fun

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Overview

This unit on Fractals was written for a Middle School Saturday Program but can be used in addition to any regular curriculum. It starts with the basics of what a fractal is and lets students explore different fractal patterns. As students are introduced to different simple fractals they begin to recognize the pattern and try to continue to expand it. They will explore similarity of the changing shape and perimeter in some cases. They will try to appreciate both the simplicity and complexity of the pattern leading them towards creating tables and equations represented by the fractals.

I am an 8th grade teacher at South Brook Middle School. South Brook is located in the south hills area of Pittsburgh. It currently houses 450 students in the 6th, 7th, and 8th grades. This is a neighborhood school, so the students are friends in and out of school and have relatively the same social experiences. They come from four different grade schools all following the same curriculum. Most of the students are on the same level, truly average or low academically.

I teach 5 pre-algebra classes. The higher achieving students are in algebra, so the pre-algebra classes are usually students of lower achievement. In our current curriculum, we emphasize launch, explore, and summarize; which can sometimes be a challenge with so many of the high achieving students in algebra. The students are not separated by ability until the 8th grade, so until this point the classes were a nice mix of abilities. When students enter the 8th grade, they no longer have the higher achieving students in the class introducing different

strategies. Now they have to start developing these skills. After a slow start to the year, the students realize that those who used to provide the answers are no longer in their classes, and they start to shine. After having 7 math classes in a 6 day cycle, they need a break from the normal math class and explore math in other places.

At South Brook we offer a Saturday program for low achieving students who are scoring basic or below basic on PA standardized tests. The day is made up of a math class, a communications class, and either an art or gym class. The students start at 8:30 am and are dismissed at noon. We start the Saturday program in October and continue through April. I teach the math class on Saturday and we work on reviewing basic math skills from prior years to help move the students to proficient on the State standardized test. The standardized tests are usually given in March, which leaves us with 4-6 weeks in our Saturday program to work on math in a different way. The students at this point are usually ready for something new and different. I think they will find fractals both challenging and exciting.

Rationale

When I applied to the Pittsburgh Teacher's Institute to be a part of "Fractals and Chaos", I wasn't sure what fractals and chaos meant, but I was intrigued and wanted to learn more. Anytime I hear about a math class I am eager to sign up, especially when it involves a subject that I know little or nothing about. This was definitely one of those subjects; and I wanted to find out what fractals had to do with math. I thrive being in a learning environment and this was something in my field but foreign as well. I am enthusiastic about learning new things and hope to pass this enthusiasm on to my 8th graders. I want them to enjoy learning as I do and help them see that math will be important to them for the rest of their lives. I want to expose these students to math in a different way.

Up until now our math curriculum has been very structured. Math as we use it everyday. We currently use "Connected Mathematics Project" or CMP, which is problem centered. It consists of interesting mathematical problems imbedded with mathematical concepts. As they explore problems, they start to develop a deeper understanding of mathematical ideas. It helps the students to recognize math as a way of thinking; making sense of situations rather than seeing math as a series of unrelated events. This mathematical process helps to connect students who struggle and usually end up with a dislike for the subject. Because of the connection that is made between math concepts and problems, the students begin to enjoy problem solving. They see a real connection between real life situations and math. They are usually familiar with the types of life circumstances in the book and relating to them helps them to find ways of solving

the problems. In looking at fractals, they are unlike anything we see in middle school math, yet the concepts learned in middle are needed to explore this unit.

By introducing non-traditional math ideas, I hope to spark something in a student that may never have been ignited. I want students to gain a new appreciation for the subject, and hook them forever. I have always been excited by new math ideas and I am always looking for something to help bring that alive in my students. Fractals may be a way of making these math connections somewhat fun and different. We look at pictures or designs known as fractals and actually take away the layers and start with a basic geometric shape. By following a pattern, or using a set of rules, students start to use symbols to continue the pattern.

By definition, a fractal is any geometric shape that is recursively constructed and has infinite complexity. When magnified, a fractal has a self-similar nature; it resembles the entire object. In mathematics, a fractal is a geometric object that has self-similarity at any level. As you look closely at a fractal you see more and more detail or a copy of the original object. Repeating a process over and over again can create fractals. Fractals can also describe objects found in nature. Coastlines, clouds, and mountains are examples of fractals that are not geometric shapes. When looking at them up close, they too can show self-similarity.

Fractals combine a number of middle school concepts in a fun design. Students can be creative while indulging in math. In the 7th grade, our students spend some time with geometry. We explore perimeter, area, proportion, and ratios. Fractals encompass all of these. The seventh grade curriculum also focuses on shapes being similar. For an object to be similar, it must have the same shape, corresponding angles must be equal, and corresponding sides must be in proportion. Similarity and proportionality are both the fundamental parts of fractals. They give students an opportunity to see similarity in a different perspective. Students look for it in simple objects, now they can see it in a more complex shape. They now have an opportunity to look for similarity in different places.

This unit was designed for 8th grade students who need additional math help. I plan on using it during our Saturday school program to help provide an opportunity for students to review concepts in a non-traditional way. I always like working with students who need the most help and this is a fun way of connecting math concepts. I use it to teach math in a way that focuses on designs. Fractals are a break from everyday math. It involves lots of pictures, which helps grab the students' attention. It takes their attention off problems centered situations and focuses on design, form instead of function. It also results in a

rigorous curriculum mandated by our state standards. These students are giving up Saturday morning and I like to involve them in learning different from what they have done Monday through Friday. If they feel that they are in a math class, much like they have been doing in all week, now matter how interesting, they will choose to do something else with their Saturday mornings. I need to provide an exciting way of developing math skills to keep them coming.

This unit can also be used with very low or gifted students because of its approach. I wanted students of all levels to be able to enjoy exploring fractals. You can keep it as simple or as complex as your audience dictates. You can start with the Sierpinski Triangle and spend time coloring the different levels. This helps young students to see how the pattern repeats and is similar. And they have done nothing so far but color. It also lends itself to a mixture of abilities. Because of it being fun in nature, the students don't see themselves at different math abilities. You can bring a number of levels together working as one. Students, who are low in math, generally look at the patterns in a different way and discover the patterns now easily seen by others. No matter how you look at it, fractals are definitely math.

The Sierpinski Triangle, which we will explore in this unit, displays self-similarity. I think it is one of the most interesting fractals and one that your students maybe a little familiar with. Its beautiful, has dimension, and can be as simple or complicated as you want to make it. The outline of a Sierpinski Triangle is an equilateral triangle and inside it is made of an infinite number of similar equilateral triangles (or copies). These inside triangles also congruent, they are all exactly the same size and shape. These are all concepts developed in the 7th grade.

In the 8th grade, the students take pre-algebra and spend most of 8th grade looking for patterns and expressing those patterns symbolically. This again is problem centered and the students solve the problems by eventually writing equations. We start with linear equations, follow through to exponential equations and end the year with quadratic equations. The students seem to have an easier time creating tables and graphs than they do writing a rule to fit the pattern. We look at patterns using numbers in our tables and eventually write an equation to fit this pattern. Writing these equations was one of the most fascinating things for me when I was researching fractals. Getting away from this traditional problem centered classroom and applying this to designs is fun for all students. Not only do we take a familiar geometric figure and operate on it to make it new and a little bit more complicated, but also in the process we actually apply rules for finding the area and perimeter as we continue to make these figures more and more complicated. Students enjoy watching the figures grow and change in a similar format. My 8th grade students have been finding rules for

different real life situations, but with fractals they can actually see the figure and watch it grow and change. Then by measuring the length of line segments or by dividing the line segments, they can write a rule or equation to predict the perimeter after any number of steps. I like the fact the students begin to find these rules while having fun with the fractal. They forget that this class is math centered.

When I decided to adapt this for my Saturday math class, I had trouble finding information for my students on a middle school level. Everything was geared toward a more sophisticated class. As I sat through my class on “Fractals and Chaos”, I found myself in a pool of knowledge much deeper than I had anticipated. The content was well above the student that I wanted to expose fractals to. I didn’t know if this would work for my students or not. The deeper I explored fractals, the deeper the content became. I knew the subject would certainly interest my students, but I wasn’t sure how to begin and how far to go. Most books were written at a much higher level. I needed to put this on a middle school level. I started this by taking a fractal and breaking it down into its simplest form and use basic concepts such as perimeter and area. These two middle school concepts always seem to give the students problems, especially those students who are basic or below. This is also a big part of our state assessment test. I needed a way to relate this to my students. This unit works well with these students. The shapes are more interesting than the standard triangle and rectangle; and the area consists of more than just finding the inside of the figure. With fractals, the area may only be the shaded part, which adds another dimension to their learning. They are not considering the entire inside of the object to refer to the area. This adds to the rigor of the math lesson.

In this unit, we will explore a number of different fractals. They were explored and discovered long before the word was coined. In 1975, a Polish/French/American mathematician, Benoit Mandelbrot coined the word “fractal” from the Latin fractus, meaning “broken” or “fractured” to describe shapes that are detailed at all scales. Fractals although simple and complex, beautiful and intricate, can be used to predict, unpredictable random things. By studying fractals, students not only explore self-similarity (containing copies of itself), but fractals also help the students understand the concept of infinity by repeating patterns over and over.

When looking at linear relationships in the eighth grade, students are able to predict future outcomes by looking at patterns and creating tables. By developing equations, they can predict any point in the future without continuing to write a list of numbers. When things become less linear and more complex, the predictions sometimes become harder. By generating simple mathematical equations, fractals can be created. As the patterns become more and more

complex, equations can help predict the outcome of any iteration (repeating a series of operations). By looking at a fractal and zooming in, you begin to see smaller similar pictures of the original shape. By using computers, you can zoom in to a point that has never been seen before. This is what makes fractals math both new and fascinating. The fractals that we will explore in this unit have iterated functions systems; they all have a fixed replacement rule, which we will attempt to write.

Fractals can be classified by three types of self-similarity. The first is referred to as “Exact self-similarity”. At any different scale, the fractal appears to be exactly the same. The iterated function system used to create the fractal also displays exact self-similarity. The second, “Quasi-self-similarity” is identical but not exactly the same at any different scale. The third, “Statistical self-similarity” are statistically similar but are weak and not exact. In this unit students will explore the first type of similarity “Exact self-similarity”. Not all objects that are self-similar are fractals. A straight line, which does display self-similarity, is not a fractal.

Fractals can also be found in nature. Even though we will not explore these types of fractals, they too display self-similarity, but at a finite scale range. They are similar in nature and can be a miniature replica of the original. Some nature fractals include ferns, clouds, mountains, and snow flakes.

My first experience with fractals was with one of the most famous fractals to be studied mathematically called the “Cantor Set” or “Cantor Dust”, named after its founder, a German mathematician, Georg Cantor in 1883. He created a subset of real numbers with unusual properties. Its construction involves the real numbers between zero and one and is uncountable. We start with a line segment and remove the middle third. Since the middle interval is internal, the endpoints always remaining leaving a set that is never empty. We now have two line segments of equal length and an equal size hole in the middle or an open interval. We then remove the middle third of both of these two remaining line segments to create another version. We continue repeating this process of removing the middle third of the remaining line segments until we have what looks like “dust”. This gives the student their introduction into fractals and they can actually go through the process of creating Cantor Dust themselves. Even though it gets harder for us to actually remove the middle third, we can continue this pattern indefinitely. It also introduces them to the idea of infinity even though they physically cannot remove another third of the line. This concept denotes a higher order of thinking and students start taking ownership of their learning.

Looking at the Cantor Sets, intervals are taken out of the unit interval $(0,1)$, leaving $(0, 1/3)$ and $(2/3, 1)$, a set that displays similarity. There are now

two copies of it. The points $1/3$ and $2/3$ are left behind and these endpoints are never removed even with subsequent steps since we are removing only the parts that lie between the endpoints.

All points that are not removed in the interval $(0,1)$ are the Cantor Set. The students may wonder if there is anything left after removing so many $1/3$ intervals. But our set cannot be empty because it contains $0, 1, 1/3, 2/3, 1/9, 2/9, 7/9, 8/9$, and so on. The Cantor Set is actually the final row of the picture that is created by the students. As they continue through the process, they notice that at each stage, the picture is two exact copies of the previous stage. Because of the self-similarity at every iteration, the Cantor Set is considered a true fractal.

Another famous fractal, the “Sierpinski Triangle” was introduced by Vaclav Sierpinski, a Polish mathematician in 1916. I have seen this triangle a number of times, but never realized it was a fractal. The intriguing design starts with an equilateral triangle and is a fun fractal for students to create. Students in middle school enjoy looking and creating different designs. Students start with an equilateral triangle and by connecting the midpoints of each side, they create four equilateral triangles inside of the original triangle. By shading in the middle triangle, or by cutting holes in the triangle, the students have created the first iteration of the Sierpinski Triangle. They can continue this process by connecting the midpoints of the remaining three triangles and shading in, or removing the newly created middle triangle. The students now have a Sierpinski Triangle made up of three smaller but similar Sierpinski Triangles. Again they can continue this process realizing that even though they have to stop at some point, this can go on an infinite number of times. It gives students a visual of the idea of infinity. This activity can be done on triangular grid paper, but I like providing the students with a large picture of an equilateral triangle and let them find and connect the mid points. The students get a true picture of how the Sierpinski Triangle is created without already seeing the lines on the grid paper. I like this particular fractal because it not only connects geometry and arithmetic, but math with art. With this fractal we are able to explore patterns, similarity, fractions, congruency, measurement, and area; all of which are part of our state standards. We also connect exponents and exponential growth and by exploring the non-shaded region, the students are able to write a rule symbolically to predict the area after any number of repeated steps or iterations. With the Sierpinski Triangle, the area of the triangles is finite because it will not go beyond the original triangle, but the number of triangles is infinite.

One of the earliest fractals is the “Koch Curve” and the “Koch Snowflake” devised by a Swedish mathematician, Helge von Koch, in 1904. It’s one of my favorites, not only for its simplistic beauty, but the students are also exposed to a perimeter that can go on forever. With the snowflake students will see that while

the perimeter can go on indefinitely, the area will again never go beyond a certain area. After developing an understanding of infinitely, the students are able to explore the idea that with this infinite perimeter there is a finite area. Even though the perimeter gets bigger and bigger, the area will stay within a confined circle. As with the first two fractals, the students will create both the Koch Curve and the Koch Snowflake themselves.

With the curve, the students start with a straight line and remove the middle third replacing it with an upside down V, with each side of the V measuring the same length as the third that was removed. One line now becomes four segments. One segment removed now becomes two. Continue removing the middle third of each segment and replacing it with an upside down V. Four segments now become 12 segments and so on. This line or curve continues to get longer and jagged with each step.

With the snowflake, the students start with an equilateral triangle and operate on the sides, this time creating a pattern on the outside of the triangle. The Koch curve is placed around the three sides of an equilateral triangle to construct the Koch snowflake. The Koch snowflake is sometimes referred to as the Koch island. Students will take out the middle third of each side and use the hole to create an equilateral triangle on the outside of each of the three sides of the original equilateral triangle. This is the same pattern they used to construct the Koch curve. This complicated pattern continues by taking out the middle third of what now appears to be 12 sides and creating equilateral triangles on the outside of these sections. The infinite numbers of sharp corners, which make up the perimeter of the Koch snowflake, are tightly packed. At each step, the students add more than one unit of length when they add triangles to the perimeter. $1\frac{1}{3}$ units or $(\frac{4}{3})$ units are added with each step. When looking at a little part of the snowflake the students see that it resembles the larger snowflake. After creating several iterations of the Koch Snowflake, the students will be able to see how both the perimeter and area continue to grow and even though the perimeter can grow infinitely, it encloses a finite area. The students can draw a circle around the snowflake and its area will never reach the area of the circle. I like that the students are able to observe both the perimeter and the area. They will also be able to write a rule for the perimeter, after any number of iterations. This is what has hooked me on fractals, the fact that we can continue a design and actually write an algebraic rule for what is happening.

The growth created with both the Sierpinski Triangle and the Koch Snowflake is exponential. I would like to have the students create a fractal that actually grows linearly. By drawing a square and attaching the hypotenuse of the right triangle to the top of the square, the students are able to create a Pythagoras Tree. With knowledge of the Pythagorean Theorem, a big part of our eighth

grade curriculum, the students will create the first iteration, by drawing squares off of both legs of the right triangle and they will soon realize that the area of these squares is equal to the area of the base. The squares off of the legs will now become the square off of the hypotenuse of the next iteration of right triangles with squares off of the legs. The area of these four squares for the 2nd iteration is equal to the area of the two squares in iteration one and also equal to the area of the square of the base. The new area continues to grow by one, or the same area of the base. By creating a table listing the iterations and area, the students will be able to recognize linear growth and write a representation in equation form.

A lot of discussion takes place while the students are creating these fractals and writing the corresponding equations. This also provides an opportunity for students to write about the math that they are doing. They start to write equations by truly looking at patterns. Each fractal that we look at involves a lot of math that is truly connected to both our curriculum and our state standardized test. Even though this will be a part of our Saturday curriculum, it could be included in our normal math curriculum. It makes a nice unit to use around the holidays when the students are usually distracted.

Objectives

Students engaging in this unit should be familiar with finding both the area and the perimeter of a geometric shape, have an understanding of scale factor, the Pythagorean Theorem, and be able to write a rule symbolically. The students should also be able to organize information into a table and recognize both linear and exponential growth. This unit will deepen their understanding of variables and their role in equations. This unit will also enrich the student in recognizing similar shapes, repeating patterns, and writing equations. They will begin to understand that the pattern can repeat an infinite number of times.

The students will meet a number of standards throughout this unit. They will be more adept to using their problem solving skills, computation, finding area and perimeter, the Pythagorean Theorem and reinforcing their algebraic concepts. They will represent fractal patterns with words, tables, and symbolic expressions.

Strategies

The students will meet the objectives of the lesson in creating famous fractals by repeating a series of instructions. They will have to record the change in both perimeter and area in organized tables and use variables to write a rule symbolically in the form of an equation. They will have an opportunity to use the Pythagorean Theorem and solve numeric expressions including fractions.

Classroom Activities

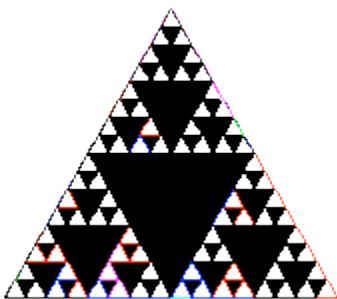
Lesson One: "What is a Fractal?"

The students will spend the day discovering what a fractal is. We will look at Cantor Dust, the Sierpinski Triangle, and the Koch Snowflake. We will compare the three and look for similarities. The students will spend time discussing what they notice about the three before we get into a formal definition of a fractal. After listing the comparisons, we will see that there is a distinct resemblance between the three. We will discuss how the three patterns repeat. Then we will talk about mathematical fractals and how they are constructed using a fixed set of rules and the concept that fractals have self-similarity. We will discuss the fixed set of rules for Cantor Dust. The students will actually write their own set of rules for creating the fractal and then switch with a partner to see if the partner can create the fractal from their instructions. Revisions will then be made to the original set of instructions and the class will write a formal set.

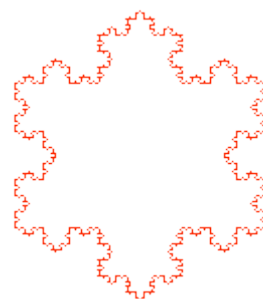
Cantor Dust



Sierpinski Triangle

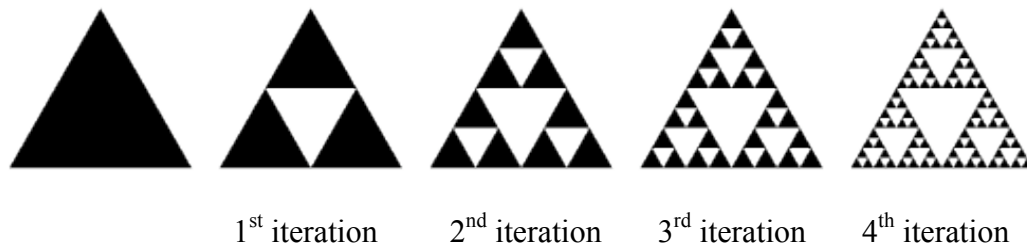


Koch Snowflake



Lesson Two: “Remaining Triangles”

In this lesson we will focus on the Sierpinski Triangle. We will start with an equilateral triangle on a worksheet and discuss the area. Next we will use our formal set of instructions to complete our first set in the Sierpinski triangle. The students will actually measure the sides to find the mid points and connecting these mid points to create four triangles inside the original triangle. After shading the middle triangle, we will be left with three.



We will continue creating similar Sierpinski Triangles and recording the number of remaining triangles. We will discuss a way of organizing our data. The students have made many tables in CMP and should be able to use a table to organize information. We will number our steps and we will record the number of remaining triangles. We will continue to follow our fixed set of instructions and continue to record our information in a table. After completing 3-5 steps, we will look for a pattern in our table. The students at this point should notice that the relationship between the step and the remaining triangle is exponential. In order to get the students to write an equation, we will talk about how many triangles will be left after step “ n ”. We will discuss how our triangles are changing from one step to the next and recognize the fact that we are multiplying by 3. We will discuss writing the number of triangles in exponential from using the powers of 3. The students will then relate the power to the step and make the connection for the “ n^{th} ” step.

Step	Triangles
0	1
1	3 $1 * 3 = 3^1$
2	9 $1 * 3 * 3 = 3^2$
3	27 $1 * 3 * 3 * 3 = 3^3$

4	81	$1 * 3 * 3 * 3 * 3 = 3^4$
n		3^n

Lesson Three: "Finding Area"

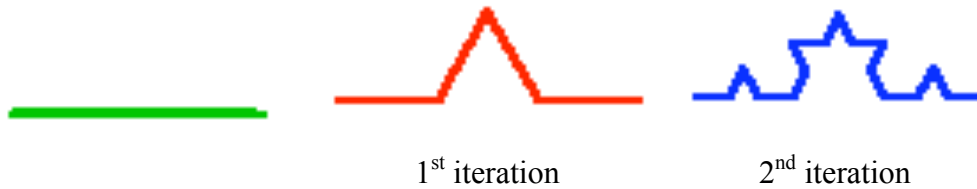
This time we will explore the area of the Sierpinski Triangle. First we will actually find the area of the triangle and talk about what happens to the area after step one. Again we will find a way to organize our information, which will involve creating another table. This time when we record the step, we will also record the area of the remaining triangles. With each step, we divide the triangle into four equal parts. We only keep 3 of the 4 triangles, so the area will be $\frac{3}{4}$ of the original area. As the students continue creating similar Sierpinski Triangles within the original Sierpinski Triangle, they will continue multiplying the remaining area by $\frac{3}{4}$. Once again they will be able to recognize the exponential relationship and write a rule symbolically for the area of the remaining triangles after n iterations.

Step	Area of Remaining Triangles
0	$1 * \frac{1}{2} bh$
1	$1 * \frac{1}{2} bh * \frac{3}{4}$ $3^1/4^1$
2	$1 * \frac{1}{2} bh * \frac{3}{4} * \frac{3}{4}$ $3^2/4^2$
3	$1 * \frac{1}{2} bh * \frac{3}{4} * \frac{3}{4} * \frac{3}{4}$ $3^3/4^3$
4	$1 * \frac{1}{2} bh * \frac{3}{4} * \frac{3}{4} * \frac{3}{4} * \frac{3}{4}$ $3^4/4^4$
n	$1 * \frac{1}{2} bh * 3^n/4^n$

Lesson Four: "The Koch Snowflake and its Perimeter"

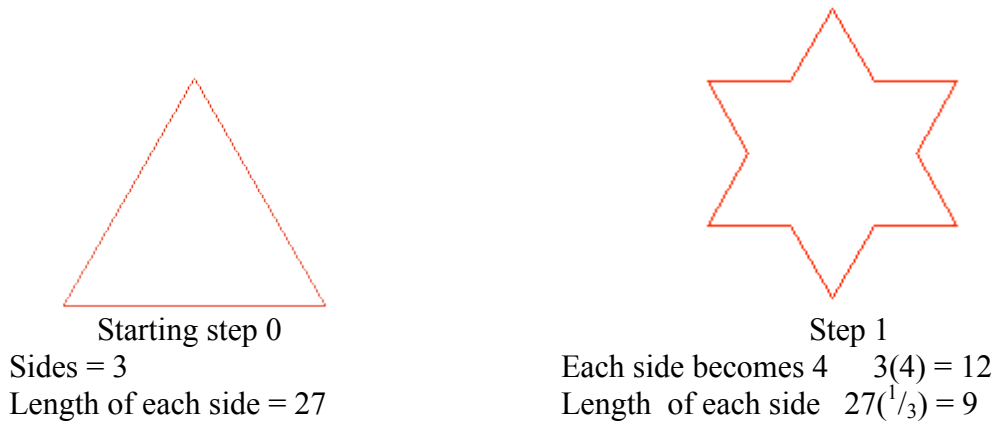
We will start this lesson by first looking at the Koch Curve and again at the Koch Snowflake. The students will start with a straight line, which they can draw themselves. In each case the students will be expected to draw the fractal involved. By removing the middle third of the line and replacing it with an upside down V,

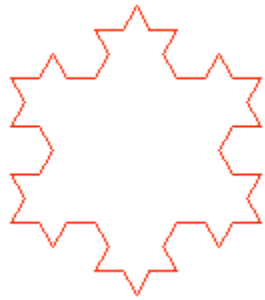
the students have completed their first iteration of the Koch Curve. The V needs to be the same length as the space created by the removal of the middle third of the line. They can continue with this pattern of removing the middle third of all four segments and replacing them with another up side down V to complete the second iteration. At this time the students could discuss how the perimeter is actually changing. The curve will help them in figuring out the change in the perimeter of the snowflake.



I usually have the students construct a fractal bulletin board using the Koch Curve and yarn. The students construct the first two iterations using the yarn and then label the changing lengths of the segment and the total perimeter after each iteration.

The students will now look at different stages of the snowflake creating their own, one iteration at a time. We will discuss how the area is changing along with the perimeter. Again we will discuss infinite and finite. The students will discover that the perimeter may be infinite, but the area is finite, meaning even though it will continue to get bigger, it will never meet or go beyond an enclosed area. As the students use their fixed set of instructions, they will go through the phases of constructing the snowflake and will calculate and record the perimeter. We will start with the same triangle that we used for the Sierpinski Triangle. On all three sides of the equilateral triangle, we will follow the same steps we used to create the Koch curve. As we follow through with each iteration, we will calculate the perimeter and again use a table to record our data.

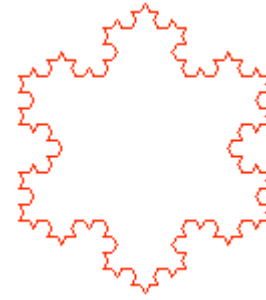




Step 2

$$\text{Sides} = 3(4)(4) = 3(4)^2 = 48$$

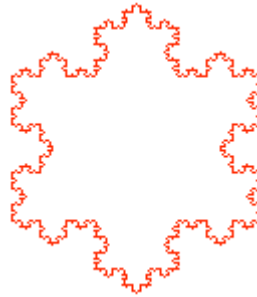
$$\text{Length} = 27^{(1/3)}(1/3) = 27^{(1/3)^2} = 3$$



Step 3

$$\text{Sides} = 3(4)(4)(4) = 3(4^3) = 192$$

$$\text{Length} = 27^{(1/3)}(1/3)(1/3) = 27^{(1/3)^3} = 1$$



Step 4

$$\text{Sides} = 3(4)(4)(4)(4) = 3(4)^4 = 786$$

$$\text{Length} = 27^{(1/3)}(1/3)(1/3)(1/3) = 27^{(1/3)^4} = .33$$

Step	Perimeter of the Snowflake (no. of sides)(length of each side)
0	$(3)(27) = 81$
1	$(12)(9) = 108$
2	$(48)(3) = 144$
3	$(192)(1) = 192$
4	$(768)(.33) = 253.44$
n	$[3(4)^n][27(1/3)^n]$

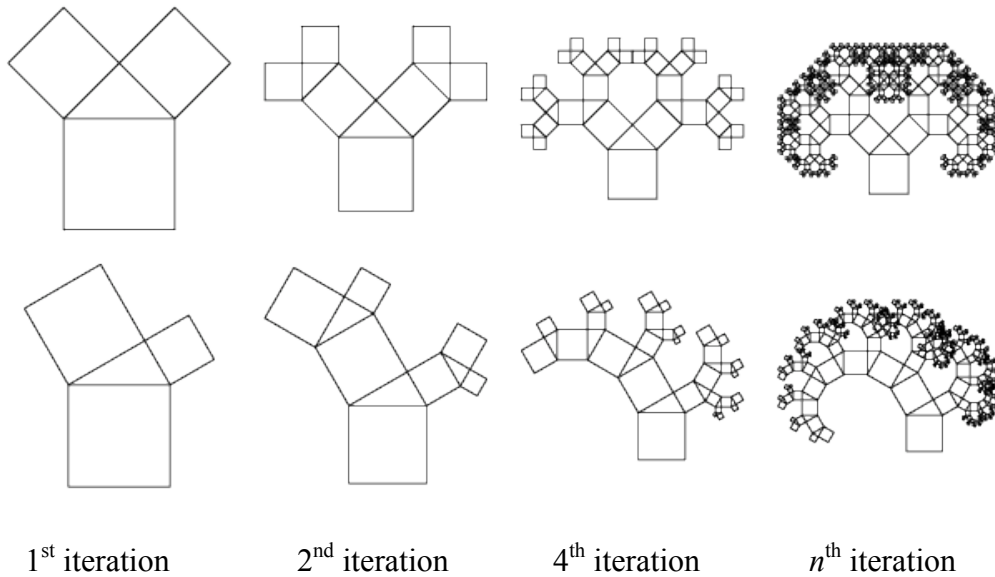
Lesson Five: “Area of the Koch Snowflake”

We will explore the area of Koch Snowflake by constructing the Koch Snowflake on a triangular grid paper. The objective here will be to observe the increasing area while keeping it confined to a limited space. Even though we can continue to enlarge the snowflake, it will never exceed the paper we are using. The students will also have an opportunity to color their snowflake, with each iteration; to help identify the steps involved creating the fractal. This will also enable them to see the relationship between the original snowflake and the additional area.

Lesson Six: “The Pythagoras Tree”

Even though the Pythagoras Tree shows linear growth instead of exponential growth, I think the construction of it is the hardest so far and therefore I saved it for last. Again I will have my students start from scratch for construction of this fractal. Diagonal dot paper may help, but I like to have my students take the time to draw the fractal on plain paper. It helps them to gain a deeper understanding of the properties of a right triangle.

The students will be required to draw a 3” x 3” square at the bottom of a sheet of paper. The top of the square will become the base of their first right triangle, and off of the two legs, the students will draw tilted squares. This is the first iteration of the Pythagoras Tree. Before the students start the 2nd iteration, have the students start constructing their table, showing the total area. This is where the students will have to identify the new area. With branches of the tree developing, it is sometimes hard to keep track of the new area as it continues to grow. The students will discuss how to find the new area. Many may go back to the algorithm $A=bh$. Having had already studied the Pythagorean Theorem, the students now have a chance to review the theorem. This leads into a nice discussion on what the theorem is and what “a”, “b”, and “c” represent. This is also an opportunity for the student to review what they know about the Pythagorean Theorem.



Step	Total Area
0	$C^2 = (3)^2 = 9$
1	$9 + 9 = 18$
2	$9 + 9(2) = 27$
3	$9 + 9(3) = 36$
4	$9 + 9(4) = 45$
n	$9 + 9n$

As the students continue to create the Pythagoras Tree, they will soon see that as the squares get smaller, there are more of them, and adding the initial area to the preceding area creates the new total area. This is a nice connection to the Pythagorean Theorem realizing that the same area is being added.

Lesson Seven: "Creating a Fractal"

At this point the students will have an opportunity to create their own fractal. Either working in pairs or in groups, the students will start with a geometric figure and try to see how they may be able to operate on the figure to create similar figures within the original. The students will write a fixed set of instructions to go along with their fractal. They will trade instructions with another group to see if their fractal could be created from these instructions. From there the students would have to create a table and an equation for their fractal based on either the perimeter or the area. This will take some time so the students should be given sufficient time to work on this project.

Example: Box Fractal



Additional Resources

Students will need an assortment of graph paper and triangular paper to help create the fractals.

Calculators

Rulers

Colored Pencils

Annotated Bibliography

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Appendices-Standards

2.1.8A Represent and use numbers in equivalent forms.

2.1.8B. Simplify numerical expressions involving exponents, scientific notation and using order of operation.

2.1.8E Simplify and expand algebraic expressions using exponential forms.

2.2.8A. Complete calculations by applying the order of operations.

2.2.8B. Add, subtract, multiply and divide different kinds and forms of rational numbers including integers, decimal fraction, percents and proper and improper fractions.

2.2.8G. Represent relationships with tables or graphs in the coordinate plane and verbal or symbolic rules.

2.3.8A. Develop formulas and procedures for determining measurements.

2.3.8G. Create and use scale models.

2.3.8E. Describe how a change in linear dimension of an object affects its perimeter, area, and volume.

2.8.8B. Discover, describe and generalize patterns, including linear, exponential and simple quadratic relationships.

2.8.8C. Create and interpret expressions, equations or inequalities that model problem situations.

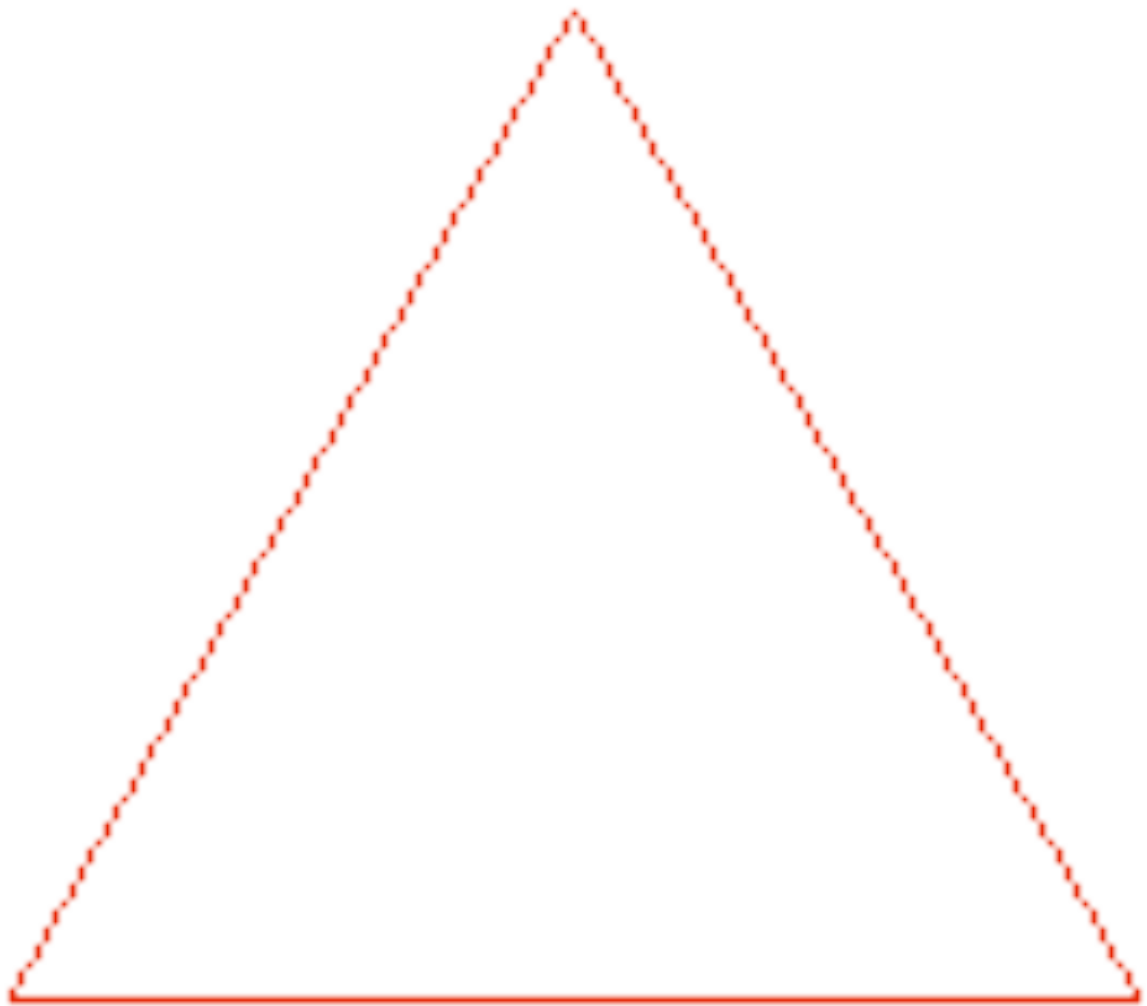
2.9.8K Analyze objects to determine whether they illustrate tessellations, symmetry, congruency, similarity and scale.

2.10.8A. Compute measures of sides and angles using proportions, the Pythagorean Theorem and right triangle relationships.

Appendix A

Name _____

Triangle Worksheet



Appendix B

Name _____

Sierpinski Triangle – Remaining Triangles Worksheet

Step	Remaining Triangles
0	
1	
2	
3	
4	
n	

Appendix C

Name _____

Sierpinski Triangle – Finding Area

Step	Area of Remaining Triangles
0	$1 * \frac{1}{2} bh$
1	
2	
3	
4	
n	

Appendix D

Name _____

Koch Snowflake – Perimeter

Step	Perimeter of the Snowflake (no. of sides)(length of each side)
0	
1	
2	
3	
4	
n	

Appendix E

Name _____

The Pythagoras Tree – Total Area of the Squares

Step	Total Area of the Squares
0	$C^2 =$
1	$C^2 + C^2(1) =$
2	
3	
4	
n	