Why Pi? and Other Math Mysteries

Sarah J. Ricketts
Robert L. Vann School

Overview

Did we create mathematics to explain science, or did we design our study of science to fit the descriptive language we already knew? Why is it that math works so well in relation to the natural sciences? While these questions have puzzled mathematicians and scientists alike for generations, many people take the relationship for granted and never really give much thought to the origins of mathematics or science as separate entities.

This unit will provide 7th and 8th grade students with an opportunity to explore the history and evolution of mathematics in relation to the natural sciences. By design, it will be available as a complete course (10-11 lessons) to be taught at the Pittsburgh Gifted Center, but could be modified for various abilities and classroom dynamics. The unit can also be used in pieces in conjunction with the Connected Mathematics curriculum (pull activities/lessons where appropriate.) It will be a broad overview and introduction to the history of some of the most poignant math concepts, but will focus on those concepts that have major applications in science.

Rationale

The 2007 PTI math seminar focused on *The Unreasonable Effectiveness of Mathematics in the Natural Sciences,* a 1960 essay written by noted mathematician Eugene Wigner. The purpose of Wigner's essay is to provoke debate and/or discussion regarding the phenomenon that mathematics has an uncanny relation to and usefulness in the natural sciences. According to Wigner, "mathematical concepts turn up in entirely unexpected connections… they often permit an unexpectedly close and accurate description in these connections."

Although Pythagoras is the first person recorded as stating that "Mathematics is the way to understand the Universe," (Hamming) Galileo expanded on the concept of math as a universal language:

> *The universe stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering in a dark labyrinth.*

The majority of examples given in Wigner's essay are related to general relativity and quantum mechanics—both subjects which are not easily grasped by the average middle school student. Fortunately, there are many simpler (but no less exciting) avenues to explore when looking for connections between science and the "language" of mathematics.

The idea that students will employ concrete applications to illustrate abstract mathematic concepts is the primary focus of this unit. Unfortunately, math is frequently presented as a theoretical subject, where formulas must be memorized and symbols replace tangible materials. This model tends to create confusion and anxiety in many adolescents. It is our job, as teachers of mid-level mathematics, to present material in tangible manner and lead our students toward their own discoveries of mathematics as a universal language.

Our own attitudes and conceptions of the nature of mathematics greatly determine our students' outlooks. We may view math as *a study of patterns and relationships*, where the focus is on recurring ideas and the determination of how one idea is like or unlike others already learned. Math may also be viewed as *a way of thinking*, or our strategies for organizing, analyzing, and synthesizing information. Perhaps math is simply *an art form,* characterized by order and internal consistency. We also use mathematics as *a language* with "carefully defined terms and symbols which enhance our ability to communicate about science, real-life situations, and math itself." (Reys, p.2) Regardless of which particular spin we put on it, math is the tool we use to solve many of the problems we encounter in life, be they abstract or practical.

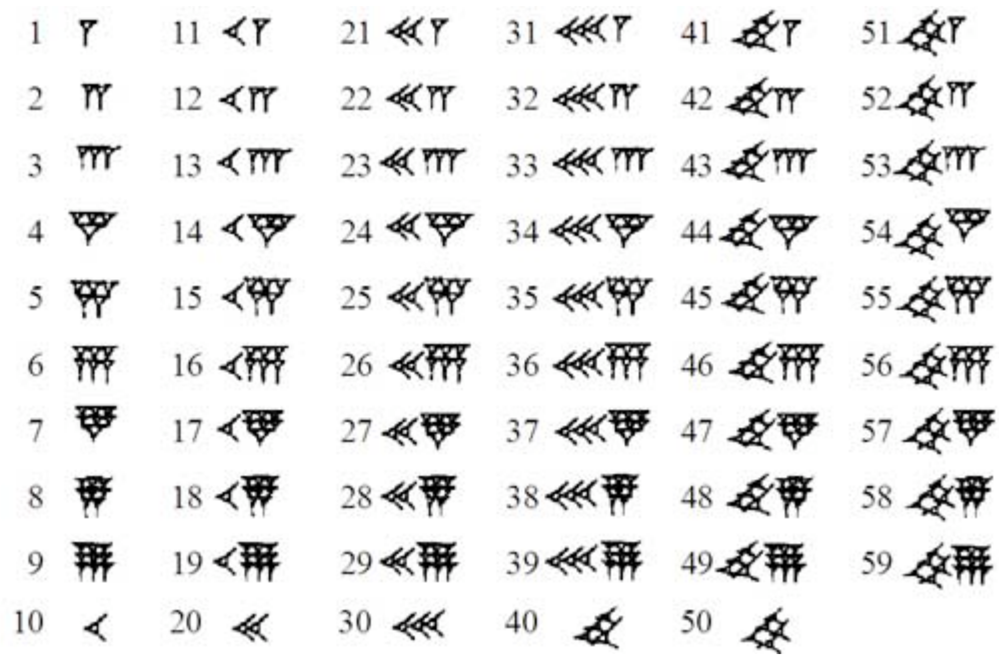
Jean Piaget, best known for his theory of cognitive development, suggested that mathematics is made, or constructed, by children, "not found like a rock nor received from others as a gift." (Reys, p.11) Students create new mathematical knowledge by reflecting on their physical and mental actions. They observe relationships, recognize patterns, and make generalizations and abstractions as they integrate new knowledge into their existing mental structure. To put it simply: Children must be actively involved in their mathematical learning for real ownership to occur. To quote the ancient Chinese proverb:

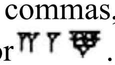
I hear and I forget
I see and I remember
I do and I understand

This constructivist view is one of the major influences of the Connected Mathematics Project, the 6th-8th grade math curriculum currently employed by Pittsburgh Public Schools. CMP supports student- centered investigation of mathematical problems and designs problem content and formats that encourage student-to-student and student-to- teacher dialogue about the work. According to the Connected Mathematics website, "…mathematical understanding is fundamentally a web of logical and psychological connections among ideas [and] sound conceptual understanding is an important foundation for procedural skill, not an incidental and delayed consequence of repeated rote procedural practice."

While the following lessons and activities are not pulled directly from the CMP curriculum, they can easily be incorporated as extensions or alternate investigations. Through the physical explorations in this unit, students will gain a stronger understanding of how and why math works, which will only enhance their regular classroom learning.

Necessity breeds invention, so it makes perfect sense that counting systems came about in response to the desire to keep track of livestock, property, and the passage of time (Seife, p. 5). Written representations of those counting systems can be traced back several thousand years, but the Babylonians are generally credited with the first place-value-based notation. Most famous for their astrological observations and calculations, the Babylonians (c. 1900 B.C.) used a sexagesimal, or base-60, numeral system. There were 59 symbols for the numbers 1-59. The most common Babylonian representations for these numbers were:



For numbers larger than 59, the symbols were repeated in different columns. For example a '2' in the second column from the right meant (2 x 60) =120 and a '2' in the column third from the right meant (2 x 3600) =7200. To use the sexagesimal notation in modern language, we separate the "columns" by commas, so that the number 7267 = 2(3600) + 1(60) + 7 would be written as 2, 1, 7 or .

The Babylonians did not technically have a digit for, or a concept of, the number "zero." They did understand the idea of nothingness, but saw it as merely the lack of a number, so a space marked the nonexistence of a digit in a certain place value. This could obviously lead to ambiguity and confusion in interpretation, so a non-number placeholder symbol was eventually developed.

The digit "0" seems like a simple notion these days; Even a young child, when asked what *zero* means will respond that it means "nothing." The actual origins of zero, however, are surprisingly complicated and tie in well with the development of written number systems. Through comparisons of ancient numerals, including those developed by the Mayan, Egyptian, Greek, Roman, and Arabic cultures (in addition to the previously discussed Babylonian numbers,) students will investigate concepts such as the mysterious birth of "zero."

These investigations should lead into a more complex history of mathematics, particularly the contributions of the Egyptians, Greeks, and Chinese. In fact, although Pythagoras (c. 500 B.C.) is credited with the discovery of the relationships of the three sides of a right triangle ($a^2+b^2=c^2$, where c is the hypotenuse,) the theorem may go back to as early as 1700 B.C. (O'Connor, 1997) It is known in China as the "Gougu theorem", or 勾股定理, for the (3, 4, 5) triangle.

Both pi [π] and phi [φ] have numerous applications in the natural sciences and should be studied at length. The idea of the ratio of a circle's circumference to its diameter as a constant can be traced back to biblical times (O'Connor, 2001). The DNA double-helix is another natural illustration of pi. Phi, or the "golden ratio" is uncannily present in natural design.

Neither pi nor phi are ordinary finite numbers with exact values. They don't have precise numerical specifications, either as decimals or as fractions. In the language of mathematics, these unspecifiable ratios are called "ir*ratio*nal"-- oddities in math's otherwise precise, predictable, perfect, analytical, and abstract conceptions of order in the universe. Nature, however makes profound use of these irrational numbers in multitudes of fundamental forms—from star systems to viruses to shell growth in mollusks.

The study of pi and the golden section inevitably leads to discussion of the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144… .) The Internet has countless articles and activities relating these concepts to the study of natural reproduction. Students will be fascinated to discover the patterns of leaf formation and how they relate to predictable number sequences.

Scientific notation is another concept that should be touched on. Most students don't learn the alternate representation of very large (or very small) numbers until high school chemistry, but its applications can be relevant in simpler scientific studies. This can also be a valuable reinforcement for exponential values, which are learned in middle school.

Numbers in scientific notation are made up of three parts: the coefficient, the base and the exponent. For example, in the following number, 8.56×10^8, 8.56 is the coefficient, 10 is the base, and ^{8} is the exponent. The standard notation of this number would be 856,000,000, which wouldn't fit on most calculator screens. In order for a number to be in correct scientific notation, the following conditions must be true:

1. The **coefficient** must be greater than or equal to 1 and less than 10.
2. The **base** must be 10.
3. The **exponent** must show the number of decimal places that the decimal needs to be moved to change the number to standard notation. A negative exponent means that the decimal is moved to the left when changing to standard notation. (Curran)

Students will be provided ample opportunity to observe practical applications of scientific notation throughout this unit. They will also get personal experience with conversions of numbers from scientific to standard notation and vice-versa.

Measurement of space and time are introduced in elementary math classes, but their correlation to planetary science is undeniable. Through study of the history of measurement, students will be introduced to Kepler's planetary laws, basic Newtonian Physics, quantum mechanics, and relativity theory. Most 7^{th} and 8^{th} graders will not have the math skills necessary for in-depth investigations, but they will be allowed freedom to explore further, if interested.

Johannes Kepler believed that "God's handiwork could be understood only through mathematics." (Hamming) His greatest achievement was the discovery of the laws of planetary motion. The first two, which govern the motion of an individual planet, are:

- Law I (the Ellipse Law)- The curve or path of a planet is an ellipse whose radius vector is measured from the Sun which is fixed at one focus.
- Law II (the Area Law)- The Time taken by a planet to reach a particular position is represented by the area swept out by the radius vector drawn from the fixed Sun. (Davis, 2006)

Time measurement has obvious connections to the sexagesimal, or base-60, numeral system developed by the ancient Babylonians. There are, for example, 60 seconds in a minute and sixty minutes in an hour. The study of the history of time measurement should be a nice wrap-up for the unit, since it will bring us back full-circle to where we began our journey.

It is important to acknowledge the skill level and educational background of the intended audience. Even the more sophisticated students will not have much exposure to advanced mathematics such as calculus and non-Euclidian geometry. Topics presented should be somewhat familiar so as not to alienate learners. There should also be a natural flow from one topic to the next, which will pique student curiosity and enthusiasm.

Objectives

Students will explore the development of counting systems, including base-2, base-5 (vigesimal), base-10, and base-60 (sexagesimal). They will then develop an original counting system of their own that is not simply a representation of tally marks or a 1-to-one correspondence. Students will also investigate the origins of zero as a number, and identify landmarks in the early history of mathematics.

Once the students have thoroughly explored the origins of rational numbers, irrational numbers (particularly pi [π] and phi [φ]) will be introduced. One possible investigation of phi could include comparing students' navel heights to their over-all heights. They will also discover the nature of pi through measurement of real-life circles and come to their own conclusions regarding the idea of constants and number patterns in nature.

Students will employ various technology tools, in addition to traditional methods, to develop geometric constructions. They will be introduced to Sketchpad software and compare outcomes to those reached through use of Euclidian tools (compass and straight-edge.)

The students will also familiarize themselves with the application of scientific notation for representations of very large or very small numbers. They will apply this alternate notation in inquiry-based activities that require multiple-step algorithms in order to properly represent collected data.

Finally, students will investigate the connections between the history of measurement (both space and time) and the discovery of many of the physical laws, including Kepler's planetary laws and Newton's law of gravity. They will discover the correlation between modern time measurement and the Babylonian's sexagesimal counting system.

Any unit planned for use in the Pittsburgh Public Schools must correlate to Pennsylvania Academic Standards. *Why Pi? and other Math Mysteries* is designed to touch every standard, at least in part. The study of rational and irrational number systems is highlighted in Standards 2.1 (*Numbers, Number Systems, and Number Relationships*) and 2.2 (*Computation and Estimation.*) Pi and phi, and their uses in measurement and geometry, align with Standards 2.3 and 2.9. While the remaining standards will all be addressed, the most pertinent will most likely be Standard 2.8 (*Algebra and Functions.*) Throughout their investigations, students will apply simple algebraic patterns to basic number theory and to spatial relations. They will also use concrete objects to model algebraic concepts.

Strategies

Motivation of students is a key factor in the success of any unit, but the study of mathematics is particularly daunting. *Why Pi? and Other Math Mysteries* is designed to create enthusiasm and interest from the very first lesson. Students will be actively involved in their inquiries and will be allowed to explore topics that interest them, while growing significantly in their comprehension of some of mathematics' more sophisticated concepts.

There will be some direct instruction in this unit, but Students will primarily be introduced to new concepts through brief videos and questioning, then jump into hands-on learning activities, with the teacher acting as a facilitator.

Technological resources will be used throughout the unit, including streaming videos from Discovery Learning and BrainPop, Java applets, geometric sketchpad software, multi-media presentation applications, and various websites. Both Discovery Learning and BrainPop require subscriptions for full access, but it is possible to register, at no cost, for a free trial of both programs. The software required for some activities, such as HyperStudio and Geometer's Sketchpad, are available at many schools, but free trials are also available online.

Classroom Activities

The following are descriptions of intended activities for *Why Pi?and Other Math Mysteries.* I have included several complete lesson plans to illustrate the progression of the unit, but other activities are described in briefer narrative form.

Lesson 1: Development of Counting Systems

Objective:
Students will compare similarities and differences of various ancient numeral systems.

Materials:
whiteboard or chalkboard with marker or chalk
Ancient Numeral Systems handout (see Appendix B)
Ancient Arithmetic worksheet (see Appendix B)
pencils

Procedures:

1. *Launch*- As students enter the room, they should observe pre-written examples on the board. This may work best if a particular number (19, for example) is represented in Babylonian, Egyptian, Mayan, and Roman numerals. After they have had a chance to share their ideas about what the symbols represent, lead a brief discussion on the origins of counting systems, paying particular attention to the reasons a society, or civilization, might develop an original counting system.

2. *Instruction*- Pass out and review *Ancient Numeral Systems* handout. Using the charts on the handout, have the students identify the numbers on the board. Explain that it is a very basic overview of four different systems. Students who want to explore these systems further should be provided with resources for research.

3. *Independent practice*- Pass out *Ancient Arithmetic* worksheet. Allow students to work with partners to complete the computations.

4. *Closing*- Review *Ancient Arithmetic* worksheets. Discuss any difficulties the students encountered. This should bring up the problem of non- base-10 systems. Explain that the class will be working with these concepts more closely in the next lesson

Assessment:
Informal, ongoing observation during discussion and evaluation of *Ancient Arithmetic* assignment

Lesson 2: Binary/base 5/base 10/ base 60 (reflect on Babylon)

This lesson should clarify non-base 10 number systems a bit. I intend to present an activity titled "Count the Dots" that makes binary code simple to understand. "Count the Dots" comes from a program entitled "Computer Science Unplugged" and involves cards with dots that represent the values of the first five places in base 2 representation (1, 2, 4, 8, and 16). After a brief introduction to the cards and their values, five students will line up in front of the class with the cards that correspond to their positions. They will work together to represent various given numbers, up to 31. The idea behind this activity is that if a card's face is showing, that place would have a "1" in binary code. If the face is not showing, the place would have a "0".

After several examples of base 2 representation, base 5 will be introduced. Students will find the values of numbers written in base 5 and attempt to add and subtract up to 3-digit numbers. This will be very difficult for some students, but others will grasp the concept quickly and assist their struggling classmates.

As a wrap-up, the class should take another look at the Babylonian numerals and attempt to clarify their understanding of base 60.

Lesson 3: Original counting system- Can you do better?

Objective: Students will work in small groups to develop an original counting system that does not simply rely on tally marks or one-to-one correspondence.

Materials:
Ancient Numeral Systems handout (from Lesson #1)
Paper and pencils
Chart paper and markers

Procedure:

1. *Launch*- Review Lesson #1. Discuss various counting systems and symbols used by other cultures. Separate students into groups of 3 or 4.

2. *Instruction*- Students should work in their groups to develop an original counting system. It is important to stress that these systems should not be tally marks. Allow approximately 30 minutes for group work. Completed projects should be written on chart paper and contain the following information:
   - Number representations up to at least "one thousand"
   - Written examples of at least five different multi-digit numbers
   - At least one addition and one subtraction problem represented in the original number system

3. *Closing*- Allow each group to present their number systems to the class. Provide time for questions.

Assessment: Use the following three-point rubric to evaluate students' work during this lesson.

• **3 points:** Students actively participated in group project; all criteria were present on chart paper; demonstrated a clear understanding of written number systems during activity and presentation.

• **2 points:** Students participated in group work; most criteria were present on chart paper; demonstrated a satisfactory understanding of written number systems during activity and presentation.

• **1 point:** Students did not participate in group work; little criteria were present on chart paper; demonstrated a poor understanding of written number systems during activity and presentation.

Lesson 4: Reflect on Babylonian/Greek, etc. contributions

In this two-day activity, students will revisit the contributions of ancient civilizations to the study of mathematics. They will use various resources, including books and Internet, to further study Babylonian mathematics.

The primary focus of this activity will be the Pythagorean Theorem. Students will manipulate a pre-programmed Geometer's Sketchpad script to discover exactly *how* and *why* $a^2+b^2=c^2$. They will also use the Sketchpad software to draw simple geometric constructions, such as angles and circles with defined radii. After they have completed the virtual constructions, they will be given an opportunity to construct the same angles and circles with traditional Euclidean tools (compass and straight edge.)

Lesson 5: Origins of "Zero"

In this mini-lesson, we will discuss what life would be like without "Zero." This will be a very difficult concept for the students to grasp at first. They will be reminded that the Babylonians were the first to recognize "zero" as a digit, originally using it as a place-holder in their numeric representations. Hopefully, the students will be able to remove themselves from their attachment to zero as a digit and truly consider how different math would be without it.

Lesson 6: Introduction to irrationals

Ideally, any lesson on pi (π) would be introduced on March 14^{th}. Students will discover the nature of pi by measuring circumferences (c) and diameters (d) of various circular objects. They will then divide c by d and note the relative consistency of their results. We will discuss the reasons that they may not have come up with exactly 3.14… (They will most definitely not.) Depending on time and interest, the class will research and report on various calculations for pi throughout history.

Once the students are comfortable with the concept of pi as an irrational (non-terminating/non-repeating decimal), they will be introduced to another fantastic irrational, phi (φ). Students will explore the National Library of Virtual Manipulatives applet "The Golden Ratio." If time permits, students will be encouraged to measure and compare various parts of their bodies, particularly the ratio of their navel heights (ground to bellybutton) to their total heights. They will also be given opportunities to research examples of the golden ratio in nature.

Lesson 7: The Fibonacci Sequence

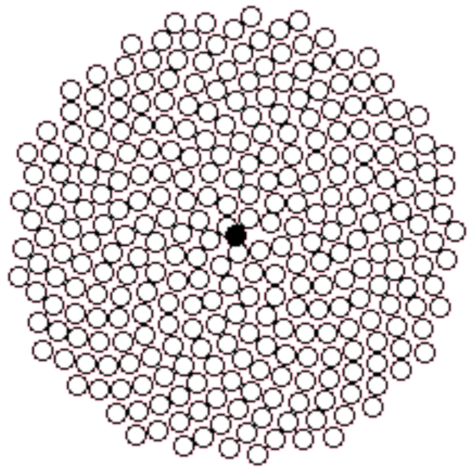
Objective: Students will become familiar with the Fibonacci sequence and its relationship with the natural world.

Materials:
Computer with Internet and media capabilities
BrainPop.com video: *The Fibonacci Sequence*
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html
Creating the Fibonacci Spiral handout (See Appendix B)
Java applet: *Fibonacci Sequence- National Library of Virtual Manipulatives:*
http://nlvm.usu.edu/en/nav/vlibrary.html
Chart paper and markers

Procedures:

1. *Launch*- Before class begins, write the first ten numbers in the Fibonacci sequence on the board: 0,1,1,2,3,5,8,13,21,34. Allow students to attempt pattern recognition. Show BrainPop video (free at brainpop.com), and complete the virtual quiz as a class. Briefly discuss the fact that the Fibonacci sequence has intrigued mathematicians for years. These numbers appear in many patterns in nature, often creating beauty. Tell students that they will look for Fibonacci numbers in objects from nature.

2. *Instruction*- Divide students into groups of three or four. Show the following diagram of a seed head from a sunflower plant. Tell students to look for Fibonacci numbers in the spirals. For more images, go to http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html



Each small circle on the illustration represents a seed, all of which form a spiral from the center. Use two different colored pencils to mark the clockwise and counterclockwise spirals. Starting with an outside circle, trace the spiral shape through the circles that define one spiral. (Note: Some circles will not be used.) Count the clockwise spirals, then count the counterclockwise spirals. The numbers should be consecutive Fibonacci numbers.

3. *Practice*- Assign the *Creating the Fibonacci Sequence* activity and have students share their drawings. Students who finish early may explore the Java applet on the National Library of Virtual Manipulatives website.

4. *Closing*- Brainstorm with students to name plants and animals that have spiral shapes, such as pinecones, snails and nautilus shells. Write these plants and animals on chart paper and display them in the classroom.

Assessment: Use the following three-point rubric to evaluate students' work during this lesson.

• **3 points:** Students participated actively in classroom discussions and worked cooperatively to complete the in-class activity.

• **2 points:** Students participated to some degree in classroom discussions; worked somewhat cooperatively to complete the in-class activity.

• **1 point:** Students participated very little in class discussions and attempted to work cooperatively to complete the in-class activity.

Lesson 7: Scientific Notation

Students will view the BrainPop video, *Scientific Notation*, and take a 10 point quiz immediately thereafter. We will also practice converting numbers from standard notation to scientific notation and vice-versa as a class.

Lesson 8: History of Measurement (space)

Objectives:
Students will discuss the importance of accurate measurements and recall how units of measurement have been calculated throughout history.

They will also use their own feet as a standard measurement, and then measure and compare distances.

Materials:

Computer with Internet and media capabilities

Forensic Detectives: Lengths and Heights video from United Streaming
rulers or tape measures (one for every two students)
quarters (one for every two students)
paper and pencils

Procedures:

1. *Launch*- Assign four groups to determine the distances between the places listed below. Use MapQuest (http://www.mapquest.com/directions/) for those in the U.S. and the Distance Calculator (http://www.indo.com/cgi-bin/dist) for international distances.

   • Distance between school and the closest shopping mall

   • Distance between school and a school in another town in your state

   • Distance between Pittsburgh and a large city elsewhere the United States

   • Distance between Pittsburgh and Paris, France, or any other foreign city

2. *Lesson*- Watch *Measure for Measure: Lengths and Heights*, and point out that the narrator says, "Measurement rules our lives. It has sliced up our world and helped us impose order and logic on our restless universe." Ask students: What do you think the narrator means? What would a world with no measurements be like? Then, have students discuss how people featured in the program (below) used measurements.

   a. Sailors calculate positions of stars and the accurate time to determine longitude and latitude at sea.

   b. Scientists measure the wavelengths, speeds, and heights of tsunamis, as well as underwater pressure to detect incoming tsunamis.

c. The rower measured his heart rate.

Have students work with a partner to measure the length of their feet without shoes. Have them record this measurement in standard inches.

Length of my foot in standard inches: _____________

Using their personal unit measurement, have students determine the length of a personal non-standard inch. They should divide the length of their foot by 12. (Example: If a student's foot measures 8 inches, a non-standard inch would be 8 divided by 12 = .6.) Students may round their answer to the nearest eighth.

Length of my personal inch in standard inches: ________

Have students draw two squares with four-inch sides, one using standard inches, the other using personal inches. (Using the personal non-standard inch example above, the second square would have 2.5-inch sides.)

Have students attach their personal squares to the board. Discuss the consequences of countries using non-standard measurements. For example, what if one country supplied parts for a machine produced in another country?

Now students will consider their personal units of measurement on a larger scale. How many personal feet would go into a mile? (5,280 standard feet = one mile) When comparing feet to miles, the numbers are very large, so it will be easier to compare standard miles and new miles using a ratio. First, determine the ratio of the standard foot to a personal foot. For example, 12 inches to 8 inches is 12:8, or 1.5. In other words, a standard foot is 1.5 times larger than the personal foot. Since ratios are constant, you can also say that a standard mile is 1.5 times longer than a personal mile. That means a distance of 100 standard miles would measure 150 personal miles. (They will multiply the distance times 1.5: 100 X 1.5 = 150))

Give students a chart to show distances between your school or town and different locations. Then have them use the ratio to determine the distances with their personal measurement system. For example, a completed chart might look like this:

Distance From	**Standard Miles**	**Personal Miles**
school to mall	5 miles	7.5 miles
Pittsburgh to Erie	128 miles	
Pittsburgh to New York, New York	370 miles	

Pittsburgh to Los Angeles, California	2430

3. *Closing*- Have students share some of the distances they determined. Then discuss the consequences of using non-standard units of measurement. Pose these questions to the class: What would happen if you gave directions to the mall using personal miles? What if a pilot used personal miles to calculate the distance on an international flight to Paris? Why are standard measurements so important? Ask students to share other examples of why accurate measurements are critical.

Assessment:

Use the following three-point rubric to evaluate students' work during this lesson.

• **3 points:** Students actively participated in class discussions about measurements; accurately measured their own feet; correctly calculated all of the lengths and distances; demonstrated a clear understanding of the importance of standard measurements.

• **2 points:** Students participated in class discussions about measurements; closely measured their own feet; correctly calculated most of the lengths and distances; demonstrated a satisfactory understanding of the importance of standard measurements.

• **1 point:** Students did not participate in class discussions about measurements; made sloppy, inaccurate measurements of their own feet; could not calculate the lengths and distances; demonstrated a poor understanding of the importance of standard measurements.

Source: Discovery Education. © 2005

Lesson 9: History of measurement (time)

Students will work in small groups to research the history of time. In their groups, they will create time lines with the dates of important breakthroughs in our understanding of this phenomenon including the dates of inventions for measuring and keeping track of time. For example, students may note when Saint Benedict made his contribution to the measurement of time, what the contribution was, and the cultural consequences that grew out of his contribution. Students will be encouraged to use illustrations from their sources to decorate the time line. They might include for the year during which the time line is constructed the cultural events and celebrations that are dependent on astronomical observations—for example, when will or did Easter occur in that year?

Source: Discovery Education

Lesson 10: Laws of physics

This activity will be an introduction to the advanced concepts of physics. We will begin by discussing what students know about Sir Isaac Newton. They will probably mention something about the apple falling out of the tree and his "discovering" gravity. We will then watch the BrainPop video: *Newton's Laws of Motion* and complete the virtual quiz as a class.

After the quiz, we will discuss equations that the students may already be familiar with ($e=mc^2$, $r=d/t$, etc) We will identify the variables and solve sample problems, then truly ponder the question: *Did we create mathematics to explain science, or did we design our study of science to fit the descriptive language we already knew?*

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Appendix A- PA Academic Standards

2.1.8 Numbers, Number Systems, and Number Relationships

C. Distinguish between and order rational and irrational numbers.

D. Apply ratio and proportion to mathematical problem situations involving distance, rate, time and similar triangles.

G. Use the inverse relationships between addition, subtraction, multiplication, division, exponentiation and root extraction to determine unknown quantities in equations.

2.2.8 Computation and Estimation

C. Estimate the value of irrational numbers

F. Identify the difference between exact value and approximation and determine which is appropriate for a given situation.

2.3.8 Measurement and Estimation

A. Develop formulas and measurements for determining measurements

D. Estimate, use, and describe measures of distance, rate, perimeter, area, mass, and angles.

2.8.8 Algebra and Functions

D. Use concrete objects to model algebraic concepts.

2.9.8 Geometry

G. Approximate the value of pi through experimentation.

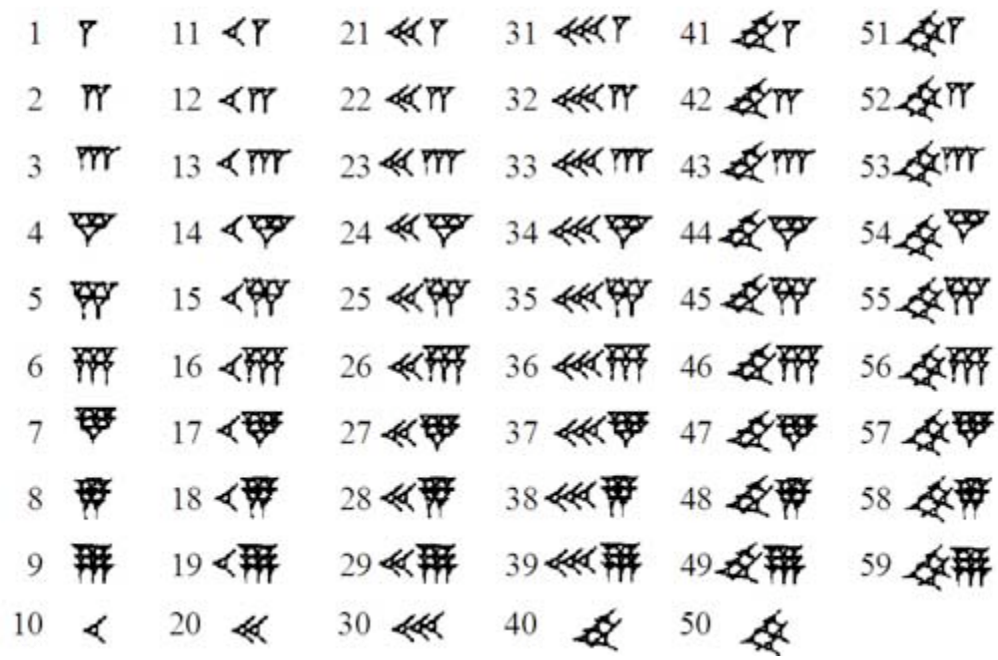
Appendix B- Handouts

Ancient Numeral Systems
Ancient Arithmetic
Creating the Fibonacci Sequence

Ancient Numeral Systems

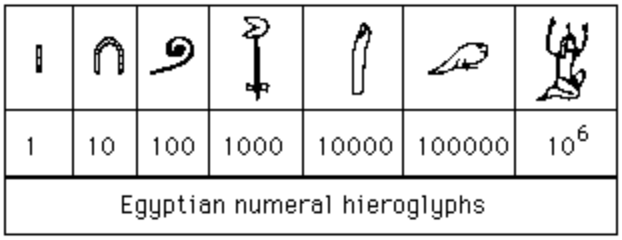
Babylonian, circa 1900 B.C.

The Babylonians are generally credited with the first place-value-based notation. Most famous for their astrological observations and calculations, the Babylonians (c. 1900 B.C.) used a sexagesimal, or base-60, numeral system. There were 59 symbols for the numbers 1-59. The most common Babylonian representations for these numbers were:

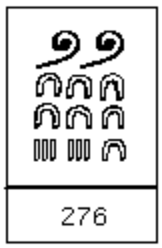


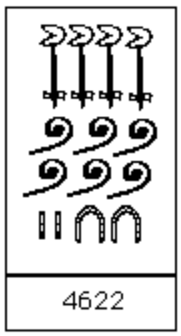
For numbers larger than 59, the symbols were repeated in different columns. For example a '2' in the second column from the right meant (2 x 60) =120 and a '2' in the column third from the right meant (2 x 3600) =7200. To use the sexagesimal notation in modern language, we separate the "columns" by commas, so that the number 7267 = 2(3600) + 1(60) + 7 would be written as 2, 1, 7 or  .

Egyptian, circa 300 B.C.



Examples:

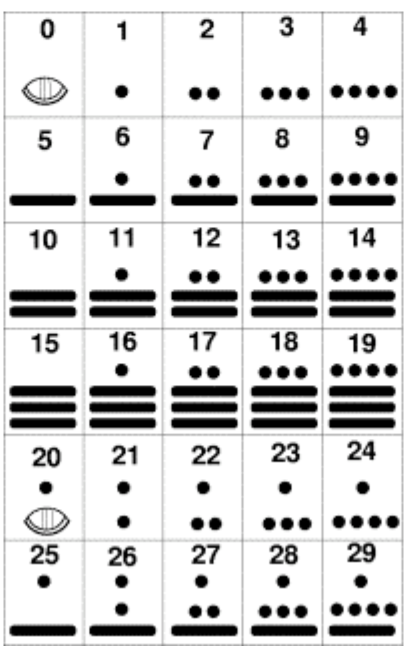




The Egyptians had a base 10 system of hieroglyphs for numerals.

This means that they had separate symbols for one unit, one ten, one hundred, one thousand, one ten thousand, one hundred thousand, and one million.

Mayan, circa 36 B.C.



The Mayan numerals are made up of three symbols; zero (shell shape), one (a dot) and five (a bar).

For example, 19 is written as four dots in a horizontal row above three horizontal lines stacked upon each other.

Digits are stacked with the higher significant digits at the top. Thus, two dots above each other would be read as 1□20 + 1 = 21.

Roman, circa 300 A.D.

Roman numerals are written as combinations of the seven letters in the table below. The letters can be written as capital (XVI) or lower-case letters (xvi).

Roman Numerals

I = 1	C = 100
V = 5	D = 500
X = 10	M = 1000
L = 50	

If smaller numbers follow larger numbers, the numbers are added. If a smaller number precedes a larger number, the smaller number is subtracted from the larger. For example:

- VIII = 5+3 = 8
- IX = 10-1 = 9
- XL = 50-10 = 40
- XC = 100-10 = 90
- MCMLXXXIV = 1000+(1000-100)+50+30+(5-1) = 1984

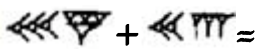
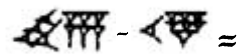
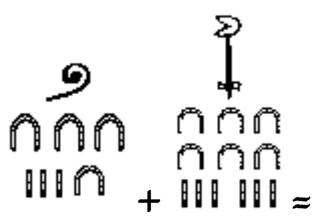
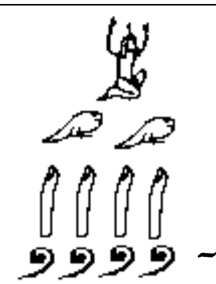
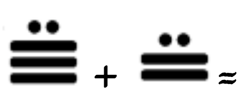
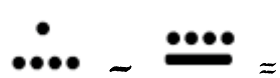
Roman Numeral Table

1	I	14	XIV	27	XXVII	150	CL
2	II	15	XV	28	XXVIII	200	CC
3	III	16	XVI	29	XXIX	300	CCC
4	IV	17	XVII	30	XXX	400	CD
5	V	18	XVIII	31	XXXI	500	D
6	VI	19	XIX	40	XL	600	DC
7	VII	20	XX	50	L	700	DCC
8	VIII	21	XXI	60	LX	800	DCCC
9	IX	22	XXII	70	LXX	900	CM
10	X	23	XXIII	80	LXXX	1000	M
11	XI	24	XXIV	90	XC	1600	MDC
12	XII	25	XXV	100	C	1700	MDCC

13	XIII	26	XXVI	101	CI	1900	MCM

Ancient Arithmetic

Use your "Ancient Numeral Systems" handout to complete the following addition and subtraction problems. Be sure to write your final answers in the appropriate number system.

1.  ______	2.  ______
3.  __________	4.   __________
5.  ______	6.  ______

7. LVIII + CCCXIV+ ______	8. CMXXI - CDXXI=_______

Creating the Fibonacci Spiral

Materials

- sheet of quarter-inch grid paper
- ruler
- compass

Follow the directions and watch the spiral emerge. (Each successive square will have one edge with a length the sum of the two squares immediately preceding it.)

1. Draw a 1-inch square. Draw a second one-inch square to its left, making sure the squares touch.
2. Draw a 2-inch square above the two one-inch squares, touching the lower squares.
3. Draw a 3-inch square to the right of the three existing squares; its left side should touch the other squares.
4. Draw a 5-inch square below these squares.
5. Draw an 8-inch square to the left of the existing five squares.
6. Draw a 13-inch square above the six squares.
7. Use a compass to complete the drawing. Within each square of your drawing, draw an arc from one corner to the opposite corner. (Each arc will have a radius equal to the length of one side of its square.) Connect each arc to the next. To begin, place your pencil in the upper-right corner of the original 1-inch square and draw an arc toward the lower-left corner. In the second square, draw an arc from that point (the lower-right corner) to the upper-left corner of the second square. Continue drawing arcs in each square, starting each arc at the point where the last one ended.

8. You will create a logarithmic spiral. What forms in nature reflect this shape?