Problem Solving, Puzzles and Patterns

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Contents of Curriculum Unit

Overview

By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages.

-- National Council of Teachers of Mathematics (Cushner 320)

Throughout the seminar *Problem Solving: Where Education Interacts with Life*, I was given the opportunity to examine numerous problem solving situations and learn various approaches to explain the solution/s. As a high school mathematics teacher, I was very enthusiastic to have the opportunity to participate in this seminar. My current teaching schedule consists of Advanced Placement AB Calculus and Center of Advanced Studies (CAS) Gifted Algebra 2. The curriculum unit I created in this seminar will be integrated into my CAS Algebra 2 class. Regardless whether or not a teacher teaches Advanced Placement, Gifted, Scholars, or Mainstream students, all can benefit from this unit. The advantage of using this curriculum unit is that teachers can incorporate pieces of it, in any level of a classroom. My students are continuously challenged with problem situations that are not in our textbook and are very receptive to different activities in the classroom. This seminar provided me with the tools necessary to enlighten my students even further. Even though I have a specific curriculum to cover, I think it is important to inform my students of a variety of techniques that can be used to solve problems. Every lesson I present has students dealing to some extent with the idea of problem solving, however it is more systematic. With this curriculum unit I plan to take the concept of problem solving a step further. However, due to staying on schedule with our district's curriculum, I do not have the freedom to venture off and add additional assignments that take up a lot of classroom time. At the same time, since I am teaching gifted students, it is expected that I incorporate assignments beyond the mandated district curriculum.

Teaching this unit to my CAS Algebra 2 students gives me the opportunity to treat it as an extension to the gifted curriculum. "The term 'mentally gifted' includes a person who has an IQ of 130 or higher, when multiple criteria as set forth in Department Guidelines indicate gifted ability. Determination of gifted ability is not based on IQ score alone. A person with an IQ score lower than 130 may be admitted to gifted programs when other educational criteria in the profile of the person strongly indicate gifted ability. Determination of mentally gifted includes an assessment by a certified school psychologist" (CAS Centers for Advanced Study Teacher Handbook 7). "The CAS program has been designed to meet the needs of the gifted students for individualized, accelerated and enriched learning. All CAS classes emphasize an inquiry approach to learning, problem solving techniques and the higher cognitive skills of analysis, synthesis and evaluation" (CAS Centers for Advanced Study Teacher Handbook 3). The CAS class sizes are limited to 10 – 18 students which helps allow more individualized attention. In addition to the regular course work for each class, students are required to complete a Long Term Project (LTP). The LTP should focus on a topic that interests the students because they have to research it in depth, work a minimum of 30 hours outside of the classroom, and then do a presentation to their class.

The main goal of this unit is to find open-ended problem solving activities that I can associate with specific topics I'll be teaching. I can substitute a day of book teaching and have students work in groups on problem solving activities, as well as incorporate problem solving activities as warm-ups. Each lesson that I develop for this curriculum unit will be independent. If another teacher would like to use a portion of my curriculum unit they will be able to easily incorporate it into one of their lessons. I know from my own experience of trying to teach what is expected for a particular subject, it is sometimes very difficult to try to add anything else into your school year. You can always do as much or as little as you see fit. Also, once you do one or two lessons, you may decide to assign some of the other lessons as homework for students to do outside of the classroom. This is still a beneficial enrichment activity and you do not have to use all of your class time to complete it.

In all of the lessons for this unit, students are expected to write and explain their findings. "When students explain mathematical concepts in writing, they learn to clearly and accurately communicate and by thinking about these concepts and writing about them in their own words, they may better understand and better remember them" (Williams 187).

Rationale

As a high school mathematics teacher, I am looking forward to incorporating more problem solving activities into my classroom. This will be accomplished through warm-ups, class work or homework assignments. Students will be given open-ended problem solving activities; they may work individually or in groups. I have noticed when students are working in groups, the answers may not always come to them quickly, but they persevere until they determine the solution. I am in awe of my students' problem solving techniques and competitiveness in the classroom. Once a group is able to arrive at a solution, the other groups feel as though they need to be the next group to complete the task. I am aware, as are they, that who completes an activity first is not important.

Why would someone reading this curriculum unit on problem solving want to try the lessons in their classroom? I am confident mathematics teachers will find the lessons and activities interesting and beneficial. I tried to ensure activities I created correspond to particular subjects and/or topics in the textbook. I also found open-ended problem solving activities that made connections between mathematics and the real-world.

This unit is not like something from a traditional textbook. Our students will be given the opportunity to learn about topics that otherwise they may never be exposed to. I cannot begin to explain how important I think it is when teaching to make connections between education and the real-world. I believe once students are exposed to this unit, they will want to pursue problem solving at greater length. I am expecting this unit to reach some students that may not have been engaged in their studies previously. If I am able to spark an interest in my students, I would be delighted.

As a teacher it is my responsibility to learn about new developments in the teaching world, particularly mathematics. Being a member of the National Council of Teachers of Mathematics, I receive a monthly subscription to the magazine, the <u>Mathematics Teacher</u>. Over the past few years I've noticed problem solving is becoming an important topic discussed in the magazine. Not only are there numerous articles published, but every month there is a calendar which contains a problem for every day. I incorporated some of the problems into warm-up assignments for this unit. Prior to this seminar, I have never attended any classes dealing with the inclusion of problem solving in the curriculum. Due to what I learned in this seminar, I will incorporate more problem solving activities in my future lesson plans. Even though I have a specific curriculum to cover in my existing classes, I think it is important to constantly challenge our students and add more activities which will promote their mathematical growth. Hopefully my students will enjoy doing more problem solving activities in their mathematics class.

In addition to daily warm-ups, this curriculum unit also includes more involved problem solving activities. Lesson 2 of this unit is the ladder problem. I incorporate this activity while teaching Chapter 4, Quadratic Functions and Factoring, where roots are discussed. Students are expected to know concepts such as the square of numbers, square roots of numbers, and the Pythagorean Theorem. Some students grasp these concepts quicker than others and share their solving problem skills with the class. Units are also an important concept in this activity. The problem is explained in feet but the answer is to be given in inches.

The next problem solving activity, Lesson 3, is the Barn Problem. I usually have my class try this problem early in September. Because it deals with geometry concepts, I assume the formulas that are needed to solve the problem are fresh in their minds from their last year geometry class. This is one of my favorite problems to give to my students. Students have to sketch a picture, and determine lengths and areas in order to calculate the correct answer. There is a great deal of problem solving taking place in this activity.

The fourth lesson in this unit is the Making Washers Problem. I have used this problem in previous years and found it to be a good problem solving activity. I was first given this task (which is a released sample open-ended task for 11^{th} grade students) while attending a workshop in the district on preparing 11^{th} grade students for the PSSA. I usually have students work in pairs during a class period to complete this task. What I like most about this problem is it asks students to calculate not only the volume of the washer, but also its weight given the metal used is 5 ounces per cubic inch. Students are required to label the figure, show all their work mathematically, and then explain the steps used to justify their answers.

In 2003, I became a Nationally Board Certified Teacher (NBCT). The National Board of Professional Teaching Standards (NBPTS) was created in 1987 to advance the quality and teaching of learning by:

- Maintaining high and rigorous standards for what accomplished teachers should know and be able to do.
- Providing a national voluntary system certifying teachers who meet these standards.

- Advocating related education reforms to integrate National Board Certification in American education and to capitalize on the expertise of National Board Certified Teachers. (NBPTS, Mission)

In order to become a NBCT, I had to submit a portfolio with four entries and pass a 3 hour mathematics exam which consisted of 6 questions ranging from Algebra 1 to Calculus. One of the entries I submitted included a 20 minute video of my Scholars Algebra 2 class discussing the difference of squares (Appendix 3 for the assignment and Appendix 4 for a table the students used to record their solutions). I incorporated this activity when I was teaching the unit on factoring polynomials. After I taught this unit, I spent an entire class period discussing the difference of square numbers. Each student was given a list of numbers ranging from one to 40. As a class, we discussed which numbers could be written using differences between two square numbers. If a number could be written in more than one way, the students were to write both solutions. After we completed the list of numbers, we started to discuss some of the patterns and properties of the square numbers. We had very engaging conversations with interesting points that enabled students to draw from the knowledge of other students.

The final lesson in this unit explores the relationship between two variables. This is an extremely valuable lesson because it prepares students for functions. During this lesson I also introduce the use of graphing calculators for graphing purposes. This is a nice refresher for students who have already been exposed to functions and it is a wonderful way to introduce students to graphing calculators and linear regressions. The lesson objectives are for students to understand the concepts of correlation and slope, how to collect data, create a scatter plot, determine the linear regression, find specific measurement of various arm spans and heights, and interpret the meaning of individual coordinates as well as the complete graph. (Appendix 5) The understanding of these concepts is essential for success in my Scholar Algebra 2 class. This course emphasizes manipulative skills of Algebra and solving real life problems. In addition to learning new mathematics, students will investigate how mathematics is used in the world around them. Technology plays an important role in this course because students are expected to solve problems algebraically and support their findings graphically. All of these goals are important for my students because the first semester is the study of linears (equations, inequalities, and systems), algebraic simplifications of polynomial and rational expressions, and related problem solving applications. For my students to be successful in this course, they have to have a thorough understanding of linear functions because the second semester builds on the first. The second semester is the study of radicals, complex numbers, quadratics (equations, functions, and systems), conics, exponential and logarithmic functions, probability, and trigonometric functions.

This lesson actually took four class periods to complete. The first day I had students form groups of four. They helped each other measure their arm span and height and recorded their data on the overhead projector and on a sheet of paper (Appendix 6). The groups had to enter the data points in list one and list two of their graphing calculator and find the scatter plot and plot the data points on their graph paper (Appendix 7). The second day students worked in their groups and continued with the worksheet (Appendix 8) I had given them the day before. Each student had to pick any two points and create a line that they thought would be the best fit and calculate the exact line. Finally, the students found the linear regression with their calculators, and discussed and compared what the linear regression line looked like compared to their line from their two points. The third day was spent discussing the concept of slope and what it means in this problem. Students were given a person's arm span and asked to find their height. Then they were given a person's height and asked to find the arm span. The fourth day was spent as a whole class discussion, with data from the day before on the overhead projector. We discussed commonly occurring topics from the group work of the previous days. Some of our discussion topics included: Why were some of the students' lines close to the linear regression while others were not? How important is slope in this lesson? What is correlation? What is an outlier? Students gave examples of other situations where a linear regression was present and explained what can be determined using a linear regression. Mastering the concept of linear regressions prepared students for future work with quadratic and exponential regressions. The assignment was time intensive and a lot of work, but the students gained valuable knowledge from this exercise. The overall objectives were successfully met while working in small groups.

The technology used in this assignment was imperative for the success of this lesson. Students were expected not only to determine a line through any two points, but they were expected to enter their data points into a graphing calculator and determine a linear regression or line of best fit. We needed to use the graphing calculators in order to complete this part of the assignment. It is the expectation that once a student leaves my algebra class they can successfully use a graphing calculator for future mathematics classes. Once students fully understand the concepts presented in this lesson, they have a strong foundation that they can continue to build on throughout the year.

When I was in college I had to read G. Polya's, *How to Solve It: A New Aspect of Mathematical Method.* I still have this book and follow Polya's ideas of how to approach problem solving. He states four steps necessary to solving problems:

Step 1: Understanding the problem
Step 2: Devising a plan
Step 3: Carrying out the plan

Step 4: Looking back

These steps may seem very basic, but I find it necessary to reinforce these steps in order to be successful in problem solving. It is essential to understand the problem before you try to solve it. "What is unknown? What are the data? What is the condition? Draw a figure. Introduce suitable notation" (Polya xvi). The first thing I teach my students is to write down all the important information that is given in the problem situation. Write what is known, unknown and sketch a picture if needed. In step 2, devising a plan, it is vital to find a connection between the data and the unknown. "Have you seen it before? Do you know a related problem? Look at the unknown! Here is a problem related to yours and solved before. Could you use it? Could you restate the problem? Go back to definitions" (Polya xvi). These are all important prompts that help students begin to problem solve. It is beneficial to students if they are able to make connections when determining their plan. In the third step, carrying out the plan, it is imperative to check each step. "Can you see clearly that the step is correct? Can you prove that is correct" (Polya xvii)? And lastly, step 4, looking back, this is the time that students can examine their solution that they obtained. "Can you check the results? Can you check the argument? Can you derive the result differently? Can you use the result, or the method, for some other problem" (Polya xvii)? When teaching mathematics and incorporating problem solving into the classroom, it is to a teachers advantage to guide students in the beginning of the year so they learn how to approach problem solving. Hopefully teachers are not just giving a problem situation to students and telling them to figure it out. The time will come when students are expected to successfully complete tasks on their own, or in groups, but when students are new to this concept, I think it is necessary to explain the steps required to successfully complete a given task. We have to help our students gain the necessary confidence needed so they can problem solve successfully on their own.

I have thoroughly enjoyed the problem solving activities that have been presented to us in this seminar. In Harold R. Jacobs's book, *Mathematics, A Human Endeavor*, we discussed problems from the first two chapters dealing with inductive and deductive reasoning as well as sequences and patterns of numbers. "Inductive reasoning is the method of drawing general conclusions from a limited set of observations. It is reasoning from the *particular* to the *general*" (Jacobs 20). "Deductive reasoning is a method of using logic to draw conclusions from statements that we accept as true" (Jacobs 32). I learned a great deal in our short time together and look forward to sharing with my students similar activities. I find this textbook appealing because it is not like any traditional textbook that I have used in my teaching career. This is problem solving at its finest. There is so much that I can make use of and include in my classroom. Again, the problem I encounter is how to fit this into my existing curriculum. Most activities that involve inductive and deductive reasoning seem to work nicely as warm ups, within the context of solving puzzles and/or brain teasers. I can present a problem situation at the beginning of the class period, have students begin trying to obtain a solution, then if they need more time I can assign it as homework.

My mathematics classes are not centered on group discussions. I primarily have to teach a new topic every day and then students practice the skills they learned. Of course discussions take place, but not in the same way as in an English or Social Studies class. I am excited to try these activities with my students. With some of the more demanding and real-world type problems I plan on including in this unit, I am looking forward to reading student responses and explanations. I am even more excited to have my students share how they derived some of their answers with each other.

Not only is the way students think and perceive things important, but I also would like them to make connections between mathematics and the real world. Being a mathematics teacher is not only about doing problems from a textbook – it is necessary to give thorough explanations of the topics you are teaching and make connections to real-world situations. I am looking forward to teaching this lesson because this is an opportunity for me to connect how mathematics is perceived in the real world.

When I was growing up I enjoyed learning mathematics. "Research has shown that many students actually like mathematics in elementary grades. Unfortunately, this enjoyment seems to decrease as students advance to high school" (Lewkowicz February 2003: 92). I am hoping this unit of study will enlighten some, if not all, of my students. I do not want them to just sit in my classroom, watch me work out problems and then practice the same concepts at home. I want them to explore and investigate other topics.

Objectives

This unit can be applied to all levels of mathematics in the high school curriculum. I am enthused about including more problem solving activities in my classroom and optimistic that my students will enjoy trying these activities in class other than the major emphasis being their textbook problems. The ultimate goal is that my students will:

- enjoy problem solving
- gain more confidence in problem solving
- become better problem solvers
- be able to work successfully with others
- appreciate writing in a mathematics class

- be able to communicate orally and by writing
- make connections between mathematics and the real world
- expand their inductive, deductive, explicit, and recursive reasoning skills

The majority of the Pennsylvania Mathematics Standards (Appendix 1) and the NCTM Communication Standards (Appendix 2) are also addressed in this curriculum unit. "In high school, there should be substantial growth in students' abilities to structure logical chains of thought, express themselves coherently and clearly, listen to the ideas of others, and think about their audience when they write or speak. Consequently, communication in grades 9 – 12 can be distinguished from that in lower grades by higher standards for oral and written exposition and by greater mathematical sophistication" (NCTM 348 – 349).

Strategies

Problem solving requires cooperation on the part of the students. It is important for them to become active participants in the classroom to enhance their learning process. To do this, I will:

- foster an environment of respect to help students feel more comfortable as they learn
- reinforce Polya's four steps of problem solving
- guide students to be more self sufficient
- ask thought provoking questions

Prior to giving problem solving activities to my class, it is necessary to make sure students have the appropriate skills to accomplish these tasks. During the first few weeks of school, I always review the TI-83 Plus Graphing Calculator. Our Geometry and Algebra 1 classes really do not use these calculators enough. It is imperative I teach my students how to perform basic operations, such as: entering equations, determining an appropriate window for a graph, finding x and y intercepts, minimum and maximum values, and points of intersection, to name a few. I also discuss basic formulas they learned from their Geometry class to refresh their memory. I always stress it is expected they remember basic area, volume, perimeter, and circumference formulas. There are always open-ended homework problems from our textbook which requires students to use certain geometry formulas.

When I first started attending seminar classes, I thought it was going to be difficult to incorporate the problem solving activities into my classroom due to the time constraints I mentioned earlier. I decided to try some of the activities with my current CAS Algebra 2 class as warm-ups. Much to my surprise, it worked out perfectly. The students were successfully completing the activities within five minutes. In fact, when I gave them problem solving activities to do as warm-ups, they did not want to turn them in when I was ready to begin class. They were excited to be working on something they considered different from the traditional warm-ups that prepared them for class. I also tried other more involved open-ended assessments from this unit which also proved to be successful.

Even though most teachers have a very strict curriculum to follow, one way to incorporate more problem solving into their lesson is through daily warm-up exercises. The first lesson in this unit are sample warm-up exercises. Some of these exercises require more time than others to complete, but all are good thought provoking problems. During our seminar we discussed topics in the textbook, Mathematics *A Human Endeavor,* such as, inductive and deductive reasoning, number sequences, geometric sequences, power sequences and the Fibonacci sequence. When determining what problems to add in this unit for warm-ups, I made sure to include problems of those types. There are no teaching strategies to be concerned with when dealing with warm-ups. Students are presented a problem situation and they are expected to draw on their mathematical knowledge and problem solving skills to correctly solve the problem.

Lesson 2 is the Ladder Problem. I usually incorporate this problem solving activity while teaching Chapter 4. Before giving the students this problem, I spend time simplifying roots as well as solving equations using the Pythagorean Theorem. The students usually complete this problem on their own, not in groups. I always have high expectations for my students and hope they are able to complete the problem with little difficulty, but it does seem to prove troublesome for some.

Before giving Lesson 3, the Barn Problem, I spent time discussing the formulas for circumference and area of rectangles and circles. I also stressed how important it is to accurately draw a picture of the problem situation in order to solve the problem correctly. The students worked together in small groups to complete the activity. I knew this would be a challenging problem for some students because they had to determine how to draw the picture correctly before they began trying to calculate the answer. The purpose of this activity was for students to use their problem solving, algebra and geometry skills. I think it is important to expose students to more problems in which they have to reason mathematically. In this particular instance, I wanted them to see the relevance of using mathematical concepts in real life situations. The formulas and theorems that students are learning have practical value in the real world and they need to be exposed to these concepts.

In the fourth lesson, Making Washers, again the topic of geometry is addressed. Because I usually assign this problem after I have already given the Ladder and Barn Problem, I do not discuss formulas again. I have students work with a partner and begin their problem solving. Like the warm-ups, there are no teaching strategies necessary. Students should be able to successfully complete this task.

When I tried the fifth lesson, the Difference of Squares, I began the lesson by talking about squaring numbers and factoring. I had students recall some of the square numbers such as $3^2 = 9$ and $12^2 = 144$. This was followed by a discussion on what we had just practiced doing throughout the chapter, factoring the difference of two square numbers, such as, $(x^2 – y^2)$ into $(x + y) (x – y)$. I then presented the problem situation to them. Some of their findings were why: some even numbers do not have a solution, an even number that is not a multiple of four has no solution, the difference among all of the patterns of odd numbers has a difference of one, the odd numbers pattern is $2x + 1$, every odd number increases by one, meaning, $1 = 1^2 – 0^2$, $3 = 2^2 – 1^2$, $5 = 3^2 - 2^2$ and so on. I was very pleased with the overall lesson and the achievement of my learning goals. I felt the students were able to factor the numbers, find patterns, think critically and work cooperatively. My students were required to think about many different patterns and make a connection between them.

The final lesson in this unit requires students to work in small groups. When assigning students into groups I always take into account the various ability levels of my students and design groups accordingly. Because each student is unique in their ability and there is such a broad range of ability within my classroom, it is imperative that I structure my classes to meet the needs of every student. I have high expectations for all my students. I encourage all of them to play an active roll in their education, to take risks, and to value mathematics. It was imperative this assignment be conducted in small groups. The assignment asked students to collect data and enter it into the lists in the graphing calculator, find the linear regression equation and sketch the scatter plot. The students in each group helped each other measure their arm span and height, and then worked together with calculators to determine the linear regression. The small groups provided more student interaction in an orderly fashion.

Classroom Activities
All solutions/answers are in Appendix 9

Lesson 1 Daily warm-ups

1. Four by Four Ranucci Page 4 Ten of the Best

Can you rearrange the sixteen numerals in the 4 X 4 array shown below in such a manner that every row and column totals 10? This can be done in many ways.

1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4

2. Four by Four Ranucci Page 27 What Comes Next? #10

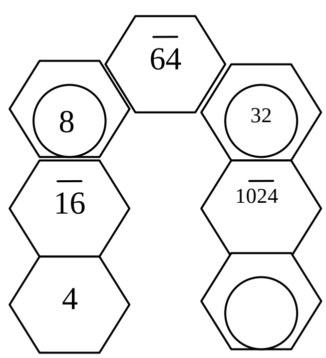
			?
		55	34
5	8	13	21
3	2	1	1

3. Mathematics Teacher March 2008 # 31

What is the sum of the first 100 positive odd integers?

4. 100 Games of Logic Pierre Berloquin Page 15 Game 12

What number belongs in the empty circle?

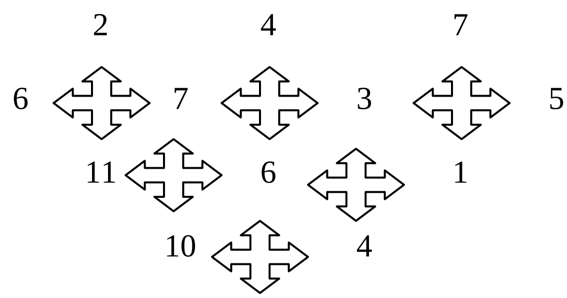


5. **100 Games of Logic Pierre Berloquin Page 44 Game 41**

1	1	8
2	5	13
3	21	

6. **100 Games of Logic Pierre Berloquin Page 51 Game 48**

What number belongs under the bottom arrow?



7. **100 Games of Logic Pierre Berloquin Page 63 Game 60**

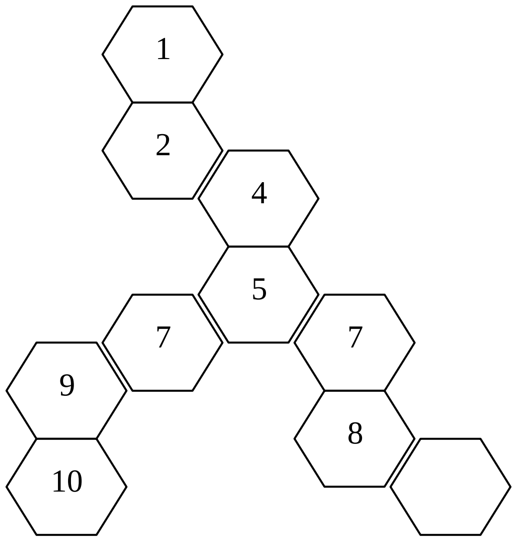
Lebrun, Lenoir, and Leblanc are, not necessarily in that order, the accountant, warehouseman, and traveling salesman of a firm. The salesman, a bachelor, is the shortest of the three. Lebrun, who is Lenoir's son-in-law, is taller than the warehouseman. Who has what job?

8. **Mathematics Teacher April 2006 #27**

For how many paths consisting of a sequence of a horizontal (forward or backward) and/or vertical line segments, each connecting a pair of adjacent letters in the diagram below, is the word *count* spelled out as a path is traveled from beginning to end?

C
C O C
C O U O C
C O U N U O C
C O U N T N U O C

9. **100 Games of Logic Pierre Berloquin Page 62 Game 59**
What number belongs in the empty hexagon?



10. **Mathematics Teacher January 2006 # 1**
Select any prime number greater than 3. Square it and subtract 1. What is the largest number that must be a divisor of the result?

11. **Mathematics Teacher May 2005 # 17**
What comes next in the following sequence: . . . , 59, 53, 47, . . .

12. **Mathematics Teacher January 2008 # 7**
So that ZIP codes may be machine readable, the Postal Service encodes them as a group of five bars, some tall, some short, in groups of five. In how many distinct ways can two tall and three short bars be arranged?

13. **Mathematics Teacher October 2007 # 23**
Find the sum of the elements in the 100th row of this triangular array:

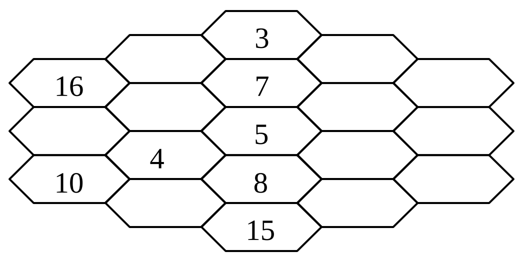
1
2 3
4 5 6
7 8 9 10
* * * * * * * * *

14. **Mathematics Teacher** **August 2005** **# 1**

Arrange ten dots in such a way that there are five rows of dots with four dots in each row.

15. **Mathematics Teacher** **November 2007** **# 20**

Find the constant sum of the magic hexagon and fill in the numbers so that every column or diagonal has that sum.



16. **Mathematics Teacher** **February 2008** **#16**

If you continued the triangular array of numbers shown in the figure, what number would be directly below 122?

1
2 3 4
5 6 7 8 9
10 11 12 13 14 15 16

17. **Mathematics Teacher** **March 2008** **# 23**

Consider the following:

1 2 3 4 5 6 7 8 9 0 = 100

Leaving all the numerals in the order given, insert addition and subtraction signs into the equation to make the equation true.

18. **Mathematics Teacher** **January 2005** **# 22**

In the sequence of numbers 1, 3, 2, . . . , each term after the first two is defined to be equal to the term preceding it minus the term preceding that one. Find the sum of the first one hundred terms of the sequence.

19. Mathematics Teacher November 2006 # 2
A digit is placed in each empty square in the grid so that each row and each column contains the digits 1, 2, 3, 4, 5. What digit is placed in the square at the bottom right corner?

	5	4		
1	3			
		5	3	
2		3	1	

20. Mathematics Teacher November 2004 # 3
The sum of nine consecutive integers is 9. What is the least of these nine integers?

21. Mathematics Teacher April 2007 # 1
What is the value of 1 – 2 + 3 – 4 + 5 – 6 + . . . – 2004 + 2005 – 2006 + 2007**?**

22. Mathematics Teacher August 2007 # 6
In the following sequence, what could be the next term?

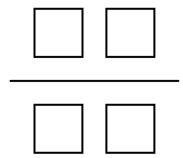
1, 4, 27, 256, . . .

23. Mathematics Teacher December 2002 # 22
Suppose that you have six weights, weighing 1, 3, 9, 27, 81, and 100 grams. How can you balance the 100-gram weight using some or all of the other weights?
Hint: Try placing one or more of the weights with the object that you are balancing.

24. Mathematics Teacher January 2003 # 7
The pattern AABBBCCCCAABBBCCCC continuously repeats. What is the 2003rd letter in the pattern?

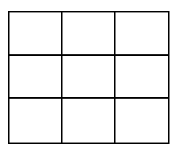
25. **Mathematics Teacher** **November 2004** **# 17**

Each of the four digits 2, 4, 6, and 9 is placed in one of the boxes to form a fraction. The numerator and the denominator are both two-digit whole numbers. What is the smallest value of all the common fractions that can be formed? Express your answer as a common fraction.



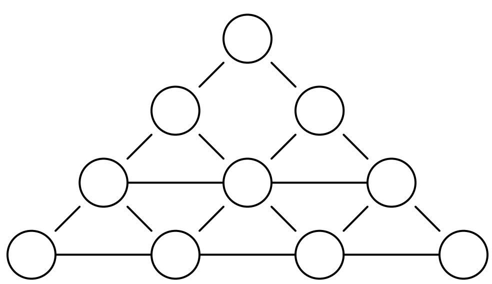
26. **Mathematics Teacher** **October 2002** **# 24**

Suppose that the numbers 1, 2, 4, 8, 16, 32, 64, 128, and 256 are placed into the 3-by-3 grid in such a way that each of the numbers appears exactly once and the product of the numbers in any row or column is the same. What is the value of the product in each row and column?



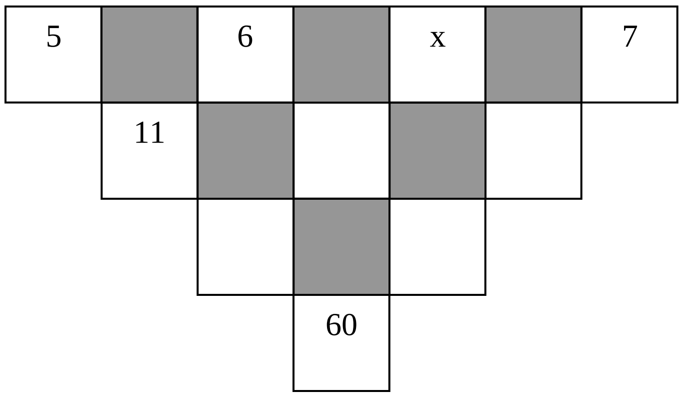
27. **Mathematics Teacher** **August 2007** **# 11**

Using the diagram, place the numbers 1 to 10 in the circles so that the sums in the rows of three circles are the same and the sums in the rows of four circles are the same.



28. **Mathematics Teacher** **August 2004** **# 9**

The number in an unshaded square is obtained by adding the numbers connected with it from the row above. (The 11 is one such number.) What is the value of x?



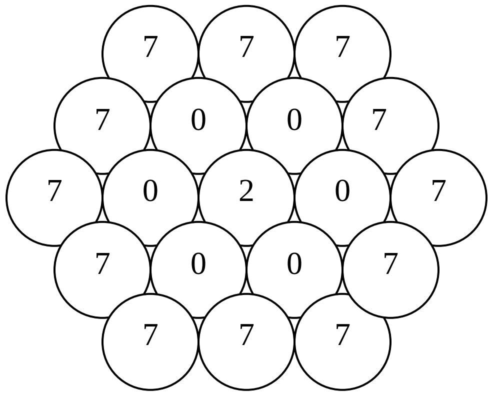
29. **Mathematics Teacher** **March 2006** **# 5**

The square shown here can be filled in so that each row and each column contains each of the numbers 1, 2, 3, and 4 exactly once. What does x equal?

			1
	2		
		x	
1			4

30. **Mathematics Teacher** **March 2007** **# 12**

Starting at the 2, the number 2007 can be formed by moving from circle to adjacent circle. How many different paths can be followed to form 2007?



Lesson 2

<u>Ladder Problem</u>

A painter has placed a 15-foot ladder against a building. The bottom of the ladder is 9 feet from the base of the building. In this position, the ladder does not reach as high as the painter needs. The painter would like the top of the ladder to reach 1 foot higher on the building. To make the ladder reach higher, the painter will push the bottom of the ladder closer to the building.

Draw and label 2 figures showing the ladder in the beginning position and in the final position.

To the nearest inch, how much closer to the building must the bottom of the ladder be moved so that the top of the ladder will reach 1 foot higher?

Lesson 3

<u>Barn Problem</u>

A cow is tied to the long side of a barn 10 feet from the corner. The barn measures 11 feet wide and 28 feet long. If the rope is 21 feet long, what is the total area of the space in which the cow can graze? Draw a picture which includes the grazing area to show how you arrived at your answer.

Lesson 4

Making Washers Problem

A machinist must make a square washer as shown in this figure. The width of the washer is to be 4 inches and the hole in the center has a diameter equal to half the width of the washer. It must be a quarter-inch thick.

Label the figure. Find the **volume** and the **weight** of the washer if the metal used to make it weighs 5 ounces per cubic inch.

Show your work and explain the steps you used to justify your answers. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

Lesson 5

The Difference of Squares

See Appendix 3 for this lesson and Appendix 4 for the table students can use to report their findings.

Lesson 6

Arm Span vs. Height

See Appendices 5, 6, 7 and 8 for this lesson.

Annotated Bibliography/Resources

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The National Board of Professional Teaching Standards was developed to help improve teaching and student learning. Teachers who achieved National Board Certification are highly accomplished educators.

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Mathematics Teacher is an official journal published by The National Council of Teachers of Mathematics. It is published nine times a year and you must be a member in order to receive the magazine. There are problem solving activities in the magazine as well as mathematical articles.

Appendices - Standards

Appendix 1 Pennsylvania Mathematics Standards

2.1 Numbers, Number Systems, and Number Relationships
Types of numbers (e.g., whole, prime, irrational, complex)
Equivalent forms (e.g., fractions, decimals, percents)

2.2 Computation and Estimation
Basic functions (+, -, x, /)
Reasonableness of answers
Calculators

2.3 Measurement and Estimation
Types of measurement (e.g., length, time)
Units and tools of measurement
Computing and comparing measurements

2.4 Mathematical Reasoning and Connections
Using inductive and deductive reasoning
Validating arguments (e.g., if . . . then statements, proofs)

2.5 Mathematical Problem Solving and Communication
Problem solving strategies
Representing problems in various ways
Interpreting results

2.6 Statistics and Data Analysis
Collecting and reporting data (e.g., charts, graphs)
Analyzing data

2.7 Probability and Predictions
Validity of data
Calculating probability to make predictions

2.8 Algebra and Functions
Equations
Patterns and functions

2.9 Geometry
Shapes and their properties
Using geometric principles to solve problems

2.10 Trigonometry
Right angles
Measuring and computing with triangles
Using graphing calculators

2.11 Concepts of Calculus
Comparing quantities and values
Graphing rates of change
Continuing patterns infinitely

Appendix 2 Communication Standard for Grades 9 - 12

Instructional programs for prekindergarten through grade 12 should enable all students to —

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely (NCTM 348)

Appendix 3 The Difference of Squares

Square numbers have many remarkable properties. In this assignment you will discover some of the patterns and properties of the square numbers.

Some numbers can be written as the difference of two squares. For example,

$8 = 3^2 – 1^2$ because $3^2 = 9$ and $1^2 = 1$ and $9 – 1 = 8$

and

$19 = 10^2 – 9^2$ because $10^2 = 100$ and $9^2 = 81$ and $100 – 81 = 19$

Try this one:

$5^2 – 2^2 =$ ____________________

Number	Square
1^2	1
2^2	4
3^2	9
4^2	16
5^2	25
6^2	36
7^2	49
8^2	64
9^2	81
10^2	100
11^2	121
12^2	144
13^2	169
14^2	196
15^2	225

Some numbers though CAN'T be written using the difference between two squares. For example, there is no way to write 6 as the difference of two square numbers. Other numbers can be solved in more than one way. For example:

$15 = 4^2 – 1^2$ AND $15 = 8^2 – 7^2$

Your job on this problem is to find all the numbers between 1 and 40 which can be written using differences between two square numbers. If a number can be solved in more than one way, make sure you record all the different ways to solve it (Appendix 4).

Appendix 4 Use this table to record your solutions for 1 – 40

1		21	
2		22	
3		23	
4		24	
5		25	
6	No Solutions	26	
7		27	
8	$3^2 - 1^2$	28	
9		29	
10		30	
11		31	
12		32	
13		33	
14		34	
15	$4^2 - 1^2$ and $8^2 - 7^2$	35	
16		36	
17		37	
18		38	
19	$10^2 - 9^2$	39	
20		40	

Appendix 5

Arm Span vs. Height

- Students should work in groups and measure their arm span and height. Tape measures and a wall chart are provided. The arm span and height should be measured in inches.

- Each student should record their name, arm span and height in the chart on the overhead (Appendix 6).

- Each student should plot their arm span and height on the graph on the overhead (Appendix 7).

Each group should work together to:

- Enter all of the data from the overhead into their TI-83 Plus Graphing Calculators under List 1(L1) and List 2(L2). L (1) being the arm span and L (2) being the height.

- Create a scatter plot to display the data.

- On the graph provided, sketch the scatter plot.

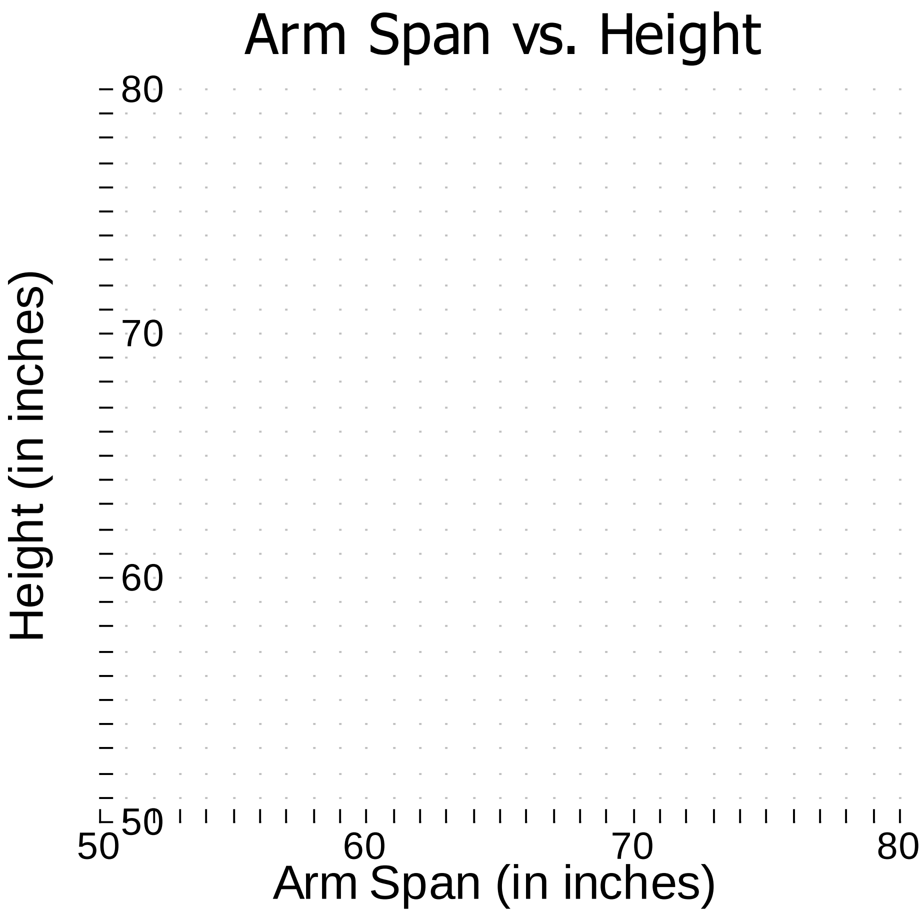
- Create a line of best fit, that is, a linear regression equation.

Once you have a complete sketch of the scatter plot and determined a line of best fit, you should continue working with your group answering the questions on the attached sheet (Appendix 8).

Appendix 6

Name	***Arm Span (in)***	***Height (in)***

Appendix 7


Arm Span vs. Height
Height (in inches)
80
70
60
50
50
60
70
80
Arm Span (in inches)

Appendix 8

Name____ Period_____
Name of Group Members____

Arm Span vs. Height

#1. Discuss the appearance and interpretation of the scatter plot. Do you notice any correlation between the arm span and height?

#2. Connect any two points on your scatter plot that you think represent a line of best fit and draw that line. What two points did you choose?

(,) and (,)

#3. With the two sets of data points from #2, determine the equation of the line.

#4. Use your TI-83 Plus Graphing Calculator to determine the linear regression (also known as the line of best fit). Write the equation that you found.

y =

#5. Sketch your line of best fit on your scatter plot.

#6. Discuss in your groups:

a. Are there any similarities between the two equations?
b. Compare the different points your group choose and explain why you picked those points.
c. Were there any outliers in your scatter plot? How would this effect your line of best fit that you found using your calculator?
d. How did your line of best fit compare to the line of best fit per the graphing calculator?

#7. What is the slope of the linear regression equation? What does the slope represent in this problem?

#8. You can use your graph/equation to make predictions.

a. How tall would a person be if their arm span was 6’11”? Explain how you found this answer graphically and algebraically (show work). Use complete sentences.

b. How wide of an arm span would a person have if their height was 7’4”? Explain how you found this answer graphically and algebraically (show work). Use complete sentences.

Appendix 9 Answers/Solutions

Lesson 1 Daily Warm-ups

1.

4	1	2	3
1	4	3	2
2	3	4	1
3	2	1	4

2. 233

3. 10, 000

4. 512

5. 34

6. 8

7. Lebrun is the accountant.
Leblanc is the salesman.
Lenoir is the warehouseman.

8. 31

9. 10

10. 8

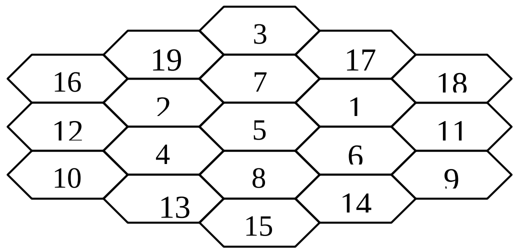
11. Possible answers 41 and 43

12. 10

13. 500,050

14. (Visualize a star)

15.



16. 146

17. Answers will vary. Here are two possible solutions.
1 + 2 + 3 + 4 + 5 – 6 – 7 + 8 + 90 = 100
1 + 2 + 3 – 4 + 5 + 6 + 78 + 9 + 0 = 100

18. 5

19. 3

20. – 3

21. 1004

22. 3125

23. 1 + 27 + 81 = 109 AND 100 + 9 = 109

24. B

25. $\frac{24}{96} = \frac{1}{4}$

26.

128	1	52
4	16	64
8	256	2

27.

			2			
		6		8		
	5		7		4	
10		1		3		9

28. 10

29. 4

30. 36

Lesson 2 Ladder Problems

The ladder must be moved 18.24 inches closer to the building.

Lesson 3 Barn Problem

The total area the cow can graze is 253π ft^2 = 794.82 ft^2 if $\pi = 3.14$

Lesson 4 Making Washers Problem

The volume of the washer is .785 in^3
The weight of the washer is 16.075 oz

Lesson 5 and 6
Answers may vary.