Designing and Modeling

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Overview

During the twentieth century mathematics education went through various changes. The National Council of Teachers of Mathematics (NCTM) created an agenda for action making problem-solving the focus of school mathematics. Problem-solving was by and large the focus of the 1980s; with most mathematics educators agreeing that a "back to the basics" approach was needed. Later, from the 1980s to the present, the NCTM's vision in the Curriculum and Evaluation Standards for School Mathematics stated that students need to become mathematics problem solvers. The NCTM suggested that problem-solving should be the central focus of the mathematics curriculum (NCTM, 1980, 1989, 2000).

This curriculum unit will provide problem-solving tasks that are challenging, motivating and very much relevant. Many of the activities will take students to places around the world in order to pique their interests. At the same time, students can get involved in the kind of tasks that require no predictable, well-rehearsed approach or pathway suggested in obtaining the answer. Through my experience, I have learned that students need to be able to reason and analyze problem situations because doing so allows them to develop a much broader perspective of mathematics. Just as important, students should be given mathematical activities that allow them to generate their own ideas and explanations.

In the problem-solving task *Making Washers*, students are required to access relevant knowledge and experiences when working through the problem. The problem is designed so that students will have to make the appropriate decisions when determining the volume and weight of the washer. To get students to explore and understand the nature of the mathematical concept or process, they will be asked to show work, explain the steps and justify answers.

To go further, students will investigate the company's break even point and profit for producing the various washers. This will allow students an opportunity to form various algebraic models and support their answers graphically.

Later in the curriculum, I will present problem-solving activities that involve a team of functions (linear, quadratic, exponential, and trigonometric) and their graphs. The activities will allow students the opportunity to describe and determine the outcome of various events such as *Pennsylvania* and *New York Populations*, San Francisco *Golden Gate Bridge*, the *Designing Engineer*, and *Navigation* around the world. As an outcome of the various activities, students will be able to suggest pathways for finding solutions and interpret the results to the problems. All problem-solving activities will require complex and non-algorithmic thinking.

Technology will play an important part in this unit as well. Multiple representations are necessary because technology broadens students' understanding of the problem. Students can support their findings by creating graphs and exploring the entire behavior of the function. The National Council of Teachers of Mathematics (NCTM) has provided standards that involve using technology for all grade-levels. Through the use of technology, students can obtain information from a whole new spectrum (visual representation).

Student involvement is necessary for the success of this unit. For each of the various activities, two days will be enough time for discussing and completing each task. When teaching families of functions, linear equations and systems should be taught before introducing the *Washer Problem*. Next, the quadratic function and its graph should be explored before involving students with the *Golden Gate Bridge* and *Fun in the City* activities. In the activity *Pennsylvania* and *New York City Population*, exponential and logarithmic functions should be explored in order to connect students' growth versus decay models. Students should have a complete understanding of the trigonometric functions before engaging the *Designing Engineer* and *Navigation* around the world.

Introduction

The term "problem-solving" has taken on many different meanings. The National Council of Teachers of Mathematics (NCTM) defined problem-solving as "a task for which the solution method is not known in advance" (NCTM 2000 52). George Polya, professor of mathematics, defined problem-solving as finding "a way where no way is known off-hand out of difficulty around an obstacle" (Polya, 1949, 1980). Problem-solving is a process of finding a solution when the path is not obvious.

Application problems can enhance a mathematics curriculum. Schoen (1988) claims, "when application problems are used for many appropriate topics, this practice will emphasize the importance of word problems." It is also important to provide students with word problems that interest students so that they will enjoy solving them. Schoen states, “in order to make word problem-solving the focus of an algebra course, it is necessary but not sufficient to include many good word problems” (Schoen 125).

Students should learn to use various strategies of problem-solving. The work of George Polya presents techniques for problem-solving that not only are interesting but ensure that the Principles of Learning in Mathematics will transfer as widely as possible. He suggests the following heuristic methods (Posamentier, Smith, Stepelman 2006):

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Look back. Examine the solution obtained.

Some problems can be solved by an intuitive (or random) trial and error method. The task below may take a considerable amount of time to reach the answer (Posamentier, Smith, Stepelman 2006)

Problem: Place the numbers from 1 through 9 into the grid below so that the sum of each row, column, and diagonal is the same (Posamentier, Smith, Stepelman 2006). *Observe the given below.*

Intelligent guessing, logical reasoning, and testing and accounting for all possibilities by organizing the data so that the answer may come quickly are the strategies that may be used in finding the result.

This shows accounting for all possibilities.

1,5,9	2,6,7	1,6,8	3,4,8
2,4,9	3,5,7	2,5,8	4,5,6

Solution

8	1	6
3	5	7
4	9	2

Next take the given problem:

Problem: In a room with 10 people, everyone shakes hands with everybody else exactly once. How many handshakes are there (Posamentier, Smith, Stepelman 2006)? *Observe the given below*

There are five solution methods which may be used in solving this problem. You may use visual representation, account for all possibilities, adopt a different point of view, look for a pattern or organize the data. The situation involves organizing data:

Person number	10	9	8	7	6	5	4	3	2	1
Number of handshakes	9	8	7	6	5	4	3	2	1	0

In everyday-life situations we sometimes subconsciously use pattern recognition to deal with a problem. For example, when you are looking for an even-numbered address on a street, you will look to the side where the even numbers are and you will search for it in numerical order. . (Posamentier, Smith, Stepelman 2006). *Observe the given below*

Problem: Find the sum of the first 20 odd numbers.

1 + 3+ 5+ 7+ 9….

You can create a table that will reveal that the sum of the first n odd numbers is n^2.

Addends	Number of Addends	Sum
1	1	1
1 + 3	2	4
1+ 3+ 5	3	9

Rationale

This unit is important because it improves students' readiness for both the state and national standards and many advanced mathematics courses. The instructional programs from preschool through grade 12 should enable all students to understand patterns, relations, and functions. It is just as important that students understand computation, estimation, measurement, mathematical problem-solving and communication so that they will be able to use mathematics in their everyday lives and in the workplace.

Students need to be challenged and given mathematical tasks that involve logical reasoning and critical thinking. I found that students' experiences in the classroom should be those that engage them in problem-solving instead of just step-by-step instruction. Just as important, the classroom teacher should focus on interesting applications and word problems in the teaching of mathematics without deleting important topics. Effective teaching involves observing students, listening carefully to their ideas and explanations, having mathematical goals, and using the information to make decisions (Schoen 1988).

Once this unit is completed, students will have explored many good problems. When I speak of good problems, they are problems that are challenging, motivating, intriguing and relevant. Good problems pique the interest of students and engage them in mathematics. Students will become acquainted with the behavior of various functions and communicate their findings mathematically and graphically. Modern technology, the TI-83 calculator, will be used to enhance what they have learned. In particular, *Pennsylvania* and *New York* population growth activities, the *Golden Gate* application, *Navigation* and *The Designing Engineer* problems will give students the opportunity to solve very sophisticated equations and observe their finding graphically.

All activities will help students become confident problem-solvers. To start, students will see the difference between the linear and quadratic functions when working on the *Washer Problem* and the *Golden Gate* activities. In allowing connections across content, all functions will be introduced along with true real-world facts. Once students become familiar with the different concepts of functions, they will be able to draw more reasonable conclusions about different phenomena.

In the state population activities students will become acquainted with increasing exponential functions. Students will be asked to investigate all of the function characteristics in order to understand the function as a whole (the axes, scale, all intercepts, and all asymptotes, entire domain and relevant domain). It is necessary for students to become involved with solving both exponential and logarithmic equations in order to support their findings. When students are able to analyze functions, they become aware of their various behaviors. Students need to apply the theory behind limits in order to understand that this function would approach infinity. This will provide connections to end-behavior and the range as well. In order to make sure that students connect to the entire model of this function, students should provide a summary of this function over its entire domain.

The *Designing Engineering* and *Navigation* activities will broaden students understanding of the trigonometric functions and their properties as a whole. This function is a very useful function to many applications. In particular, the trigonometric function has many uses in real life situation, especially the graph of the sine wave that connects many periodic events such as a person's biorhythm cycle, sound waves, and some living organism activities. Once this function is introduced, it is important to provide connections to the triangle first, and later connect it to the unit circle, and even later to their graphical behaviors.

All lessons will require students to use technology; technology has given educators a true challenge to move mathematics away from the routine and toward new ways of teaching and student learning. In my own classroom, I have found modern technology allows students to investigate mathematics globally. Using the TI 83 Plus calculator as a support device has made a true difference in the way students see and learn mathematics. Student can deal with the mathematics directly instead of just listening. In most of my lessons you will see technology used as a support tool to enhance student learning. Learning is definitely viewed in a much broader perspective than just symbolic representation.

Objectives

The warm-up activities are chosen in order to build problem-solving skill and to revisit concepts that have been taught. The *T problem* provides students the opportunity to become engaged in constant manipulation. The *T problem* is great before introducing the *Washer Problem*. *The Radio Problem* and the *Flying Squirrel* give students an opportunity to connect to the graph of the sine function. Any one of the problems can be used to introduce the *Designing Engineering and Navigation* Activities. Finally, *The Golden Gate* is a great warm-up that shows connections to the quadratic function, it may be used before teaching *Fun in the City*. The *Golden Gate* task can be used to broaden students understanding of the vertex, the minimum and maximum values and their uses in order to determine the height of the bridge's cable.

The students will be able to:

- Explore various problem-solving activities that involve functions and their behaviors. This team of functions include: linear, quadratic, exponential and trigonometric with real world events. The activities include: *Making Washers, the Golden Gate, New York City Population Growth, Pennsylvania State Population Growth, Fun in the City, The Designing Engineer,* and *Navigation.*

- *The Washer Problem* will allow students to visit concepts such as slope, equation of a line, x and y intercepts, and systems of equations. Students will compute the volume and weight of a device when exploring the *Washer Problem. Fun in the City* gives students a chance to explore quadratics functions and equations, concepts such as vertex, minimum, maximum value, zeros, domain and range will be explored. Both *New York* and *Pennsylvania Population* activities allow students the opportunity to become acquainted with the exponential functions and their complete behaviors. Last, the *Designing Engineer* and *Navigation* get students involved with the trigonometric functions and their measurements.

- Use the TI- 83 Plus graphing calculator to make a complete graph of various functions and analyze their graphs completely.

Strategies

In the classroom, students should be able to ask questions, assist in defining problems, suggest paths of solutions and help interpret the results. The teacher should guide the process in an organized way toward goals that represent the content of the subject. When I started this unit my primary focus was to provide my students with connections that help to elicit and build their mathematical thinking. It was also important that my students observe multiple representations of various functions.

The *Washer Problem* is excellent for allowing students to revisit linear equations and systems of linear equations. Students can solve linear models when asked to find cost, revenues and profit. Connections to systems of linear equations are made when students find the company's break even point. Students should use their graphing calculator to explore tables and line graphs. With the use of technology, students can concentrate almost immediately on how the values change for each of the various functions.

Once students are given the quadratic function $y = ax^2 + bx + c$, $a \neq 0$, the emphasis should be placed on certain attributes of the graphs and the function's behavior. Students should learn to find the roots, vertex, the maximum value and minimum value, where the functions are increasing and decreasing before this activity is presented. The *Fun in City* activity will allow students to revisit quadratics from a real world perspective**.** When teaching this activity, students should be made aware of the role of initial height and how it will affect the behavior of the function as a whole.

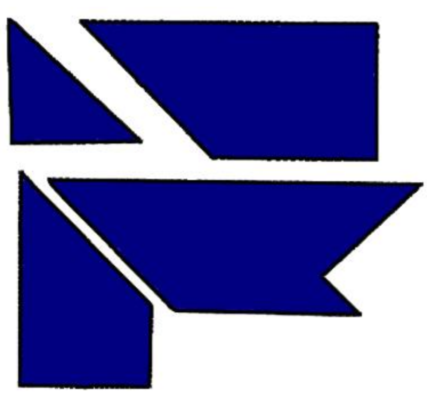
Teaching *New York* and *Pennsylvania Population Growth* activities provide several challenges for students. Students must have an understanding of powers, exponents, logarithms and roots. To ensure understanding, students must be given a quick review that involves evaluating roots and simplifying exponents. Students also experience difficulties graphing exponents and finding reasonable solutions. To ensure that graphing is understood, students must be reminded that most graphs use a large range of y-values due to the behavior of the exponential function. Once students can view the graphing table and can see the y-values, they may be able to create a comfortable window. When approaching equation solving, students must be reminded that simple equations (e.g., $8^x = 32$) can be solved by creating the same base (if possible) or switching from exponential to log. They must also be reminded to keep track of the domain of each expression in the equation; a particular algebraic method may produce solutions that are extraneous.

The trigonometric function has a deep and rich connection to geometry and modern application. This function should be taught with connections to the triangle first, and later connected to the unit circle, and other behaviors (waves and periodic). The *Designing Engineer* activity will provide a focus on angle of elevation and depression. It is important that students work on other applications to the right triangle before attempting this problem. Once student get involved with the application part of trigonometry, they will see connections to many other topics such as civil engineering, mechanical design, architectural design, space flight and building height.

The *Navigation* activity is excellent for small group work; first students will have an opportunity to discuss how the triangle was generated through the use of bearing. It is very important that students know where the right angle is located so that they can use trigonometry to solve the problem. Students may use different methods for finding the missing angles. Students may apply the Pythagorean Theorem or trigonometry when asked to find the distance from Cincinnati to Syracuse. As simple as it may seem, students have to use a great deal of reasoning and thinking to determine the measure of the various angle and the bearing at Cincinnati for the problem situation. This problem is great for revisiting trigonometric concepts.

Problem-solving Warm-up

The T Problem



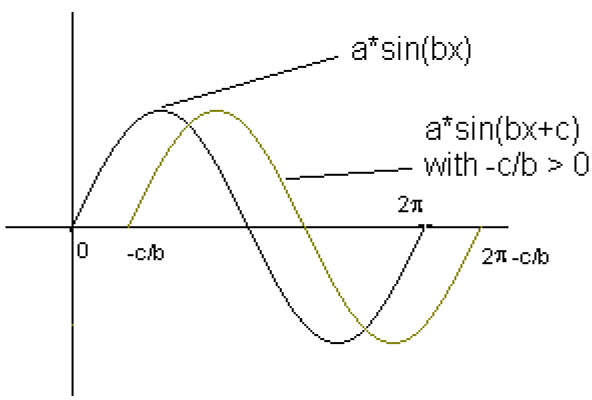
Using these 4 pieces you must make a CAPITAL LETTER 'T' with no jagged bits, no gaps, no overlaps, no bumpy bits. The solution must be all one colour ie don't turn pieces over. Constant manipulation is a problem-solving strategy that may help you.
Bruce A (2007). *Puzzles to print out then Solve*.

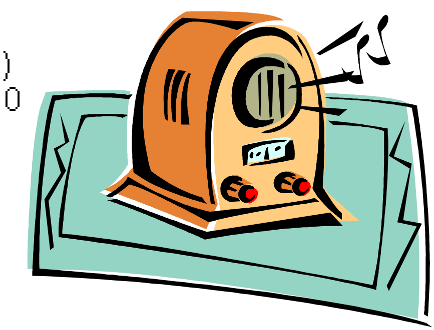
Teacher Hints:

*
You can use this puzzle on the first day of school with any class. It is helpful to enlarge the pieces and have students initial all their pieces and store them in an envelope. This task demonstrates the problem-solving strategy of 'constant manipulation'
* make some of the puzzles from wood/poster board and begin your problem-solving collection.

The Radio Problem Warm-up

The graph of the sine function as an angle travels from π to 2π.





When a musical sound wave is changed into visual image by an oscilloscope, it has a regular pattern that repeats itself many times each second. The graph produces sound of wave that is similar to the graph of a sine function. A typical equation for such a curve is

For y = sine 3x

The amplitude is 1, its frequency is 3, and its wavelength is $360^{o}/3 = 120^{o}$
The greater the amplitude of a sound wave, the louder the sound. The greater the frequency of a sound wave, the higher the pitch

Y = 4 sin 2x y = 2 sine 5x y= 3 sin ½ x

a. Which graph corresponds to the loudest sound? Which equations correspond to the softest sound?

b. Which equation corresponds to the sound having the highest pitch? Find the lowest pitch.

c. What happens to the wavelength of a sound as its frequency increases?

The Flying Squirrel Problem

For the sine function $y = a * \sin(bx)$, the dimensions of a sine curve are measured with two numbers: the amplitude (a), which is the maximum distance of the curve from the x-axis and the wavelength is $2\pi/b$ ($360^o/b$), which is the distance along the x-axis required to complete a wave. In the case of the curve $y = \sin x$ the amplitude is 1 and the wave length is 2π or 360^o.

1. Many of the activities of living organisms are periodic. For example the graph below shows the time that flying squirrels begin their evening activity.

```
Time

8:00 |            *
7:30 |        *
7:00 |                 *
6:30 |
     | * Apr  Jun  Aug  Oct * Dec  Feb *  Apr
5:30 |                         *
5:00 |                                *
4:30 |
4:00 |                            *
                       Month
```

Find the amplitude and period of this graph. Describe the activity of the flying squirrel.

Classroom Activities

The Washer Problem

Part I

A machinist must make a square washer. The width of the washer is to be 3 inches and the hole in the center has a diameter equal 1 inch wide. It must be a quarter-inch thick.

1. Design and label your figure. Find the volume and the weight of the washer if the metal used to make it weighs 5 ounces per cubic inch.

2. Show your work and explain the steps you used to justify your answers. Do all work for this problem in the area below. Remember to show all the steps you used to solve the problem even if you have used a calculator.

Part II

The company can make each washer for $0.10 each and sell the item for $0.30 with an overhead cost of $500,000.

A. Write a model for the cost and revenues.

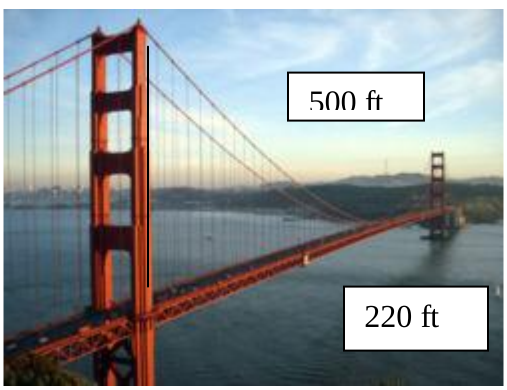
B. Find the cost of making 20,000 washers. Next, find the amount of money generated in revenues.

C. How many washers must be sold to make a profit of $20,000?

D. How many washers must be made to break even? Show your work algebraically and graphically.

Warm-up Activities:

The Golden Gate



The Golden Gate Bridge is 1.7 miles in length, is about 220 feet above water, weight is 887,000 tons and is 746 feet in height. Unfortunately, while the bridge was built, 30 people fell from the bridge. The Golden Gate Bridge was included among the "Monuments of the Millennium". Even walkers enjoy walking over the bridge. Thousands and thousands of vehicles commute over the bridge and it acts as a great convenience for many.

http://en.wikipedia.org/wiki/Image:GoldenGateBridge-001.jpg

This is a photograph of the Golden Gate Bridge. The cables that are hung between the two towers from which the roadway is suspended form a curve that is very close to the shape of a parabola

Each cable joining the two towers on the Golden Gate Bridge can be modeled by the function.

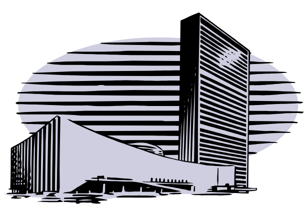
$$Y = \frac{1}{9000}x^2 - \frac{7}{15}x + 500$$

Where x and y are measured in feet. What is the height h above the road of a cable at its lowest point?

Classroom Activities:

New York City Population Growth

The Industrial Revolution was a time of dramatic change, the population of New York has been increasing since the city's early history. **Assuming the increase has been exponential and given that the 1950 census population was 14,830,192 and increasing at an average rate of .5% annually, create a model that represents 1950 as the initial report.**



a. Sketch a complete graph of this function including any asymptotes.

b. State the domain and range of the algebraic function:

c. Highlight or box the portion of the graph that is relevant to this problem situation.
   How is this portion of the graph different from the domain of the function?

d. Tell how would you will find the y-intercept algebraically and explain what it means according to this problem situation.

e. Based on the model given, about how many people were in New York in 1960, 1970, 1995 and 2000? Used the model given to show how you will find the answer both algebraically and graphically.

f. In 1995 the Census Bureau reported 18,136,000 people and in 2005 reported 18,227,000 people. Find the growth rate during those ten years. Create a new model and predict the population for this year (2008).

g. Based on the function, when will the population grow to 20 million people? Show or explain how you found your answer.

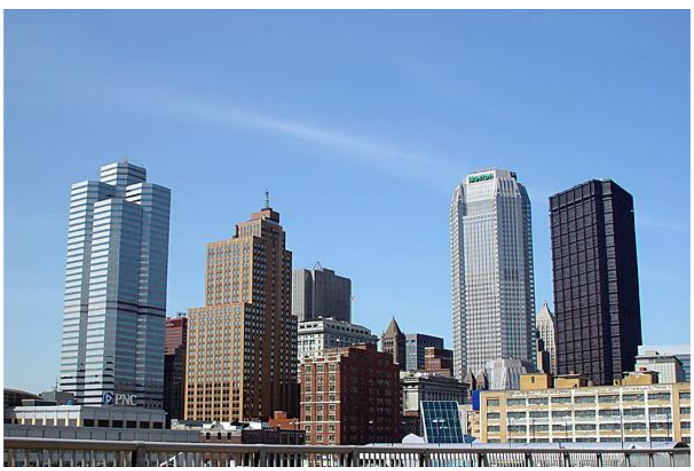
- Based on the Model, as the time (years) increase, will the population approach a limit? If yes, what is that limit? Explain your reasoning. If no, explain why the population has no limit.

State :New York

In 1790 the United States Census Bureau reported a total of 340,120 people and in 1900 reported 7,268, 894 people present in the state. The estimated population for New York is as follows: 2000 18,976,821, 2005 19,315,721. **Year 2010** 18,250,000 year 2025 19,830,000

Early in its history, the population in Pennsylvania has been increasing due to the migration of various settlers. In 1900 the United States Census Bureau reported a total of 6,302,115 people present in the state. Assuming the increase has been exponential and increasing at an average rate of 1.0266% annually, create a model that represents this problem situation.

a. Sketch and find a complete graph of this function, including any asymptotes.



Label and explain the following:

- x-axis and y-axis
- scale on each axis
- all intercepts

b. State the domain and range of the algebraic function:

c. **Highlight or box** the portion of the graph that is relevant to this **problem situation.**
How is this portion of the graph different from the domain of the function?

d. Tell how you would find the growth rate from 1995 to 2005 algebraically. In 1995 the United State Census Bureau reported 12, 071,000 and in 2005 reported 12,281,000 people.

e. When will the population grow to twenty million people?

f. Based on the Model, as the time (years) increase, will the population approach a limit? If yes what is that limit? Explain your reasoning. If no, explain why the population has no limit.

Classroom Activities:

Fun in the City

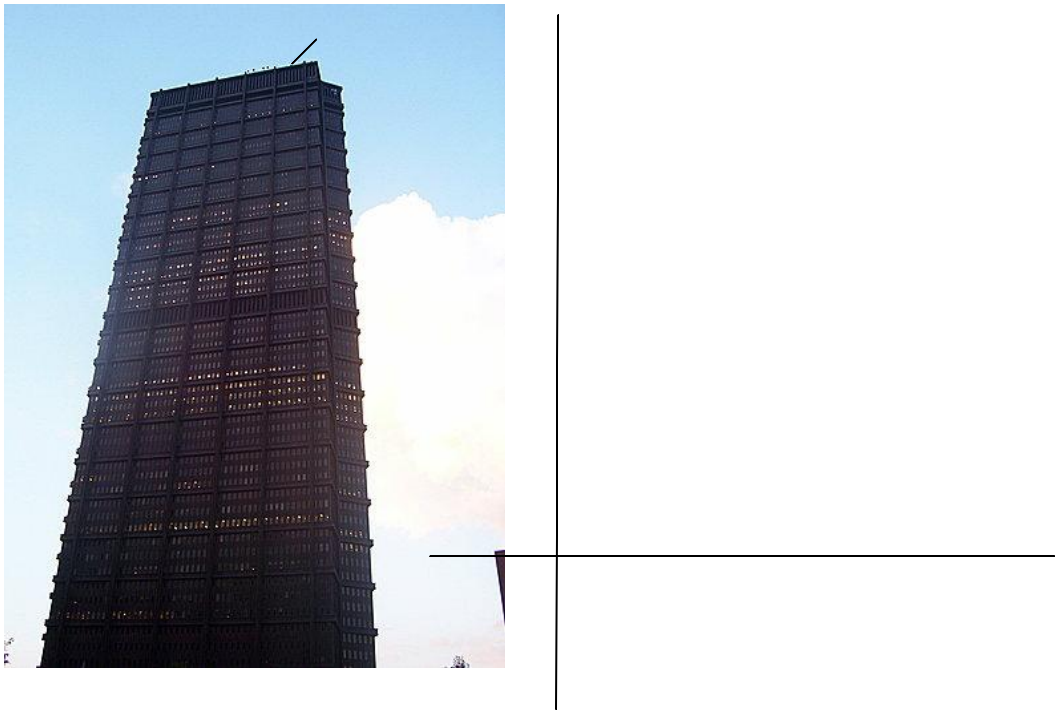
When an object is thrown straight up into the air, the relationship between the height of the object above the ground and time after it was thrown can be modeled by the quadratic function

$$H = - 16\, t^2 + v_o t + h_o$$

Where the variable h represents the height of the object , the variable t represents the time after it was thrown, the leading coefficient g represents the force of acceleration due to gravity, the linear coefficient v_o represents the object's initial velocity, and the constant term h_o represents the object's initial height above the ground.

Problem: Fireworks are shot from the top of the USX Tower with an initial velocity of 150ft/sec.

USX Tower height 831ft/225.7m built in 1970, 164 floors



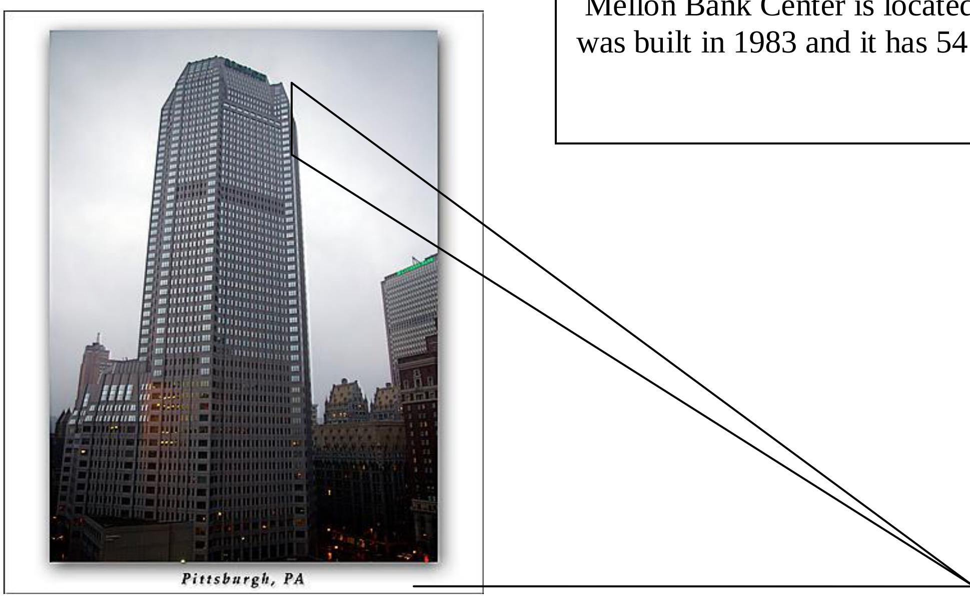
1. Write an equation that represents the height of the fireworks.
2. Sketch a complete graph of the function and highlight the portion that represents the problem situation.
3. Give the Domain and Range of the Problem Situation.
4. Find the height in 10, 20 and 30 seconds.
5. Determine the maximum height of the fireworks.
6. When will the firework reach 400 ft?
7. Determine when the firework will reach ground level.

Classroom Activities:

The Designing Engineer

Kendal, an engineer, noticed that the angle of elevation from an observer to bottom edge of Mellon Bank Center's observation window located 200ft from the base of the building is 74 degrees and 22 minutes. .The angle of elevation from the observer to the top of the observation window is 74 degrees and 34 minutes.

Mellon Bank Center is located in Pittsburgh was built in 1983 and it has 54 floors.

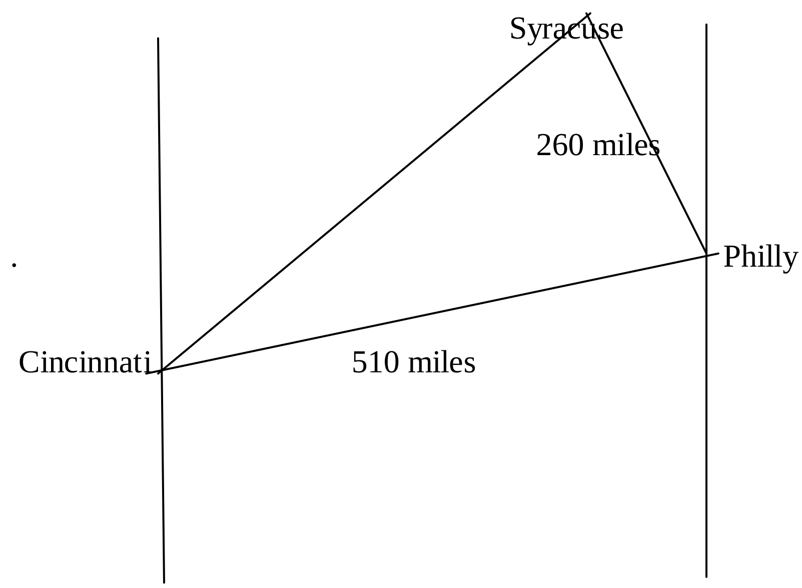


Problem: Determine the height of Mellon Bank Center and the observation window.

Classroom Activities:

NAVIGATION

The airline distance from Philadelphia to Syracuse is 260 miles, on the bearing of 335 degrees. The distance from Philadelphia to Cincinnati is 510 miles on a bearing of 245 degrees. Find the angle from Cincinnati to Syracuse.



1. Which angle forms the right angle and why?
2. Find each of the other angles in the diagram.
3. What is the distance from Cincinnati to Syracuse?

4 What is the bearing at Cincinnati?

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Scott, Foresman, (1983) “Trigonometry”, Scott, Foresman and Company p. 117-120. This text is a great for teaching beginning trigonometry. Also this text has rich application problems.

Stigler, J.W. & Hieber, J. (1996). Understanding and Improving Classroom Mathematics Instruction. *Phi Delta Kappa*. v 79 n1 p14-215.
Technology is integrated into various classroom activities in this article. Students can start with the linear function and develop others from the basic function.

The NCTM 1988 yearbook. Reston, VA: *The National Council of Teachers of Mathematics.* This manual is great for reviewing standards from early years in order to see various changes that have been done throughout time.

The National Council of Teachers of Mathematics, (2000). *Principles and Standards for School Mathematic.* Reston, VA: The NCTM INC.
The national standards for teachers and students are the main focus of this book. There are example and guidelines for both students and teachers.

Web sites:

Dendane A, (2007). *Math Software*. Retrieved April 2008 at http://www.analyzemath.com/math_software.html
This site contains math resource for many high school courses.

Dendane A. (2007). *Graph of Sine, a*sin (bx + c), Function.* Retrieved April 2008 at http://www.analyzemath.com/Graphing/GraphSineFunction.html.
This site gives useful information about the sine function.

Bruce A (2007). *Puzzles to print out then Solve*. Retrieved April 2008 at http://www.adrianbruce.com/problem-solving/problem-solving.htm
This site contains a great deal of problem-solving material.

Compare Infobase Limited (2008). *Golden Gate Bridge*. Retrieved April 2008 at http://www.travelsiteshub.com/america/golden-gate-bridge. . This site gives information on bridge design and construction of the Golden Gate Bridge.

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Science Net Links,(2007). *Technology: Past, Present, and Future*. Retrieved March 2007 http://www.sciencenetlinks.com/lessons.cfm?DocID=355
This site shows how technology has been one of the greatest influences on society and social change.

USCENSUSBUREAU (2004) .*Census of Population and Housing*, Retrieved April 2007.at http://www.census.gov/prod/www/abs/decennial/index.htm.
This article shows the United States' national population. The article travels back to the early 19th century.

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Appendix A: Answers

The Radio Problems

a. Y = 4 sin 2x loudest, y = 2sin 5x softest
b. Y = 2 sin 5x highest pitch and y = 3 sin (1/2) x lowest pitch
c. Its shrinks; pitch gets higher.

The Flying Squirrel Problem

About two hours amplitude and the period is about 12 months

The Washer Problem

1. Volume is about2.05 cubic inches

   Weight is about 10.25 oz.

2. Use the volume of rectangle solid V =lwh and volume of cylindrical hole is v= $\pi r^{2.}$

Part 2

a. C(x) = .10x + 500,00 and R(x) = .30 x
b. $502,000
c. $6,000
d. Need to sell 2,500,000 washers use window setting [0, 500,000] by [0, 1,000,000] graph both functions and find the point of intersection.

The Golden Gate

The height above the road of the cable at its lowest point is 10 feet. Use he concept behind the vertex (h, k) and substitute x for h.

New York City Population

F(x) = 14,830, 192 $(1.005)^x$

b. D: $(-\infty,\infty)$ and R: $(0, \infty)$
c. D: $[0, \infty)$ and R: $[14,830,192,\infty)$
d. Substitute x = 0, initial population

e. 1960 – 15,588,600, 1970- 16,385,813, 1995- 18,561,777, 2000- 19,030,485.
f. .05%, in 2008 New York is expected to have 18,254,238 people.
g. In 60 years around year 2010.
h. Continue to grow.

Pennsylvania State Population Growth

$P(t) = 6{,}302{,}115(1.01026)^t$

b. D: (-∞,∞) and R: (0, ∞)
c. D: [0, ∞) and R: [6,302,115,∞)
d. Use the equation $12{,}281{,}000 = 12{,}071{,}00(1+R)^{10}$ and solve for R, R= 1.726%
e. In about 113 years
f. No limit base on the equation the population will keep growing.

Fun in the City

1. $H = -16t^2 + 150t + 831$
2. Use the information provided to sketch a graph with the TI-83
3. D: [0,13.28] and R: [0,1182]
4. 731ft, -256 ft, - 9069 ft
5. In 4.69 sec 1182 ft
6. 11.68 sec
7. 13.28 sec

The Designing Engineer

Mellon Bank Center is 715 ft tall and the observation window is 10 ft tall.

Navigation

1. The angle at Philly.
2. At Cincinnati 27 degrees
3. About 573 miles
4. 38 degrees.

Appendix B

This curriculum adheres to both the Pennsylvania State and National Standards:

Academic Standards for Mathematics

2.1 Numbers, Number Systems, and Number Relationships
Types of numbers (e.g., whole, prime, irrational, complex), Equivalent forms (e.g., fractions, decimals, percents)

2.2 Computation and Estimation
Basic functions (+, -, x, /), Reasonableness of answers, Calculators

2.3 Measurement and Estimation
Types of measurement (e.g., length, time), Units and tools of measurement, Computing and comparing measurements

2.4 Mathematical Reasoning and Connections
Using inductive and deductive reasoning, Validating arguments (e.g., if...then statements, proofs)

2.5 Mathematical Problem-solving and Communications
Problem-solving strategies, Representing problems in various ways, Interpreting results.

2.5 Statistics and Data Analysis
Collecting and reporting data (e.g., charts, graphs), Analyzing data

2.6 Probability and Predictions
Validity of data, Calculating probability to make predictions

2.7 Algebra and Functions
Equations, Patterns and functions

2.8 Geometry
Shapes and their properties, Using geometric principles to solve problems.

2.9 Trigonometry
Right angles, Measuring and computing with triangles, Using graphing calculators.

2.10 Concepts of Calculus

Comparing quantities and values, graphing rates of change, Continuing patterns of infinitely

The national standards for school mathematics describe the mathematical understanding, knowledge, and skills that students should acquire from prekindergarten through grade 12(NCTM 2000).

- Algebra
- Geometry
- Measurement
- Data Analysis & Probability
- Process Standard
- Problem-solving
- Reasoning & Proof
- Communication
- Connections
- Representation