Recycling Math: Engagement and Pragmatism in Algebra Instruction

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Overview

As a public school teacher I am constantly caught in an internal debate. On a daily basis I question the fundamental premise compulsory education which – in most cases – is a general course of studies that have been greatly influenced by the mandates of public funding. I fear that without more choice and purpose in the course of studies, we may be failing to provide quality education from the outset. To make this situation all the more confounding and my situation comical, the subject I teach is math. So, as I ponder the ethics of the entire structure of public education that I am part and parcel to, I also actually delve into the numbers that make it all "work."

While rigor can't be all fun, I want to make the hours that my students spend in school the most informative, positively challenging, and significant parts of their days. Since they are compelled to be there, I want to present them with eye-opening truths about how things work and what part they can take, as they choose to, within the grand scheme of universal and human actions. I particularly take this instructional attitude with freshmen.

As freshmen, students are entering into the socially and politically educational environment of a public high school (as much or more as they are entering an academic environment), it is a perfect step in their acclimatization to and investment in their school that they should be given a rigorous series of exciting assignments. This is all the more impactful for students in mainstream academic courses who may typically feel disenfranchised from the academic environment and need to see that the work in the classroom can and should be associated with the realities of their world, in which the classroom exists.

As a means of engaging students in meaningful work, I hope to make recycling (and energy consumption) the centerpiece for study in the Algebra 1 course that I teach. CAPA High School, like many in the district, does not have a recycling program. As a real life problem worth exploration, I propose the eventual research and institution of recycling programs within our schools by our students. Because math is such a staple of all our public education efforts, yet is largely regarded as tedious and of limited merit by a large number of students, I believe that we could actually instill a sense of ownership and purpose within every school community via a sense of mathematics utility.

If students can internally say "what I am being asked to study and learn in school actually matters in the here and now for myself and my community" we can expect greater achievement based on this improved efficacy.

Rationale

All of the writing in this paper is actually devised to help me answer just one, but one very important, question: "When am I going to use this in my life?" It is not a question that I am asking myself. It is a question that I have been asked dozens of times by many Algebra 1 students. (ninth graders who are typically 14 years old and taking the course within the mainstream curriculum because of poor performance in previous mathematics courses or tests)

Too often my answer to students has taken the form of "someday…" As in: maybe someday if you are interested in architecture you'll use this knowledge; someday when you have to manage a mortgage, car payment, insurance, phone bill, and grocery tally all at the same time; maybe someday when you are managing the budget for your own theatre company; someday when you are studying higher levels of mathematics you will need these basics; maybe someday you will want to plan a trip to Australia and need to calculate your costs in multiple currencies and your schedule in multiple time zones… there is more than one someday in the week when you are trying to answer this question.

Yet, for all its malleability and adaptability to students' interests, I have come to realize that "someday" is the wrong answer. Perhaps not in a universal, objective sense, since these responses have on the whole pulled students through lessons that they were avoiding, but in a subjective sense that actually is much more meaningful for its specificity, "someday" is as bad or worse an answer for me to give to students as "never." I realize this now, despite long adherence to using someday answers – I've often attempted to make an analogous link between developing basic math skills through training and the types of training in the skills of creative and performing arts that my students are engaged in at Pittsburgh CAPA High School. They understand on a deep level that in order to be competent in real, true performance at some point in the future they must hone their skills (of course this is as relevant to sports training or other forms of preparation for students with different interests or extra-curricular activities).

While I know, and any mathematics teacher who reads this will know, that the skills developed in an Algebra 1 course are essential basics, fundamentals, that do require some intensive practice and a wealth of continuous practice in order for students to retain their skills for bigger problems and more complex math in the future, I also have come to know that treating Algebra 1 as a tentative study, or as an activity that is only precursory to what a student might do later, simply "will not fly" with the generation of students whom are currently being enrolled in freshman math classes. For the coming school year, and most certainly in years to come, I need a new answer to this "When…" question.

For these students, coming into the beginning of their high school experience and the beginning of their physical, emotional, and cognitive adulthood concurrently, the only suitable answer to them, when they ask questions about any anticipated event, is "Now!"

This may seem like surrender to many teachers; as though we are giving in by not instilling a sense of patience and maturity in ninth grade mathematics students as they embark on a journey into the profundity of academia. Oddly, most of these same "many teachers" also consider themselves "hard realists" and will espouse pragmatism in all things from household management to government spending… in addition to their planning of curriculum and their classroom management. I retort that, in order to be pragmatic and realistic, both of which I consider myself to the fullest extent, we needn't surrender to any sense of entitlement or desire for instant gratification that we perceive students to have, but we absolutely must reduce conflicts between the work we assign to adolescent mathematics students and their need to be actively making use of, rather than merely absorbing, knowledge.

In his highly reasoned and comprehensive book *How to Solve It*, G. Polya introduces a pedagogical methodology by which students are challenged with problem solving in a way that continuously builds their competence, continuously connects their existing skills and strategies with new "bright idea" moments – the minor and major epiphanies that come of synthesizing multiple touchstones of past experience into new problem situations to break through their barriers and discover solutions. This relies, to the utmost, upon students' identification with the problems and students' ability to see the reality within the theory.

A large portion of Polya's book is a dictionary of terms, but this lexicon is structured in a self-referential and web-oriented way that it is possible to read through it – either sequentially in alphabetical order or as if it is a flowing discourse, moving from one entry to another holistically much like a "choose your own adventure" novel – and absorb the mathematician's discourse on instinctive problem solving. The beauty of this is that, in whatever order the reader takes in Polya's method it becomes quickly apparent that it is extremely rational in its profound simplicity and doesn't rely on anything more than the student, a basis of reality, and the teacher. It is equally effective in a classroom of the future, in a classroom with no more than a chalkboard, or in a student-teacher dialogue carried out sans written or visual materials.

In my own studies to be a teacher, and in the common knowledge of contemporary educators, there is absolute value in relating content to students through context. Polya provides the clearest, most logical arguments for this that I have ever read – giving a humanist argument through the most stoic and unemotional means. A particular example of this is his relation between geometric figures and features of a classroom that can bring an abstract mathematical skill set to bear on an observable, true to life space. This is a step toward using a math concept "now" with immediate results that the students can not escape acknowledgement of. However, geometry lends itself to this physicality a good deal more frequently than algebra does. Polya's method, without a doubt, bears on effective algebra instruction, but algebra in its very essence and utility is an abstract subject. It is crafted by mathematicians, scientists, economists, and all who use it, to create intentional abstractions through which change can be observed and some complexities can be ameliorated.

So, how am I going to answer "Now. You are going to use this in your life right now," and offer my students legitimate work to which they can apply the principles and skills of early level algebra studies? In order to answer "now" the content and context of student assignments must be timely and relevant to the school, community, city, state, nation, and world at large these students are living in. In my experience, this is a point more often made in the pedagogy of literature than in that of mathematics. Probably because math has an air of universality, which seems self-justifying, as numbers are a part and parcel to human activity.

Math is part of everything we do within the present day world. But, while this is worthy of reiteration and deserving of frequent exemplification within the math classroom, it still falls short of serving to make students treat their algebraic study with the respect that they give to more tangible and accessible projects or personal interests. The dilemma here is that, while it may actually engage some students and facilitate their deeper understanding of algebra to connect our academic discourse with context that they can observe in the world around them, we are limited in our ability to instruct and limiting of their ability to internalize instruction if we merely give lip-service to the "real world."

We have made strong and effective use of word problems and multiple representations (verbal, expression/equation, table, graph, etc.) in the Algebra 1 curriculum in past years, with an effort to reference scenarios and situations students can relate to. My experience and the quality of work generated by my students have convinced me none the less that we should not rely on contrived problems, as good as they may seem and no matter how based in reality, as the mainstay of our algebra course content.

We may, with varied success, pull-in students' attention and gain a great deal by way of having them generate the numbers they work with in their classroom or "math lab" by having them flip coins, measure the length of their limbs, or assess fabricated phone plans (all of which have been part of the Pittsburgh Public Schools mandated Algebra 1 curriculum in the past year). These types of activities may help students get-by until "someday" they eventually use the same or similar math skills in application to their own lives, but they do not teach students an important lesson that should be part of the implicit hidden curriculum: that this study really matters because we are using it to accomplish something right now. Such problems as we have been making use of play on reality, but do not deliver a real payoff or result, quite so much as they show one or more gimmicks for working out what could possibly be a real problem. As a result these word problems can quickly devolve to the level of parlor tricks in students' valuation, and thus lose impact.

How can I introduce students to tangible math? What context can I utilize, not only to create the façade of real issues and actuarial data, but also to give substance to the algebraic skills my students are putting into practice and deliver them a result that impacts their learning and appreciation for algebra in more than a superficial way? What will be the right frame of reference for students to work within in order to develop math skills, what do students care about, what are they concerned about and wish they could change if only they knew how to. How can I have students "use it now?"

Algebra and the 14-Year Old Mind

I can anticipate many teachers throwing their hands up and exclaiming "do we have to put everything on a silver platter?" "Don't we cater to students enough?"

The notion that curriculum needs to be geared to the interest and lifestyle of students is downright offensive to many educators who grew up with education systems that demanded their attentiveness, adherence to rules, and completion of work regardless of their excitement about the subject matter. I would argue that there are some holes in the rationality of this exasperation… we became teachers, so maybe we were already better rule followers and/or more intrinsically motivated to learn in any given subject than the average student?... we were also all born in a time when there wasn't a computer (or multiple computers) in most American households, when cable television wasn't readily available, and certainly when there were not mobile media devices in everyone's hands.

The very grounds of our expectations are shaky if we look to students today to sit and wait for us, the teachers, to give them information. Everyday of their conscious lives, they have had the choice of a fast and diverse menu of informative options, which they make extensive use of. Ultimately, we would all acknowledge that the quality of education provided in schools today should be better than it was in the past. Given that students have this vast array of “inputs” already working for them, there is a very real urgency that the inventiveness and delivery of our curricula be geared to compete for students’ attention as they enter our classrooms.

I, for one, can not reinstruct my students to behave as students were expected to a decade ago and then move on to instructing them in mathematics. I don’t have the wherewithal to conquer and reform the basic identities of my students prior to teaching them the problem solving skills that will help them face life. I have to get right to the heart of skill development without attempting to make students do things “my way.” My best option is to find a way to deal with the form that adolescence has taken in America (and in particular to Pittsburgh) and teach to my students as they are today. (This approach is highly influenced by the writings of William Glasser, the eminent psychologist, in his book *Choice Theory*, which I unflinchingly recommend to every teacher and most everyone else). Again, this means I can be most successful by demonstratively teaching my students to “use math now.”

As noted in a Creedance Clearwater Revival song, “someday never comes.” In that song, the notion is that an empty promise one generation often makes to the next is “just do as we say and someday you’ll understand why.” In actuality, the next generation may likely repeat the behavior as instructed and modeled, implicitly or explicitly, but they’ll never really comprehend why.

To a teen-aged mind that is bio-chemically, physiologically geared to take thoughts and put them into immediate action, this is all the more true. This also plays into what we should be more proactive in recognizing about our students, based on fundamental Piagetian principles of cognitive and emotional development. Within the theory of Piaget, there is no skipping of stages in development, and even as an outward physical development happens, the mind, as a codependent process of cognition and affective reasoning, must be respected as an independently growing part of the individual. As teachers, we too often look at teenagers in contrast to children and take an attitude of “stop being a baby,” in which we expect them to settle down and be serious about their academic work. They are starting to look grown up, but in terms of emotional intelligence they are only one year older than a middle school child.

We are right in setting an expectation of the latter, that they be serious about their work, but if we truly expect the former, that our ninth grade students will "settle down," we are somewhere on a precarious spectrum… with "incompetent" at one endpoint, "insane" at the other endpoint, and "negligent" at the midpoint. We would be better off recognizing some of the seemingly manic behavior as being much more a sign of spontaneous intelligence than a sign of rebelliousness.

If I can borrow Polya's clout to bear on this, in *How to Solve It* the definition for "the intelligent problem solver" emphasizes:

> *The open secret of real success is to throw your whole personality into your problem.*

For an intelligent problem solver, engagement is obviously crucial. I believe that we need to assume more intelligence of all of our students at all levels – the self fulfilling prophecies of "I'm just not good at math" are ever more infectious and detrimental for early-teen adolescents. If we don't sell the students on both the value of study of algebraic reasoning and on their capabilities to gain some mastery of that skill base, we are silently reinforcing self defeating attitudes that mainstream ninth graders have brought with them from middle school.

Polya goes on to give the definition "the intelligent reader of a mathematical book desires two things:"

> *First, to see that the present step of the argument is correct.*
> *Second, to see the purpose of the present step.*
>
> *The intelligent listener to a mathematical lecture has the same wishes. If he cannot see that the present step of the argument is correct and even suspects that it is, possibly, incorrect, he may protest and ask a question. If he cannot see any purpose in the present step, nor suspect any reason for it, he usually cannot even formulate a clear objection, he does not protest, he is just dismayed and bored, and loses the thread of the argument.*

This can, of course, be taken on face value as a statement Polya is making about a given mathematics problem, raw math skills that a student might deal with. But I read this, in addition, as a more basic statement about the very choice of content to present to students – only interpreting this concern about students' ability to follow the arguments within the mechanics of technique is putting the cart before the horse. If math teachers do not provide students with meaningful, highly active (extremely cognitive and, at least occasionally, kinesthetic) problem solving activities to take part in, students will zone out in the classroom. We will have lost them before they ever get an opportunity at skill acquisition.

Recycling as Content and Theme

In order to be timely enough to capture student attention, yet purposeful enough that students are not merely processing fads or “of the moment” context that will slip away from what they might otherwise internalize as meaningful, I am turning to the very significant and numerically-rich topic of recycling.

Recycling, and the ecological issues surrounding the practice and industry of recycling, are now, and for the foreseeable future will be, ever present in most every subject area and field of media that students are exposed to. Beyond the obvious of social/political, science, and business reporting: human interest stories are often tied to materials used in manufacturing and the ethics of manufacturing practices; travel and leisure articles will be increasingly connected with reflection on the expense and availability of services and goods relative to their ecological and thus economic vitality; even sports are being heavily linked to ramifications of human energy consumption – as environmental change accelerated by greenhouse effects brings on warmer averages and chaotic seasonal patterns. A very tangible example is the prevalent focus on conditions in China as related to Beijing’s hosting of the 2008 Summer Olympics.

Pop culture and celebrity profiles also offer frequent reference to ecological awareness and progressive development in sustainability, green energy, green architecture, and materials recycling. So, beyond the personal interest that students exhibit in the topics of environmentalism and humanitarian concerns, their pop-culture icons often further encourage and/or stand the scrutiny of students for their stances on environmental and social issues, making the context of a recycling program all the more poignant as a means to engage fourteen year old Pittsburgh high school students, and American high school students at large.

I expect the theme of recycling to be hugely advantageous in overcoming the challenges of engagement, active learning, and a sense of ownership in their mathematics studies, for the reasons stated above. Many students I have had in the past few years have decried their schools’, and the school district’s, lack of ecological responsibility in not having the infrastructure for paper, plastic, and metal recycling. Encouraging this spark of intrinsic motivation from my students, I would like to propose fanning it into a flame of activity through which students are concentrated on practical math work.

What is more, taking such a focus early in their high school career can quite legitimately prepare students for expanding their thoughts within mathematics as it applies to the other social issues that will define the era in which their generation will, itself, be defined. They will be "recycling" the very basis and utility of mathematics (particularly the profoundly expansive field of Algebra) as it has been studied for centuries in numerous civilizations to promote and manage human development. For all of the reference to the ancient philosopher mathematicians who used their math discoveries to develop cities, technologies, and medical innovations, we haven't had our students following suit enough – ideally we can organize math education in our schools so that they will be reapplying, repurposing, and reinvigorating math in order to solve the social, ecological, and economic problems of their own day and age.

Objectives

I believe the true, summative objective for our students in an Algebra 1 course should be stated as this: Students will be able to make effective decisions about how, when, and why to use basic and algebraic math skills.

There are many formative objectives that are being addressed in our curriculum for Algebra 1. But I see a need for explicit purpose to the course as a stand alone study. The foundation of math skills involved, in terms of computation, properties of operations, comprehension of different forms of notation (fractions, decimals, etc.), and the development of expressions and equations are all currently within the curriculum presented to eighth grade students of Pittsburgh Public schools. In many ways, the Algebra 1 curriculum for ninth grade students has been identical to that presented to them in the two years prior as middle school students.

Not every student of Pittsburgh Public Schools will go on to be a mathematician of course, and the truth is that for some of the specific mathematical skills that are emphasized in freshman algebra courses my most honest answer to "when am I going to use this?" would have to be "now, in your future math classes, and then probably never again." Algebraic math skills, particularly as applied in problem solving, are essential to a person's survival and learning at large, but some points covered in the course are not of the greatest value to all walks of life, or, at very least, they are presented as raw skills and lose internalizing effect. This is exacerbated in a school for the Arts. I'm not off the hook for instructing all aspects of the Algebra 1 course to the best of my abilities just because I have a differing recognizance about some of the content. However, this recognition does demand that I give some consideration to how I would prioritize the skills students should learn within the script handed to me.

Three-Fold Guidelines

My guidelines for creating objectives are a three-fold combination of mandates and what I've gathered from professional and academic experience. They consist of Pennsylvania State Standards, standards from the National Council of Teachers of Mathematics, and a set of common sense principles based on *Mathsemantics*.

Pennsylvania State Math Standards

Within the spreadsheet of PA state standards for math, the skills pertaining to actually making a decision based on uncertain, non-routine situations is fairly limited. This means that much of the standards we base our curricula upon are "lab standards" meant for building the skills that can be applied to a static situation with known values and/or "easy" whole number or integer answers. It begs the question as to whether it is desirable to acquire computational skills to solve a math problem without more emphasis on acquiring the skills to recognize when these problems might occur and how they can be identified. If the practical application of the computational skills is mastered entirely in absence of the pragmatic situational experience, the hidden curriculum will continue to teach students that the only place they'll ever use math skills is in the classroom. This fundamentally restricts what I or any math teacher might present to students and can relegate students' estimation of math skills to purely theoretical knowledge.

I'm on a precipice of being over-dramatic perhaps. The state standards are quite comprehensive and serve as invaluable touchstones for myself and other math teachers to organize and articulate our educational goals for students. However, in action, at very least for mainstream math students, an alternative set of lessons and activities needs to be supplemented into their curricula to give students the reality based context they require to meet these standards. The consequence of not providing such supplemental learning opportunities will be failure of our system, or at best, without some other means of getting students to invest personally in math skills, we will have an output of pseudo competent test takers with little or no true competency as decision makers.

The PA state standard that best matches the notion of "decision competency" is *2.5.11.A*:

> *Select and use appropriate mathematical concepts and techniques from different areas of mathematics and apply them to solving non-routine and multi-step problems.*

This standard is exemplary of my main objective, but as a level of mastery not easily evaluated via multiple choice exams it often goes underemployed. Parts B and C of the set of PA Math 2.5.11 are incredibly valid statements about use of proper mathematical notation and clearly demonstrated process., yet they are outshined by the profoundly “high level” rigor expressed in standard *2.5.11.D:*

> *Conclude a solution process with a summary of results and evaluate the degree to which the results obtained represent an acceptable response to the initial problem and why the reasoning is valid.*

This standard speaks to a priority I see for students – the ability to breakdown a process into parts. From my own professional business experience as a technical writer and assistant project manager in the IT industry, I know that it is vitally important to be able to “conceptualize a process” Across any professional entity it is incredibly important for all parties, from technicians and computer programmers to sales representatives and client relations managers, to each have the same mental picture of how a process works when they communicate about it. It is obviously more important for some career paths and positions to have strong skills in building and manipulating the actual algorithmic, numeric structures of processes, but everyone involved, on any level, needs skills in analyzing a process, comprehending on a basic level how it functions, and being able to articulate what it does.

Recognizing a problem is only possible if you know how things are supposed to function, or, in the case of creative development, recognizing problems requires you know what it is you’d like to create and what you’d like it “to do.”

Having students master raw algebra skills in the ninth grade is not a bad thing. It’s wonderful in fact for students to be able to tackle a dozen computational problems proficiently, it will serve them throughout academia and is superb exercise for the mind independent of any other concern. The big “However…” here is that, without knowing why these skills matter “off paper” or how they might be of any use in dealing with problems that concern our lives, a large portion of students, particularly in mainstream courses, will never put forth the effort to master algebraic reasoning and computation – let alone retain those skills for use in legitimate work.

To be less of an idealist, I have to admit that not everyday can be a math fieldtrip that allows students to go into our city for observation of, and interaction with, situations that bring their developing skills into true application. However, there can be some days like this – and far more frequently the approach taken in the classroom can be geared to real uses of the math skills learned by students. I deem the 2.5.11 set of PA Math Standards worth serious emphasis in this regard.

National Council of Teachers of Mathematics Standards

I became aware of NCTM only through this PTI seminar. It is striking to me that the organization is not more integrated into the Pittsburgh Public Schools or at minimum is not referenced more within our mathematics departments. Of course, as an organization that requires a paid membership, it may compete with other educational contracting, however, the principles and standards of NCTM are readily accessible and mostly free online. I have found that they offer profound guidance through simple statements, very much in keeping with Polya's style of delineating a problem solving methodology.

In the NCTM *Principles and Standards for School Mathematics*, there are very succinct statements that serve as excellent primers for objectives and allow a teacher to build the structure and logistics of lessons with a clear sense of what skills students should become capable of performing. What is the more significant about these principles and standards is that they contain points that address the need for mathematics comprehension that extends beyond academia.

There are five general sections of these standards: Problem Solving; Reasoning and Proof; Communication; Connections; and Representations. They actually read almost like mantra from Lao Tzu's *Tao Te Ching* because they are universal enough to apply to whatever area of mathematics a teacher maybe involved in, yet they are not so esoteric as to lose value in their detachment from specifics.

NCTM also provides specific standards per math subject area, of which the Algebra standards are extremely effective as a very concise (fitting on one page quite legibly), yet comprehensive list that can serve as a regular reflection tool for teachers to self evaluate their instruction and course work assignments. This section on Algebra is broken into four subsections that effectively segment types of reasoning and skills.

Within these six sections, the five general and Algebra, there are numerous points of particular importance and utility to me in setting objectives, which work toward my ultimate goal of decision making competency in students. Perhaps foremost of them is the simple statement from the Problem Solving Standard:

> *build new mathematical knowledge through problem solving;*

Having students develop knowledge and skills by actually doing something beyond rote repetition or direct instruction goes very far in the direction of the kind of authentic learning I am advocating. As an example: ninth grade students are capable of deriving the means of performing algebraic substitution deductively from their prior knowledge of math operations if they are given tasks and prompting that inevitably require that they account for an equality between differently expressed quantities. This kind of discovery learning has been pushed into the core of our curricula in the past several years, but what I believe has been lacking is a level of tangibility or concrete context to finalize the connection between the abstraction of algebra and its basis in reality.

In terms of keeping my instruction of algebra thus grounded and applicable to the students' perspective of the world outside of the math classroom, two other statements from the NCTM are particularly apt:

> from the Connections Standard
> *recognize and apply mathematics in contexts outside of mathematics*
>
> and from the Algebra Standard
> *identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships*

The purist in me loves math for math's sake, but the pragmatist in me cannot ignore the fact that without its execution upon the concerns of the wider scope of human activity – including the humanities and arts, especially in the environment of the CAPA magnet program – it is an endangered species of academia. I go so far as to think that we need to approach math from the outside… that we need to take students through a process of seeing real problems in real situations first and then looking at how mathematical reasoning and skills can help solve it.

Mathsemantics

The term "mathsemantics" has a red, wavy line beneath it on the computer screen as I write this paper. I consider that a great misfortune for us all that it is not recognized by the Word application that relies so much upon the logic embodied within the mathsemantic ideology. The term comes from the title of the invaluable book *Mathsemantics: making numbers talk sense* by Edward MacNeal. If you have ever read a piece of nonfiction that really "spoke to you" by articulating a notion you had thought or felt a good deal but didn't quite put your finger on, then you'll understand how I took in MacNeal's book. It should be standard reading for every teacher of mathematics in the US and could be used for student reading.

Fundamentally, mathsemantics argues that numbers without context cannot be viable and that the semantics in which we house mathematics is of equal or greater importance to the numbers or symbols themselves. Let's say a student completes a system of operations exercise proficiently, finding the values of x and y intercepts in each of two linear equations, as well as the solution or intersection of the lines of the two equations. So what? The student can calculate. Can the student tell us whether the solution is a good one or a bad one? Can the student tell us if it would be favorable (or possible) to adjust one of the axis intercepts to change one of the equations and thus solution? None of these questions even makes sense without context, and the context is what would give the answers.

In connection with the PA Math standards and NCTM principles and standards mentioned above, we need to always be asking students questions that break the "absolutism" of math studies and provide common sense rationality to their work.

Strategies

Students in mainstream math courses tend to express a lack of affinity with the subject. This understatement is meant to set up my reasoning for strategies, including literary exercises for mathematics. Many of the students struggling in math excel in other subject areas, particularly humanities, which are no less strenuous in cognitive demands on logic, memory, and organizational reasoning.

In a program for the creative and performing arts this scenario has played out somewhat stereotypically. However I'm not discouraged as to any of these students' abilities to succeed in mathematics studies, I am simply convinced that the presentation of math, particularly for the Algebra 1 course, require an approach that makes use of the skills with which students are flourishing, rather than relying on traditional "here are some problems on paper" approach. To whatever degree such problems may be well crafted into exercises, they can only work as drill, or not work very effectively at all with students who aren't "convinced of the purpose" of the study, as Polya puts it.

I began my teaching career in volunteer positions as a mentor for an after school, high school student writing program and as a summer camp swimming instructor. In these experiences I quickly saw the value of students trying something for better or worse and then reflecting upon it, the longer they thought about doing something before attempting to do it the more difficulty they seemed to have becoming comfortable and proficient with skills. So from the outset my attitude toward instruction favors linguistic and metacognitive exercises and activities. But objectively, as I have since been an English teacher and now a math teacher, I continue to see the value in both literary and hands-on approaches to student learning for mathematics. I believe that part of why these styles of presenting mathematical concepts and skill development to students are not more prevalently espoused or employed is the stereotypical polarization of math away from humanities, of abstract thought as a practice done away from the temporal continuum.

So, in short form, my strategies are to use "hands on" literary exercises and metacognitive activities in order to meet the objective of having students make effective decisions about how, when, and why to use basic and algebraic math skills… and having them begin fulfilling this objective from the very outset of their Algebra 1 course. In long form, I give some basic premise and additional rationale below.

Literary exercises

Other than gym class, I don't think that there is another subject in which high school students do not typically write papers, complete long term research projects, and/or have serious reading assignments that inform them about the given subject area… besides mathematics. Of course, some teachers of math do fit these assignments in, but it seems they must swim against the current of curricula in order to do so.

I simply suggest that more reading and writing can happen in the Algebra 1 classroom and that this will not only ingratiate students to the course more, but it can most definitely make them more successful in their comprehension and application of the concepts and skills of algebra. Perhaps it is a default of alphabetical order in *How to Solve It* that Polya's short dictionary of terms begins with "Analogy," but I find it very significant. Not only does he express presenting other, similar, but more easily solved, math problems to help deal with a present difficult problem – the notion of literary analogies to help student thinking move with the situational logistics of a math problem is also supported by Polya.

Above I have referred to contrived word problems in a less than praiseful tone. However this is not to disregard their quality in terms of either engagement or effectiveness, it is simply to point to the certainty that even more can be accomplished with project based activities that apply skills, not to imagined scenarios but, to situations that are actual, observable, and factually quantifiable within the real world, most ideally if these situations bear some significance to the students', school's, and community's efficacy.

Literary supplements can be used in the minimal for a warm up exercise – many numerically based, one to two paragraph blurb articles appear in popular magazines and on the covers of newspapers; these are real situations that might be read by students, with the assigned task of extrapolating the numbers from the article and discussing their significance – or literary supplements can be more substantial excerpts from longer articles that provoke logical and numerical reasoning. Every issue of National Geographic is full of numerous selections.

Dependent upon the time allowable to a math teacher (I realize, of course, that the ability to creatively conduct curricular changes is severally limited in Pittsburgh Public these days), the real ideal would be the use of significant book excerpts that might open student debate about how numbers are used and how the skills of the Algebra 1 course are applicable to the issues of recycling, energy consumption, and the "footprint" of human activity on earth. Of the many interesting books of recent publication there are many such titles, but two that I can personally recommend from my reading list are *Cradle to Cradle* by William McDonough & Michael Braungart and *Small is Beautiful* by E.F. Schumacher.

Without routing this paper toward a book report, I'll briefly say that *Cradle to Cradle* can be a very versatile source of readings for students in a mathematics classroom because the authors, a chemist and an architect, relate many practical and very real details about the practices of manufacturing and materials that interact with the lives of every living person and thing on the earth. Their writing is highly relevant to thinking about the value we place on numbers and the realities of how our expectations are often disillusioned by outcomes. A prime example is when the authors relate how the oil spill of the Exxon Valdez actually increased the GDP of Alaska for 1991 because of the influx of clean up and relief workers. A GDP only accounts for a level of activity; it doesn't identify good from bad. A competent student of mathematics should be able to discern when a quantitative finding or result is good or bad, and why.

Small is Beautiful was written by an economist and its focus is on the relative human benefit or harm that results from decisions made in the name of being "economically responsible." His writing chiefly addresses the scale of systems and the way in which burgeoning systems that try to pass over individual concerns and autonomous systems that don't take a bigger picture into consideration can both be dangerous to the function of human society. In terms of introducing students to the need for understanding how math is used to make arguments that direct politics, industry, economies, and, ultimately, human lives this book is invaluable.

There are also great opportunities to use literary skills as a means of generating math exercises and activities from students' own personal observations of the world around them. While this can be tricky to manage as instruction, having algebra students present problems with quantitative and logical aspects from their own daily concerns (e.g. transportation to and from school) is one way to use a literary approach to build a mathematical skill set. A great resource for such pieces is *Hands On Math Projects with Real-Life Applications* by Judith A. and Gary Muschla. The book is broken into project guides, from which teachers may build math activity-based lessons. In terms of recycling my top choice is Project 17 - Rating consumer products. I also see great value in several projects that are geared toward students self-exploration as users of math in everyday life and in monitoring their own activity in their math courses: Project 27 – A mathematical autobiography; Project 31 – Keeping a math journal; and Project 32 – Math

Portfolios. Each of which also connects to my second overarching strategy, using metacognitive activities.

Metacognitive Activities

The red, wavy line is back on my computer screen for “metacognitive.” I find this just criminal. How the operation of a human mind or the manmade operational mind of a computer can function without the notion of metacognition is inconceivable to me. This is a tongue-in-cheek way of introducing the term, but for simplicity, I usually introduce the definition of metacognition as “thinking about thinking.” It is a grossly neglected practice in public education at large, but has been addressed in some ways within the mathematics curricula that I have seen Pittsburgh Public use for Algebra 1 and Geometry in the past two years.

What, for me, has continued to be problematic is that we do not explicitly discuss being “metacognitive” with students. By the very nature of being metacognitive, leaving metacognition as an implicit subject does not make sense. Students should be challenged to question themselves in a productive way about how they are working through problems and what they are thinking when they arrive at solutions, as well as when they are having difficulty finding solutions.

I began the practice as an English teacher, and have continued as a math teacher, to have every new group of students go through an exercise of considering themselves in terms of three definitions of learning styles: Gardner’s Multiple Intelligences; visual/auditory/kinesthetic; and sequential/holistic behavior. I present students with these concepts and then ask them to consider themselves in terms of these definitions in order to start the class with the idea that students should be able to know their own thinking, reasoning, understanding, abilities, and ultimately math competency for themselves. It is also informative to me as a teacher in getting to know the students, as students.

At a time in their lives when adolescents are, perhaps most acutely, experiencing the development of their identities – as well as making a cognitive procession from a concrete operational mentality to the ability to comprehend and utilize metaphoric logic – the study of algebra should be, perhaps most acutely at this time, an ideal subject for them to expand their neurologically newfound abilities in symbolic reasoning and abstract representation. I often encourage students to work from their strengths toward what they find more challenging, in order to make use of what they are successful in while trying to achieve in realms where they have not had success. With many of my students this has gone from non-math subjects or interests toward algebraic reasoning.

‘Teen angst’ or the channeling of hormonal energies in adolescence need not be antithetical to more rigorous and intellectually challenging mathematics – as noted by Polya in definition of “the intelligent problem solver.” If the study is couched in a representation of and practical application for the students’ environment, the study of algebra should be particularly stimulating and can provide the same kind of creative outlet that writing, musical study or other artistic interests are more frequently credited as offering.

If they are given the chance and confident support I believe students can make this a reality. As it relates to instituting recycling programs, a freshman class of algebra students could be taking on the task of choosing, developing, organizing, and administering a school improvement project, at the center of which would be their own practical math research projects and adaptation of the established and approved curriculum of the school district.

I also find being metacognitive as most useful in another, simpler sense than everything stated above. To have students perform metacognitive exercises and activities is inherently making use of the kinesthetic learning style. Even if the physicality of exercises is limited to seemingly passive cognitive work, this work becomes active by the nature of self and social exploration that is inseparable from an individual considering the functionality of her or his own active mind.

Lesson Primers

For the length of my Rationale, Objectives, and Strategies above, I have a fairly simple set of proposals for lessons that I plan to use and would recommend to other teachers. The first lesson plans I recommend, as suggested above, are readings on recycling and energy consumption that provide quantitative information for students to draw conclusions from and build models (expressions & equations, graphs, etc.) from. This can be most productive to student learning if the assignment involves them articulating, via some extent of writing, the processes and systems they can recognize within their readings.

Beyond this, my strongest recommendations are for activities that involve actual interaction with the school environment (and the city environment to whatever extent possible). The following two examples are only a starting point of imagination and can themselves be amended to a particular school’s facilities; they each encourage metacognitive, kinesthetic approaches.

Buy a Big Blue Can and Rubber Gloves

Within my budget as a teacher (or a bit beyond as is typical for us all) I plan to assemble the following materials: a 36 gallon blue recycling bin, bio-degradable clear plastic bin liners, and rubber gloves to handle post-consumer waste. I will introduce the concept to the class that we are going to perform research by collecting recyclables, in order to provide data on the volume of recyclable materials that pass through our school and to build a case to develop a recycling program in our school – for later presentation to the school administration.

In terms of how to conduct collection of data, I would most recommend use of the school's cafeteria, however significant preparation of this recycling collection should be carried out in advance to prevent non-recyclable garbage from being placed in the bin. Having students take a short portion of class time to make signage for the recycling bin – ideally with a detailed set of posters explaining the purpose of the project – would be a good start to the student's active involvement. The bin, with signage (and ideally with student volunteers assigned to monitor the bin and explain the purpose to other students) will be placed where the student body can deposit metal, plastic, and glass containers.

In order to conduct the collection of recyclables with some regularity, which will be useful for a number of possible applications of the data on the gathered materials, a schedule of times (likely lunch period(s) on a set given day(s)) will be set. Decisions about the intervals of collection can be crafted to the unit of study in the Algebra 1 curriculum students are engaged in.

From this point the direction of activities in which students use the results of recycling collection are also dependent upon the unit of study in the Algebra 1 curriculum students are engaged in. If, for instance, students are working on ratios and proportions, the volume of materials gathered in one day could be projected into one week, one month, one semester, etc.; to a further degree the ratios of plastic containers to metal containers, types of plastic, etc. could be delineated into ratio proportions. If students are currently engaged in studying central tendency, separate bags of recyclables could be collected during each of several lunch periods – from which averages for the volume per period could be computed. If students are focused on graphing or other representations of functions, the data can be plotted in various ways to show students visual representations of what they themselves have seen as real activity.

The beauty of this open-ended-ness is that true quantitative information will be available from environmental behavior that students have observed and participated in. This entire process could be a once per week addendum to studies… "how can we think about the recycling collection today?" This would also provide long term tracking of week to week comparisons of the collection. It is also possible, given the logistics that are practical for myself or any teacher, that this collection could be used on a daily basis over one week only – the data from which could then be referred back to as new elements of the curriculum come up for study. Naturally this entire model could be altered to measure recyclable paper collection (for a classroom, photocopy room, office or library).

Among all the flexible applications of this lesson primer, the one absolute that I believe will make or break the success of its execution as a tool for engagement and pragmatism in instructing algebra is that student work must be presented to the school's administration in some form that will allow for feedback and assessment. For students to connect with the value of their work, this final stage, be it conducted formatively, mid-year, or end of year, is vital.

Research Recycling Services in the City

The volumetric data that students can collect and make use of in the above lesson primer would require an additional phase to present findings that might drive an active reform or institution of a recycling program. The services of a materials recycling contractor would be needed to bring reality to a proposal.

A great way that I would like to go about this research is to have my students use computer lab time with tools such as Google Maps. While writing this paper I have another window open on my desktop from the following search:

> Enter "maps.google.com" into the URL address bar;
> Enter "Pittsburgh, PA" into the *Search the map* prompt;
> Click on the *Find businesses* tab;
> Enter "recycling" into the *Find businesses* prompt

The results for this query are ten recycling processing businesses within the metro Pittsburgh area, labeled *A* through *J*, but this is merely the first page of search results. Using the *more info>>* links next to each businesses listing gives details about what types of materials they process and other information about their operations. This is but one way to have students conduct research and the considerations within the research are multivariable – distance from the school, size of recycling businesses (as a consideration for the volume that is determined to be produced by the school), and modes of recycling collection or drop-off offered by these businesses might all be assessed by students online.

Another, ideal step in this research may be a math focused field trip to some of these businesses, having guest speakers from these businesses and/or having guest speakers from local universities to discuss the logistics and quantitative properties of recycling infrastructure and the ways in which algebra is applied to it. This research could lead to a ranking of potential recycling vendors, in which students working in groups would analyze numerous factors that would determine the best vendors of services and articulate their reasoning by evaluating those factors.

I'd like to finish with a final optimistic thought… it is not an impossibility that a far more attractive set of research findings could be assembled for school administrators to assess. As the school has saleable recycling materials, one of the main factors that students could consider is which of the vendors would pay the school the most money for its recyclable collections throughout the year.

Bibliography / Resources

Glasser, William. Choice Theory. New York: HarperCollins, 1998
Perennial writings on relationship management from professional to personal.

Jacobs, Harold R. Mathematics: A Human Endeavor. 3rd Ed. New York: W.H. Freeman and Company, 1994
A great text for introducing math concepts in engaging, non-traditional ways

MacNeal, Edward. Mathsemantics: Making Numbers Talk Sense. New York: Penguin, 1995
A dynamic book of profound thought, the author was an airline consultant through the early years of the airline industry's boom

McDonough, William & Braungart, Michael. Cradle To Cradle: Remaking the Way We Make Things. New York: North Point Press, 2002
This book delves much deeper into the reasoning of materials and energy, opening doors to more complex and precise considerations about recycling and materials manufacturing.

Muschla, Judith A. and Gary. Hands On Math Projects With Real-Life Applications. San Francisco: Jossey-Bass, 2006
A great series of practical and meaningful project starters for students.

Polya, George. How to Solve It: a new aspect of mathematical method. 2nd Ed. New Jersey: Princeton University Press, 1985
I am still reading and re-reading this book by way of "floating" through the "Short Dictionary of Heuristic" section. Each entry contains references to one or more other entries, some numerous, such that it is possible to recross your own path through its content and read an entry over again with broadened and/or sharpened perspective. I've read no other book that so elegantly or universally presents what it takes to be an effective teacher of mathematics – for whatever level of study.

Schumacher, E.F. Small is Beautiful: economics as it people mattered. London: Blond & Briggs Ltd, 1973
This elegant collection of essays combines Gandhi's philosophies with real quantitative arguments for more intelligent fiscal policies in governments and industries around the world.

Appendix 1: 2.5 PA Math Standards

2.5. Mathematical Problem Solving and Communication
2.5.11. GRADE 11
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:
A. Select and use appropriate mathematical concepts and techniques from different areas of mathematics and apply them to solving non-routine and multi-step problems. B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results. C. Present mathematical procedures and results clearly, systematically, succinctly and correctly. D. Conclude a solution process with a summary of results and evaluate the degree to which the results obtained represent an acceptable response to the initial problem and why the reasoning is valid.

Appendix 2: National Council of Teachers of Mathematics Standards

NCTM: Problem Solving Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

NCTM: Reasoning and Proof Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

NCTM: Communication Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

NCTM: Connections Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

NCTM: Representation Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;

- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

NCTM: Algebra Standard for Grades 9-12

Understand patterns, relations, and functions	• generalize patterns using explicitly defined and recursively defined functions; • understand relations and functions and select, convert flexibly among, and use various representations for them; • analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior; • understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions; • understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions; • interpret representations of functions of two variables

Represent and analyze mathematical situations and structures using algebraic symbols	• understand the meaning of equivalent forms of expressions, equations, inequalities, and relations; • write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases; • use symbolic algebra to represent and explain mathematical relationships; • use a variety of symbolic representations, including recursive and parametric equations, for functions and relations; • judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
Use mathematical models to represent and understand quantitative relationships	• identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships; • use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts; • draw reasonable conclusions about a situation being modeled.
Analyze change in various contexts	• approximate and interpret rates of change from graphical and numerical data.