Teaching Problem Solving Skills to Students with Special Needs

Natalie Kratsas
Pittsburgh Arlington

Overview

The data that I have collected on my students' strengths and weaknesses, shows that they have the most difficulty when asked to problem solve. This unit can be used as a tool for teaching problem solving skills to learning support students. When a student has a learning disability, if the teacher sets the bar too high, the students flounder and become frustrated. On the other hand, if a teacher scaffolds a problem too much, the "problem-solving", is taken out of the problem. This unit will allow the teacher to use a structured approach. It will offer tools to tailor instruction to learning support students and challenge them to become efficient problem-solvers.

This unit will be divided in two parts. Connected Mathematics II contains two units that will be ideal places in the eighth grade curriculum to place my unit. *Looking for Pythagoras* and *Growing, Growing, Growing* are two curricular units that offer many opportunities to problem solve. In the past, my students have lacked confidence when working within the curriculum due to poor problem solving skills. This unit will begin with strategies for problem solving prior to the students' beginning *Looking for Pythagoras.* The unit will continue by offering students the opportunity to use the skills they have acquired to solve challenging problems prior to beginning *Growing, Growing, Growing.*

Rationale

One encounters many difficulties when teaching problem solving skills to students with specific learning disabilities. First-hand experience and frustration with teaching problem solving led me to choose the seminar, *Problem Solving: Where Education Interacts With Life.* One issue that reoccurs is that many learning support students have significant reading disabilities. Additionally, "…the subject matter (algebra, geometry, trigonometry) demands a new vocabulary (for example, *sin, pi*), very sophisticated logical and perceptual reasoning, the learning of new symbols, and the solution of complex word problems." (Smith, 1994, p. 470)

This prevents many students from entering the problem. Another problem is the time students are able to attend to the task. Students who are unable to attend to the task are often also easily frustrated by challenging academic work. Other students express frustration at being unable to begin a problem. These students often lack confidence in their ability to accomplish an academic task. For these reasons, the majority of my students are unwilling to attempt problems that require critical thinking and sustained, individual student work. This teaching unit is designed to develop students' problem solving skills. The goal is for learning disabled students to become independent problem solvers.

This unit is designed to fit into the Connected Math II curriculum used with 8^{th} grade students. It is designed for learning support students who have difficulty approaching problems that require inductive and / or deductive reasoning. This unit could be adapted to be used with 6^{th}, 7^{th}, and 8^{th} grade learning support students.

I plan to insert the unit into the existing curriculum of Connected Mathematics II. Before beginning *Looking for Pythagoras*, students will have an introduction to problem solving techniques. Students will be able to practice these techniques and begin to use inductive and deductive reasoning. When studying *Looking for Pythagoras* students have the opportunity to "discover" the Pythagorean Theorem. The text offers multiple points for the inclusion of additional complex problems. This unit of study lends itself to the use of diagrams, drawings, and visual representations of solutions. Prior to studying exponential growth in *Growing, Growing, Growing*, students will be asked to solve more challenging problems. When studying *Growing, Growing, Growing* students have the opportunity to experiment frequently. There are problems of increasing complexity throughout the text. I will be able to link these problems to the students' prior knowledge and arrange the problems in a logical sequence.

Students with reading disabilities must first find a way to comprehend the problem. These students must develop techniques to be able to read for understanding. The issue becomes translation. “First, the problems must be read. Second, not all the information is usually given. Students must apply some additional knowledge to solve the problem. The necessary math vocabulary, which is embedded in the problem, must be translated to a math operation, and then the problem’s solution must be translated back in to language.” (Smith, 1994, p. 472.)

My aim is to develop an approach that learning support students can use to ensure that they comprehend the problem. This is equally important for non-disabled students, however, in my experience, it is the learning disabled student who continues to struggle with this concept year after year. This will require integrated vocabulary development that is specific to each unit. Students will also synthesize techniques from Communications classes.

Students with difficulty attending to task must be able to break up a task into manageable chunks. Students often give up too soon, or write down an incorrect answer just to please the teacher. These students will later express frustration with the task. These students often feel that the task has too many steps. They are overwhelmed by the task and accept failure early on. These students need techniques to self-scaffold complex problems into manageable parts. Students will need to learn some procedures to approach problems. They will acquire methods and techniques to identify key information in the problem, use pictures or diagrams, estimate the answer, and paraphrase the problem.

Some students do not feel that they are able to enter the problem. These students lack confidence. They will need techniques to access their prior knowledge in order to approach the problem. I will attempt to link the problems in this unit together with this goal in mind. The problems in the unit will be arranged according to the difficulty and the processing demands of each. The students will gain confidence as they continue through the unit. As the students gain confidence, the problems will increase in difficulty. The students will be able to build on the knowledge and techniques from the previous lesson.

Through this unit, I hope to attack the main issues that learning disabled students encounter when learning problem solving skills. I will address their ability to approach the problem, to access prior knowledge, to read and comprehend the problem, and to break the problem into manageable parts. Students will become independent problem solvers and be able to use the strategies they learn in the future.

Objectives

This unit seeks to improve students' willingness to try problems and improve their perseverance when solving problems. It will improve students' confidence in their ability to solve problems. The unit will give students problem solving strategies and make students aware that many problems can be solved in more than one way. Students' abilities to select appropriate solution strategies and students' abilities to implement solution strategies accurately will improve. Finally, this unit seeks to improve students' abilities to monitor and evaluate their thinking while solving problems. (Adapted from Problem Solving Experiences in Mathematics, 1985.)

Students working in this unit will encounter many of the 8^{th} grade mathematic standards. Students will initially work on the standards that pertain to problem solving. Students will also work with standard 4, as they will be working to solve multi-step problems. As the students begin *Looking for Pythagoras*, they will become very familiar with all of the standards that concern the Pythagorean Theorem. Finally, students will work on the standards that address exponential growth, linear, and non-linear relationships.

Strategies

Students working in this unit must first be able to decode the problem. New vocabulary will be presented to the students. Some attention will be given to the root words, so that students can apply this knowledge to future mathematics vocabulary. Students will be prompted to employ reading comprehension strategies to word problems. One strategy is to read and re-read the problem for understanding. First, students will read the entire problem through silently. Next, students will read the entire problem aloud.

Finally, they will read one sentence or piece of the problem at a time, and then restate each piece to be sure they understand what is being asked. It should be noted that the teacher may want to attend to the printed page that is given to the student. Some students have difficulty with visual perception. It may be helpful to increase white space on the page, reduce the amount of text per page, or to increase the font size. A student may use a tool as simple as a blank page to limit his visual field, or removable highlighting tape to isolate the important information. Pre-made graphic organizers may be used for students who have difficulty organizing their written work.

Next, students must be able to categorize problems and recognize clues that let them know what type of problem they are dealing with. Students will be shown how to identify several types of problems: problems that require the student to choose which operation must be used, problems that require students to recognize a pattern, problems that require students to continue a pattern, and problems that have data that can be organized on a table or graph. Students will work with problems that require both inductive and deductive reasoning.

At this point, the students will answer the question, “What do you know?” Prior knowledge will be activated. First, students will identify the type of problem. Students may answer this question using several techniques. Students may choose to answer this question in a narrative form. Students may list the facts presented in the problem. Students may choose to go back, re-read the problem, and highlight the important information. They will then take the highlighted information and translate it into usable data or facts.

Students will go about solving the problem. Students may choose from different techniques when solving the problems. They will be required to show all steps during their work. Students may then later go back and analyze their own work. Students will identify which strategies were successful and which strategies led them to incorrect solutions. Diagrams or pictures may be needed by some. Some students may choose to use manipulative pieces to aid in solving. Students may use tables to organize the data.

Using a workshop model, students will then share their results with the class. During work time, the teacher will circulate, question, and interact with the students. He or she will make note of concepts to be discussed in the closing of the lesson. The teacher will choose the order of presenters in a meaningful way. He or she will choose students to present who have work that promotes good discussion. Students will take turns presenting their solutions to the class. The class will discuss the solutions together. They will note different methods of solving the same problem. Students may critique each other, and offer suggestions to those who did not solve the problem.

Finally, students will create a reference for problem solving. Students will add to their problem solving reference after each lesson. This reference section will be kept in their math notebooks. Students will add to this reference; vocabulary, problem types, examples of each type of problem, and techniques such as diagrams, tables, or lists. Students will frequently be encouraged to use this reference when encountering a new problem.

Classroom Activities

Day One: Exploring Houses

Goals

To assess students'…..

- understanding of pattern development;
- ability to use a table to organize information;
- choice of strategy (e.g., draw a picture, use recursion, write an explicit rule) to make a prediction.

Materials and Equipment

- A copy of the blackline master "Exploring Houses" for each student.
- Enough pattern-block triangles and squares for each student to make the first four houses in the pattern.

Activity

The students should work individually. Have each student build the first four houses in the sequence in figure 1.1 as needed and then complete the following tasks:

1. For each house, determine the total number of pieces needed. How many squares and triangles are needed for a given house? Organize your information in some way.
2. Describe what house 5 would look like. Draw a sketch of this house.
3. Predict the total number of pieces you will need to build house 15. Explain your reasoning.
4. Write a rule that gives the total number of pieces needed to build any house in the sequence.

Discussion

As the students explain how they determined the pattern, take note of their strategies. Do they build each pattern and notice the changes from one pattern to the next? Do they organize the information in a table? When they make predictions about other houses in the pattern, do they "add 3" multiple times (a recursive strategy) to predict the number of pieces needed or do they see a relationship between the house number and the number of pieces needed to make the house (an explicit strategy)? Do they talk about the total number of pieces, or do they talk about the total number of squares and the total number of triangles? Can they describe their rules in words? Using symbols? Answers to these questions as well as an overall analysis of students' written responses provide insights into how comfortable and capable students are in working with patterns and variables.

(This lesson was taken directly from *Navigating through Algebra,* *Chapter 1: Understanding Patterns, Relations, and Functions, pp. 9-11.*)

Students with learning disabilities often benefit from repeated practice of a new concept. After assessing the students on this task, the teacher may choose to assign one or all of the *Selected Instructional Activities* that follow this lesson.

1. Make growing patterns using the initials in your name. What is size 5 in your pattern?
2. Make tiling patterns for garden areas. The tiles surround each garden. (teacher-made handout features two different tiling patterns) Two different patterns are shown. For each pattern, draw garden 5.

3. Use isometric dot paper to make shapes. How many unit line segments are in each shape shown? A unit line segment connects two adjacent dots.

Day Two: Building with Toothpicks

Goals

- Explore pattern development
- Use a table to organize information
- Generalize a rule that describes how to find the perimeter of the *n*th shape

Materials

- A copy of the blackline master “Building with Toothpicks” for each student
- Enough toothpicks for each student to make each of the firs four shapes

Activity

The students may work as partners. The shapes that are shown on the handout are made with toothpicks. The students should look for patterns in the number of toothpicks used to make the perimeter of each shape. Present them with the following tasks:

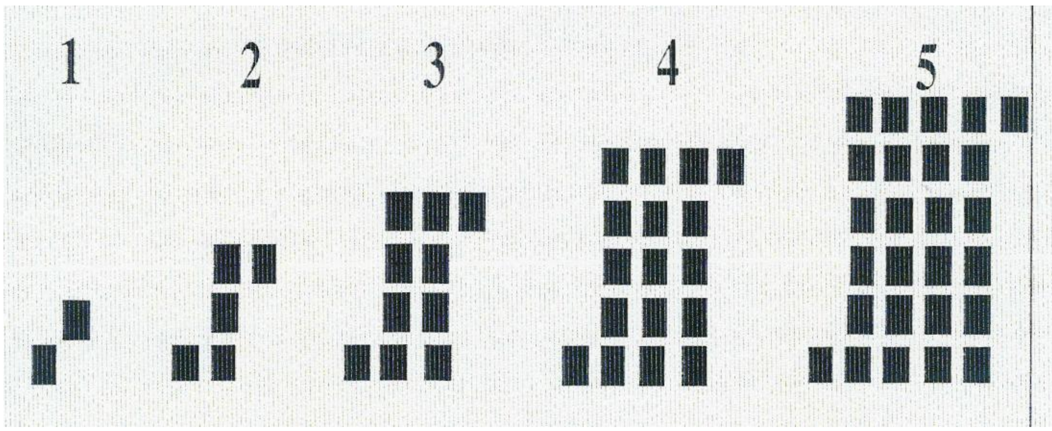
1. Use a pattern from the sequence of shapes to determine the perimeter of the fifth shape in the sequence. Show or explain how you arrived at your answer.
2. Write a formula that you could use to find the perimeter of any shape *n*. Explain how you found your formula.

Discussion

The process of generalizing involves identifying patterns in shapes 1 – 4, making predictions for a next case (e.g., shape 5, not shown), and eventually describing a general rule for predicting that is expressed in symbolic notation. Growing patterns that involve geometric models provide a variety of contexts in which students can explore reasoning using variables and the relationships among two or more variables. It is helpful to encourage students to connect their explanations to the geometric models. If students choose a recursive strategy, they can be asked to justify how the rate of change they have identified occurs as they move from one shape to the next.

(This lesson was taken directly from Navigating through Algebra, *Chapter 1: Understanding Patterns, Relations, and Functions, pp. 13-17..*)

You may decide to assign *The S-Pattern Task: Student Version* to provide further practice with pattern development.



1. What patterns do you notice in the set of figures?
2. How many square tiles are in figure 7? Your description should be clear enough so that another person could read it and use it to think about another figure.
3. Determine an equation for the total number of squares in any figure. Explain your rule and show how it relates to the visual diagram of the figures.
4. Find a second way to describe the pattern and write the equation that matches the description. Compare the two equations and show in the visual representation how one equation is equivalent to the other.
5. If you know that a figure had 9802 square tiles in it, how could you determine the figure number? Explain.
6. Does the pattern describe a linear relationship between the figure number and the total number of squares? Why or why not?

(Learning Research and Development Center, 2007, University of Pittsburgh)

Day Three: From Stories to Graphs

Goals

- Identify the independent and dependent variables in problems.
- Sketch a graph to represent a story context that involves change over time.

Materials

- A copy of the blackline master "From Stories to Graphs" for each student

Activity

Distribute copies of the blackline master, and have the students complete the following tasks presented on it:

1. In a walking experiment, Josephine walked a total distance of 40 feet. At the halfway point, she had walked for 25 seconds. She stopped for 5 seconds to tie her shoe and then continued walking for 25 more seconds. Sketch a graph that shows Josephine's distance from the starting point over time.
2. You are gathering data in the school cafeteria from 8:00 AM to 3:00PM. Sketch a graph that tells a story about the number of cans of soda in a vending machine over that time. Write a paragraph that tells the same story in words.
3. You are mowing the lawn. As you mow, the amount of grass to be cut decreases. You mow at the same rate until about half the grass has been cut. Then you take a break for a while. Then, mowing at the same rate as before, you finish cutting the grass. Sketch a graph that shows how much uncut grass is left as you mow, take your break, and finish mowing.

Discussion

You may find, when it comes to drawing graphs, that the students are challenged by the request to "sketch a graph" when just a few or now numbers are specified. Students may want to plot points and connect them to make a graph and may be confused without a numerical label on each axis.

As students gain a greater understanding of analyzing change by making graphs, they can be asked to make up stories for graphs they are given or to read graphs to answer questions about a story "in progress".

(This lesson was taken directly from *Navigating through Algebra,* *Chapter 1: Understanding Patterns, Relations, and Functions, pp.27, 28, and 78.*)

Day Four: Tiling Tubs

Goals

- Write equations to describe the relationships among variables
- Determine when expressions are equivalent

Materials

- A copy of the blackline master "Tiling Tubs" for each student

Activity

Have the students complete the following tasks, which are presented on the blackline master:

Hot tubs and in-ground swimming pools are sometimes surrounded by borders of tiles.

1. How many 1-foot square tiles will be needed for the border of a square hot tub that has edge length *s* feet?
2. Express the total number of tiles in as many ways as you can.
3. Be prepared to convince your classmates that the expressions are equivalent.

Discussion

Unlike problems in which students may have looked at a pattern of change as pools increase in size, in this task, students must be able to state the relationship that is determined by knowing a variable side length, *s*, and the context of the problem. The results can be represented by a variety of equivalent expressions. Students can discuss ways to determine whether one expression is equivalent to another.

(This lesson was taken directly from *Navigating through Algebra,* *Chapter 1: Understanding Patterns, Relations, and Functions, pp.63, 64, and 86.*)

Day Five: Sharing Cookies

Students will follow the workshop model that is described in the *Strategies* section of this paper. The students will begin with the following problem:

Two sisters, Molly and Sarah, came home from school one day. Molly was really hungry, so she went into the kitchen to find something to eat. ON the kitchen table she found a plate of cookies along with this note from her mother: "I had to go to the store. Please share these cookies equally with your sister. Be back soon. Love, Mom." Molly ate one-half of the cookies on the plate. A little later, Sarah came into the kitchen, found the cookies and the note, and ate one-half of the cookies left on the plate. When the girls' mother came home, she saw 3 cookies on the plate. How many cookies did Molly first see on the plate? How many cookies did each girl eat? Did they share them equally? Explain how you solved the problem. If there were 4 cookies left on the plate when the mother came home, how many cookies did Molly first see on the plate?

After the students close this problem, they will tackle a more complex version.

Molly and her sisters, Sarah and Stephanie, arrived home from school one afternoon. Molly was really hungry, so the first thing she did was go to the kitchen to find something to eat. On the kitchen table was a plate of cookies along with this note from their mother: "I had to go to the store. Please share these cookies equally with your sisters. Be back soon. Love, Mom." Molly was so hungry that she quickly ate one-third of the cookies on the plate. A little later, Sarah came into the kitchen, found the cookies and the note, and ate one-third of the cookies on the plate. Finally, Stephanie came into the kitchen, found the cookies and the note, and not realizing that her sisters had already eaten some cookies, ate one-third of the cookies on the plate. When the girls' mother came home, she found 8 cookies on the plate. How many cookies were originally on the plate? How many cookies did each girl eat? Did they share them equally? Explain how you solved the problem. Find another number of cookies that the mother could leave on the plate so that the sisters could share them equally without having to break them into parts.

(This lesson was taken directly from Teaching Children Mathematics, Problem Solvers, the Sharing Cookies Problem.)

After students complete each problem in the unit, they will close the problem by sharing their answers, strategies, and come to agreement about the most efficient ways to solve the problem. Students will then add strategies, techniques, vocabulary, and examples to their problem solving reference guide.

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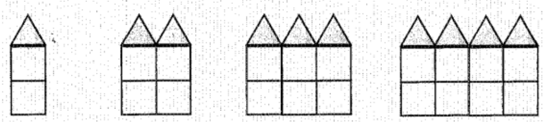
Appendix 1: Blackline Masters

Exploring Houses

Name ______________________

Build these houses using the orange squares and the green triangles from pattern-block pieces:



House 1 House 2 House 3 House 4

1. For each house, determine the total number of pieces needed. ______________

   How many squares and triangles are needed for a given house? ______________

   Organize your information in some way.

2. Describe what house 5 would look like. ______________________________

   ____

   ____

   Draw a sketch of this house.

3. Predict the total number of pieces you will need to build house 15. ______________

   Explain your reasoning. ______________________________

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4. Write a rule that gives the total number of pieces needed to build any house in this sequence.

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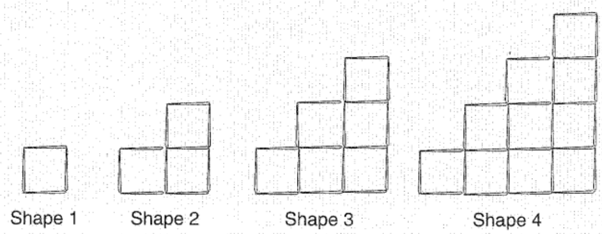
Building with Toothpicks

Name ______________________________

The shapes shown below are made with toothpicks. Look for patterns in the number of toothpicks in the perimeter of each shape.



1. Use a pattern from the shapes above to determine the perimeter of the fifth shape in the sequence. Show or explain how you arrived at your answer. ______________________________

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2. Write a formula that you could use to find the perimeter of any shape *n*. Explain how you found your formula. ____

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This activity has been adapted from Burkhardt et al. (2000)

From Stories to Graphs

Name ________________

1. In a walking experiment, Josephine walked a total distance of 40 feet. At the halfway point, she had walked for 25 seconds. She stopped for 5 seconds to tie her shoe and then continued walking for 25 more seconds. Sketch a graph that shows Josephine's distance from the starting point over time.

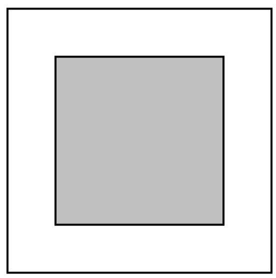
2. You are gathering data in the school cafeteria for 8:00AM to 3:00PM. Sketch a graph that tells a story about the number of cans of soda in a vending machine over that time. Write a paragraph that tells the same story in words.

3. You are mowing the lawn. As you mow, the amount of grass to be cut decreases. You mow at the same rate until about half the grass has been cut. Then you take a break for a while. Then, mowing at the same rate as before, you finish cutting the grass. Sketch a graph that shows how much uncut grass is left as you mow, take your break, and finish mowing.

Tiling Tubs

Name______________

Hot tubs and in-ground swimming pools are sometimes surrounded by borders of tiles. This drawing shows a square hot tub with sides of length ***s*** feet. This tub is surrounded by a border of square tiles. Each border tile measures 1 foot on each side.



1. How many 1-foot square tiles will be needed for the border of a square hot tub that has edge length ***s*** feet?

2. Express the total number of tiles in as many ways as you can.

3. Be prepared to convince your classmates that the expressions are equivalent.

Appendix 2: Standards

Grade 8 Mathematics Standards

The following standards will be included in this unit of study:

4.1 Solves problems that require logic and mathematical reasoning skills.
4.2 Makes conjectures about a problem or picture.
4.3 Solves multi-step problems involving whole numbers, money, fractions, decimals, percents and integers; and can explain the procedures used for estimating or computing the answers.
5.1 Solves problems that require reading carefully to find the important information and make a plan.
5.2 Explains all work mathematically, with words, pictures, tables, equations, graphs, and / or diagrams.
5.3 Revises work using a rubric as a guide.
8.1 Uses algebraic reasoning to describe and generalize linear patterns, exponential patterns, and quadratic patterns.
8.2 Creates an expression or equation that models a problem situation.
9.1 Analyzes geometric patterns (e.g., tessellations, sequences of shapes, etc.) and develop descriptions of the patterns.
9.2 Determines rotations, reflections, or translations of figures.
10.1 Computes the measures of sides and angles using proportions, right triangle relationships, and the Pythagorean Theorem.
10.2 Calculates the missing length in a triangle given two similar triangles and the lengths of corresponding sides.