

Problem Solving: Where Education Interacts with Life

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Pittsburgh Lincoln K-8

Overview

Rationale

Objectives

Strategies

Classroom Activities

Annotated Bibliography/Resources

Appendices-Standards

Overview

The focus of this unit plan is on giving fifth grade students the opportunities to embrace some key problem solving strategies as they study the mathematical concept of measurement. The chance to use these strategies will be rooted throughout the unit plan. Fifth grade students will select and use appropriate instruments and units for measuring quantities (e.g., perimeter, area, weight, etc.). They will estimate, refine and verify specified measurements of objects, as well as, convert linear measurements within the same system. Students will develop strategies for estimating the perimeters and areas of a figure, as well as applying appropriate standard units and tools to measure length and weight.

The purpose of this unit is to have student become proficient at developing a plan for analyzing problems, taking note of the important information, and solving the problem using the strategy of choice. The students must determine if their answer is reasonable, and be able to explain their solution both verbally and in written form. The students will be given the chance to display their work in various manners including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models.

The unit plan will entail a series of lessons that will take real life situations and lace them with challenges that will require students to explore the use of problem solving strategies such as making a table, an organized list, drawing a picture or diagram, etc. Fifth grade students will be given opportunities to place ownership of the various problem solving strategies. The unit plan will conclude with a major project that will focus on the organization and design of Pittsburgh Lincoln's annual garden display. Pittsburgh Lincoln's annual garden display will require students to create a design for the garden using grid paper. They will need to go to the store (Home Depot) and purchase the wood, soil, and plants to complete the project. The students will need to figure out the following:

- How large the area and perimeter of their garden display will be.
- How many bags of soil will they need to cover their garden (in both ounces and pounds), and the cost.
- How much wood (will need to convert feet to yards) will be needed and cost.
- How much will the total project cost.

The student will be allowed to spend a maximum amount and retain a minimum (which will dictate possible sizes of the garden display and accessories). The students will complete the project with a proposal to the principal explaining the thinking and rationale behind their garden display. Problem solving is a deficiency that many American students struggle to overcome. This project will challenge the student's thinking on many levels. They will have the opportunity to make use of key problem solving strategies in an engaging type of activity that is enriched with many aspects of mathematics, especially measurement.

Rational

As the population in the United States continues to increase and the number of jobs for its citizens decreases, attaining a quality education is vital for heightening one's ability to be productive in this society. Education must involve teaching and learning specific skills and possessing a standard of knowledge needed to operate productively in the society one resides in. Our society is becoming more global. Our educational systems are also becoming more global. One of the main components of a global education is mathematics. As the society becomes more global, educational reform in the United States, continue to place increasingly higher demands on teachers and administrators. These demands have been graphed into mandates and laws such as No Child Left Behind Act (NCLB, 2001), which attempts to maintain a focus on high standards and accountability for student learning. On a global level, the United States has fallen short of preparing its students for a math and science orientated global economy. According to the Washington Post on December 7, 2004, "American high school students have a poorer mastery of basic math concepts than their counterparts in most other leading industrialized nations..." This statement comes from the results of a study that is compiled every three years amongst all major industrialized nations. Students who participated in the survey were asked to solve real-life math problems. Based on the students' scores, the United States ranked 24th out of 29 countries in the 2004 study.

From a practitioner's perspective, if US students are performing poorly on solving real-life math problems, teachers in the US are not equipping students with the skills and knowledge needed to compete in a global economy. As a practitioner, I am always looking to increase my knowledge and improve my craft at teaching mathematics. A course in Mathematics Problem Solving could assist me in developing a unit on real-life math problems in the world of measurement.

This unit is designed to address critical focal points in measurement, which are embedded in real-life circumstances. In teaching problem solving, one must structure situations where students are required to face a new question, define it in specific terms, and evaluate potential solutions to the problem. These situations would require students to access prior knowledge, use reflective reasoning, and various problem solving strategies to arrive at their conclusions. What comes out of an approach of this kind is an emphasis on process rather than on substance (Henton, Marotz-Baden & Kieren 1979). The National Council of Teaching of Mathematics points out that developing the disposition within students to analyze more deeply enhance their understanding of mathematical situations. Considering this as a principle of math education, students must be given many opportunities to experience math in the world around them. With the use of interesting problems, students' thinking can be challenged. The overall purpose of getting students to question their thinking is to advance their understanding to another level.

Problem solving is the vital component of mathematics. Possessing mathematical knowledge and skills alone is imperfect without the ability to apply those skills in the form of solving problems. Students must know the appropriate situations for multiplication or dividing. When students develop and carry out a plan to solve a mathematical problem, they show a level of knowledge that goes beyond simple computation.

When students problem solve in mathematics, they strengthen their cognitive abilities and exercise determination. Problem solving gives students the chance to solidify and extend what they know, in order to stimulate mathematical learning.

At school and in their home environment, students use measurement in many different ways, for example: comparing the temperature in Fahrenheit between six different cities, running laps around the gym to account for their daily two mile requirement during soccer practice, or helping mom convert cups to pints to quarts as she prepares her famous fruit punch. Many students are not given the opportunity to explore these strategies in interesting situations. It is important for students to embrace the various problem solving strategies and make them a part of their mathematical repertoire.

Objectives

Through multiple problem solving activities, students will utilize appropriate mathematical terms, vocabulary, as well as, graphs and diagrams to explain their solutions in a clear and logical manner. Students will validate the methods, materials and strategies they used to solve problems. They will estimate, refine and verify specified measurements of a garden that serves as a replica of a larger garden, and convert linear measurements within the same system. The unit will build new mathematical knowledge through problem solving, allow students to apply and adapt a variety of appropriate strategies to solve problems, and have students monitor and reflect on the process of mathematical problem solving.

Strategies

Drawing a Picture or Diagram

For some students drawing a picture or diagram helps them make something abstract concrete. These types of representations aid students with seeing situations that are not obvious after simply reading a problem. When the situation is not easy to visualize, a simple diagram can be use to represent the situation. This method may assist students with clarifying the problem for them. The pictures and diagrams strategy makes the various stages of multi-step problem easier to manage.

Making a Organized List

A useful problem solving strategy is organizing information into some type of list, a technique that may serve a variety of purposes. When a problem requires you to generate a large amount of data, a list may help you account for all possibilities and avoid repetitions.

Making a Table

When a problem involves data that has more than one characteristic, an effective problem solving strategy is to organize the data into a table. A table displays data so that

it is easily located and understood, and missing data becomes obvious. If you are not given the data for a problem and must generate it yourself, a table is an excellent device for recording what you have done so you don't repeat your efforts, a table also can be an invaluable aid in detecting significant patterns.

Work Backwards

With some problems, you know the end result but you need to find out something that happened earlier. You can use the strategy of working backward to solve problems like this. To use this strategy, start with the end result and *undo* each step.

Solving a Simpler Problem

When faced with a problem that appears difficult or complicated, you may find it helpful to first solve one or more similar problems that have simpler conditions. Sometimes the solutions of simpler problems may lead to the solution of the more difficult. At other times, solving a series of simpler problems may lead you to a pattern that provides a basis for solving the original problem.

Finding a Pattern

One of the most frequently used problem solving strategies is that of recognizing and extending a pattern. As we shall see, there are many times that this is used as a strategy in conjunction with other problem solving strategies.

Guess and Check

An effective way to solve certain problems is to make a reasonable guess of the answer, then check the guess against the conditions of the problem. Sometimes your first guess will yield the correct answer, and the other times you will have to go through many guesses before you succeed. However, even when your guess is not correct, you make progress in solving the problem by eliminating one possible answer, thus obtaining other information that may lead to the correct answer. This method is also known as "Guess, check, and revise."

Acting Out The Problem

There may be times when your students experience difficulty in visualizing a problem or the procedure necessary for its solution. In such cases, they may find it helpful to physically act out the problem situation. They might use items that represent the people or objects. Acting out the problem may itself lead them to the answer, or it may lead them to find another strategy that will help them find the answer. Acting out the problem is a strategy that is very effective for young students.

Classroom Activities

Day 1-2

The start of this unit will begin with the game Knights N' Dazes (see Sheet A attachment). After explaining to the class how to play the game, the teacher should use a transparency of the game board and overhead projector to model how the knight moves around the board. Students can take turns making moves on the transparency. Once the students understand how to fill in the board, allow them 10-12 minutes to complete the task. After the allotted time the teacher should regain the class's attention. Ask the students to discuss their thinking as they tried to complete the entire board. What strategies did they use? Did they notice any patterns? Summarize the activity as an opportunity to practice strengthening their problem solving skills. The teacher should

briefly discuss the importance of developing strong problem solving skills in mathematics and in real life situations. In order to assess the students' knowledge of problem solving strategies, ask the class to brainstorm various problem solving strategies previously learned. Their responses should be listed on the chalkboard. Any strategy that the students fail to mention should be added to the list and briefly explained. At this point, inform the students that they will be given a list of various problems, and they are to categorize each problem from Sheet B1-B2 under the problem solving strategy that could be used to solve the problem. The students should be given time to work individually for 10-12 minutes. When the allotted time has expired, group the students in threes to compare answers and complete the task. This should take approximately 10-12 minutes. As students finish categorizing the problems, pull the class back together to discuss their results. Have the class share their answers and ask for agreement or disagreement on each category. The last part of the activity will allow the students to illustrate their thinking behind categorizing the problems.

With the students in the same group as before, they are to select one of the three problems from each category to solve. The teacher should monitor the group's progress as they work to solve the problems. As the activity progresses, each group should be given a poster sheet to illustrate one problem and the strategy used to solve the problem. The teacher should select the problem to be displayed for each group based on the quality of the group's work. The class should be pulled back together to post and share the strategies used to solve the problems. After every group has shared their work, ask students if they would rearrange any of the problems they categorized previously. Remind students about the importance of developing strong problem solving skills in mathematics and in real life situations, and inform them that they will be exploring these strategies throughout the next few lessons. The students' posters should remain up as reference material for problem solving.

Day 3

The class will begin with each student receiving a copy of game Sheet C. Read aloud the puzzles the students are to solve. After the puzzles are read, give each student a sandwich bag containing ten toothpicks. On an overhead projector, model the initial arrangement of the first puzzle using the toothpicks. Give students 10-12 minutes to complete the puzzles. After the time expires, collect the baggies and prep the students for today's lesson. Once they are ready, give each student five square plastic pieces and Sheet D of graph paper (3/8" grid). Place a transparency sheet of the graph paper on the overhead projector. As the teacher traces the frame of the square plastic piece, the students should also trace one of their square units on the graph grid. Once finished tracing ask the students what the lines represent, and what other ways to describe these lines are. Have students label the length of each side as 1cm. Ask the students what the distance around the square unit equals, and how do you find the distance around the shape. The teacher must highlight the method for finding the perimeter and how to identify the perimeter. The students should write next to the figure P for perimeter and the formula ($2L + 2W$). With a different color mark, the teacher should shade in the space inside the square unit that has been outlined. Identify the shaded space as the area, which represents one square unit. Students should write an A inside the square and write the formula for area ($L \times W$) on the outside with an arrow pointing to the A. Have students take out two squares and arrange them on the graph paper so that at least one pair of sides touch. The students should trace the two units. Ask the students how many square units are present now, what the perimeter of the two square units is, and is there a different way to arrange the two squares that would change either the area or perimeter or both. At

this point, have the students do the same thing with three squares using various positions and find the area and perimeter for each arrangement. Remind the students not to repeat arrangements (the use of the same position of the squares flipped or rotated). After they have found the area and perimeter for the positions of the three squares have them do the same with four and five squares. Student should continue to trace their arrangements on the graph paper and identify the perimeter and area for each. Students should determine which arrangement of squares has the largest perimeter and area. Have a few students illustrate their arrangement with the largest perimeter and area on a transparency graph grid. Summarize the distinction between perimeter and area and how each is found. Ask the students to talk about the problem solving strategies they used to complete the task.

To finish up the lesson, pass out task Sheet E. The students are to complete this task individually. Once students have completed the task allow them time to work on toothpick puzzles or Knight N' Dazes challenge.

Day 4,5,6

The class will begin with the students receiving a copy of Sheet F. Students should be given calculators to aid them in the challenges. After 10-12 minutes have passed, get students to share their findings. Talk about the strategies they used to figure out the solutions.

At this point the teacher should discuss with the students the special project they are going to participate in. To start the project, distribute copies of Project A to each student. The information on the sheet must be discussed with the students. Explain to the students that they can work with a partner to complete the task. Once they have completed all of the requirements of the project, students will be given a large poster size sheet of graph paper to recreate the garden design for the voting session. The posters that are submitted for the contest will be judged by staff members, including administrators, and parents. Each person will vote on their favorite garden design based on the criteria, rubric points, and creativity. The garden design that receives the highest vote will be selected as the upcoming garden layout for the school. Hand out a few two sided copies of Sheet D (3/8" graph grid) to each student. Remind the student about the various problem solving strategies we have for them to reference around the room. Rulers, crayons, and markers should be made readily available for the students.

Content Standards

PA Standards 2.3.

- A. Select and use appropriate instruments and units for measuring quantities (e.g., perimeter, volume, area, weight, time, temperature).
- B. Select and use standard tools to measure the size of figures with specified accuracy, including length, width, perimeter and area.
- C. Estimate, refine and verify specified measurements of objects.
- D. Convert linear measurements within the same system.
- E. Add and subtract measurements.

PA Standards 2.4.

- B Use models, number facts, properties and relationships to check and verify predictions and explain reasoning.
- C. Draw inductive and deductive conclusions within mathematical contexts.
- D. Distinguish between relevant and irrelevant information in a mathematical problem.

PA Standards 2.5.

- A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense and explain how the problem was solved.
- B. Use appropriate mathematical terms, vocabulary, language symbols and graphs to explain clearly and logically solutions to problems.
- C. Show ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models.
- D. Connect, extend and generalize problem solutions to other concepts, problems and circumstances in mathematics.
- E. Select, use and justify the methods, materials and strategies used to solve problems.
- F. Use appropriate problem-solving strategies (e.g., solving a simpler problem, drawing a picture or diagram).

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Polya, G. (1945). *How to solve it: a new aspect of mathematical method*. Princeton, N.J. Princeton University Press.

Polya, G. (1962). *Mathematical discovery; on understanding, learning, and teaching problem solving*. New York, Wiley.

Shuell, J. T., *Teaching and learning as problem solving* (1990). *Theory into practice*, 29(2) 102-108.

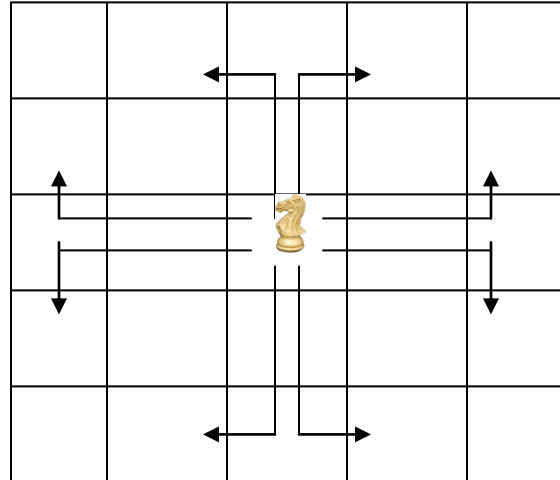
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Knight 'N Daze

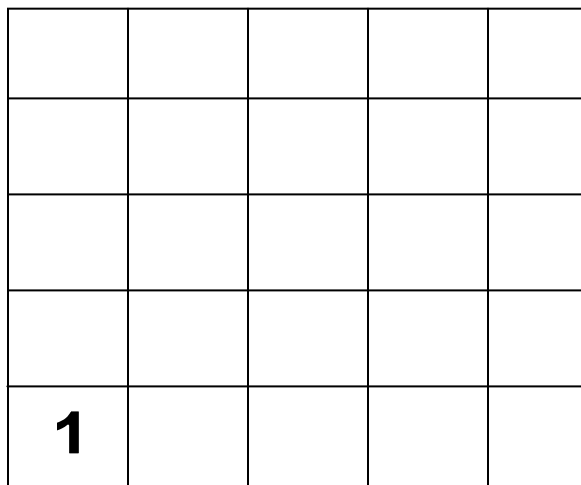
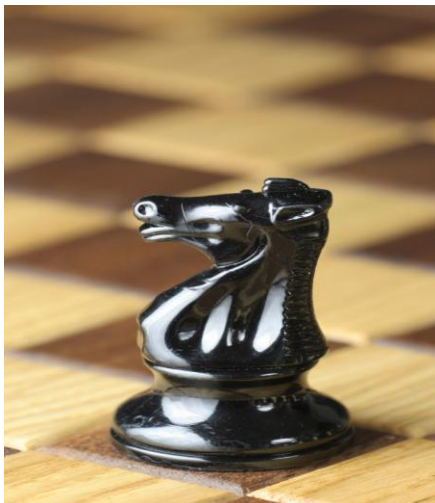
A knight in chess moves two over and to the right or left.



IF YOU WERE IN THE CENTER, YOU COULD MOVE TO ANY OF THESE SQUARES.



It is possible for the Knight to move in every square on this 5x5 grid without landing in the same square twice. Number the moves- 1, 2, 3, 4,...up to 25.



The number 1 must always start in the bottom left hand square.

Appendix B

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Appendix C

Directions: students are to cut each box out, decide which problem solving strategy would work best to solve the problem and place that problem under the appropriate category.

Sheet B1

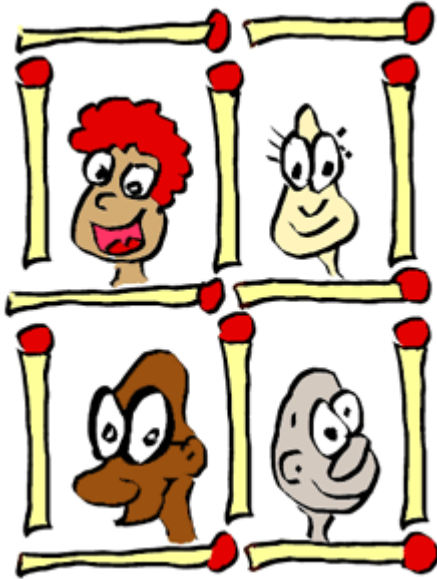
<i>Sample Word Problems</i>						
A disc jockey has 5 special requests. One of the requests is her own favorite song. If she plays her favorite first, in how many different ways can she play the other songs?	A lion trainer needed 5 large rings linked together as a large chain for his circus act. In order for the rings to be put together, some had to be cut and welded back together. What is the least number of cuts needed to link the 5 metal rings together?	Mike gave the store clerk \$1.65 in nickels and quarters. If there were 3 more nickels than quarters, how many nickels did Mike give the clerk?	Ben collects baseball cards. Every day he gets 10 new cards. Every third day, he gives some to his brother, the same number each time. If he has 9 cards on the first day, on what day will Ben have exactly 100 cards?	Tim & Mike are fishing in the river. Each time they turn the reel 2 turns they lower their bait 3m. A school of fish are swimming 18m below the surface of the water. How many times must they turn the reel to reach the fish?	A squirrel found some acorns. He stored half in a hollow tree. He gave half of what he had left to his brother. After that he had 16 acorns. How many acorns did the squirrel find?	Larry, Max, Rhonda, & Ellen play in the school band. They play the clarinet, flute, tuba, & drums. Ellen plays the clarinet, and Max does not play the flute. If Rhonda plays the drums, what does Larry play?
In April 2003, it cost \$0.37 to mail a letter weighing 1 ounce. A letter weighing 2 ounces cost \$0.60. A letter weighing 3 ounces cost \$0.83, and 4 ounces cost \$1.06. What was the weigh of a letter that cost \$2.21?	Shawn had 7 white mice and 2 cages. How many different ways could she place the mice in the cages?	A 4-digit number has a 0 as one of the digits. The thousands digit is 2 less than the ones digit. The tens digit is 9 more than the hundreds digit. The ones digit is 4 less than the tens digit. What is the number?	Monica left her house and biked 5 blocks east to the bank. Then she biked 3 blocks south to the store. After that she biked 5 blocks west the bakery and 7 blocks north to a friend's house. How many blocks from home is she?	Doug has 2 pairs of pants: black and green. He has 4 shirts: white, red, gray, & stripe. How many different outfits can he put together?	Kevin used 4 cans of paint to paint her basement floor. Each can of paint covered 91 square feet. Her basement is 14 feet wide. What is the length of Kevin's basement?	Dave bought a piece of elastic that is 270cm long to make swim shorts. Each pair needs a piece of elastic that is 30cm long. How many cuts will she have to make?
Jane is	quarters,	Eve set a	Pumpkins	Old	Darlene	A robin

twice as old as Monica. The sum of their ages is 42. How old are they?	dimes, & nickels. If the snack cost \$0.35, and if she must use the exact change, how many different combinations of coins can Tammy use to buy a snack?	goal of hiking 100 miles during July and August. In the four weeks in July, she hiked 11 miles, 12 miles, 7 miles, & 15 miles. How many miles does she have to walk during each of the remaining 5 weeks to meet her goal?	cost \$5.00 each, and bags of plums cost \$3.50 each. Fern spent \$37.00 and bought a total of 9 items. How many of each did she buy?	McDonald had 3 kinds of animals on his farm. Of the animals, $\frac{1}{2}$ were cows, $\frac{1}{4}$ were horses, & the rest were pigs. He had 8 pigs. How many animals were on his farm?	saved a quarter, a dime, and a penny every day until he had an exact number of dollars, not one penny more or less. How many days did it take him and how much money had he saved?	comes to Tanya's birdfeeder every 5 days and a blue jay comes by every 3 days. Today the robin and blue jay both came to the birdfeeder. How many days will it be before the two birds come on the same day ?
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Sheet B2

Drawing a Picture or Diagram	Making a Organized List	Making a Table	Solving a Simpler Problem	Finding a Pattern	Guess and Check	Acting Out The Problem

Match Stick Puzzle



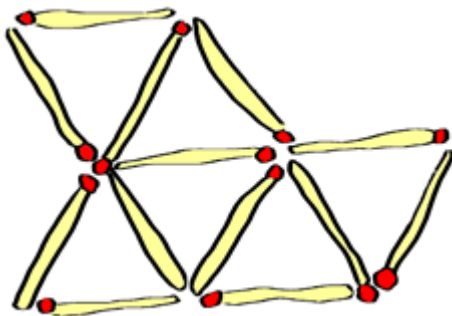
Match Stick Puzzle One: You are an office supervisor and the matchsticks show the movable office walls. One of your workers gets a promotion so you need to rearrange the walls so that each person has a square office.

NB You must use all of the matches. 



Match Stick Puzzle Two:

Using ten matches make two squares.



Match Stick Puzzle Three:

Remove only three matches and leave three triangles.

Appendix F

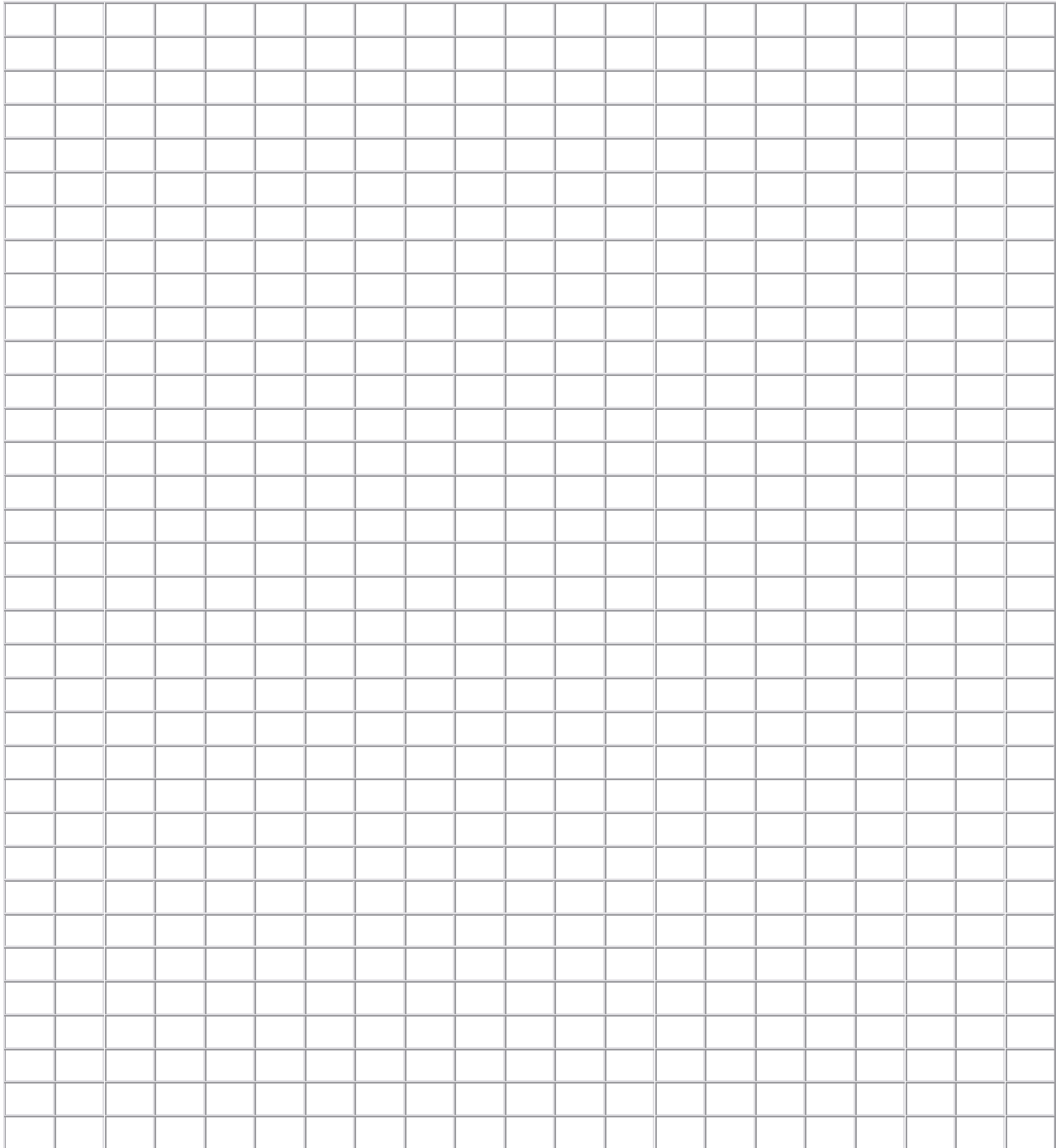
Sheet E

Garden

1. You have 16 feet of fencing to create a garden behind your tree house. How can you design a garden so that...

- a) it will give you the maximum area?
- b) it will give the maximum perimeter?

Draw your garden designs on the grid below.



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Appendix G

Sheet F

Use a calculator to solve the following problems.
Find two consecutive numbers with a product of;
1) 1332

- 2) 702
- 3) 2652
- 4) 1482
- 5) 7310
- 6) 9120

Find a pair of numbers with a product of;

- 1) 54
- 2) 72
- 3) 99
- 4) 121
- 5) 252
- 6) 420

Appendix H

Project A

Pittsburgh Lincoln's Spring Garden

Name-

Date-

Pittsburgh Lincoln K-8 is getting ready to plan its spring garden. Our math class has been chosen to provide the design and selection of the plants for this year's garden. The Lincoln family (including community members) has asked us to help them figure out how to design a garden to brighten up our community. Each person will vote on their favorite garden design based on the criteria, rubric points, and creativity. The garden design that receives the highest vote will be selected as the upcoming garden layout for the school. The class will journey to the store and purchase the fencing and soil to complete the project. The St. Benedict Church will provide you with the flowers you desire for the design. The fencing at the local Home Depot cost \$0.98 per foot, and a bag of soil cost \$1.59. One bag of soil covers 1 square foot.

QUESTIONS YOU MUST ADDRESS

What is the area and perimeter of your garden? How many bags of soil will you need to cover your garden? What will be the cost for the soil? How much fencing will you need in yards, and how much will you pay? How much will the total project cost? The school administrator wants you to bring back less than \$5 in change. They have given you \$80 to spend. Summarize your design by identifying the fraction and percentage of the different flowers you have in the garden. In the form of a letter, explain to the stakeholders how you accomplished the task.