Introducing Calculus in Physics Class

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Overview

This unit deals with accelerated motion and how it can be used to introduce the integral and differential in a main stream physics class for high school juniors

Rationale

In thirty years of teaching high school physics, it has been my experience that most students do not strive to understand the topic they are studying. They would much rather memorize facts and equations so that they can solve physics homework problems. Once they study the material for an exam, it is many times forgotten. This is in part due to the teaching style to which many students have become accustomed. Due to the limitations of space and budget placed on the urban teacher, lecture is the logical teaching method to use. However, from my experience, students have many different styles of learning. Some students learn faster with a hands on approach, which can take the form of individual lab experiments or group projects.

A traditional high school physics course usually consists of a lecture followed by homework. Sometimes the homework is a series of questions to develop understanding, usually the homework consists of word problems dealing with the current topics of the lectures. This usually is scheduled into five periods a week, one class period per day. Some school districts impose a laboratory requirement of one or two lab classes per week. My district requires five periods of lecture plus two periods of lab a week. The lecture classes usually consist of lecture, discussion, demonstrations, and solution of sample physics problems.

Many of my students are accustomed to math class where they see several examples of a mathematics process before they are assigned 10 to 15 problems that imitate the example. The homework problems usually differ from the examples only in the numbers used in the homework problems.

Physics instruction is a little different. Several sample physics problems are usually worked out in class. Unlike the mathematics homework, physics homework usually requires the students to synthesize a solution from several of the example problems. If the student does not understand the concepts, then the solution requiring a synthesis of concepts ends up looking like magic.

The students' lack of understanding occurs in both the application of physics principles and the use of physics equations to solve for the unknowns in the physics problems.

This paper describes an attempt to work around these difficulties by fully discussing new concepts in class and relating the concepts to daily life. Redesigning lab experiments that can be performed inexpensively with existing physics lab supplies, and teaching the mathematics necessary to the understanding of physics concepts.

Objectives

This unit enables the students to: demonstrate their knowledge of the concepts of accelerated motion. (Pittsburgh Public Schools Standard #2), to explain how technology is used to measure very small time intervals. (Pittsburgh Public Schools Standard #4), to gather data and organize it for analysis (Pittsburgh Public Schools Standard #6), and to develop an understanding of the concepts of calculus. (Pittsburgh Public Schools Standard #5).

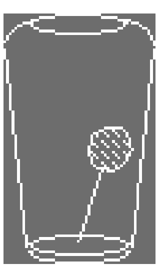
Strategies

My physics classes start out with the idea of the fundamental properties of mass, time and displacement. Each term is defined and thoroughly discussed. Through this process, the students learn that terms used in physics have a different meaning than they do outside of the classroom.

The students have experience measuring mass in chemistry. They still need some discussion about the difference between mass and weight before they become comfortable with the idea. Eventually they come to the understanding that mass is the measure of the amount of matter and weight is a force caused by gravity. One example is the amount of stuff an astronaut puts into a space suit stays the same as long as the space suit is sealed. However, the weight of the suit and its contents can change depending on where the astronaut wears the suit. In orbit, the suit and is contents weigh nothing, while the amount of mass in the sealed suit stays the same.

The discussion shifts to the idea of displacement being a change in position measured in a straight line from the starting point to the ending point. The students learn the difference between distance traveled and displacement by moving around the classroom and measuring the distance they travel and comparing this to the displacement from the starting point. They are instructed to keep track of the direction of the displacement and they quickly learn that a two-meter displacement can result from a number of different trips that differ in the distance and directions traveled. This lays the foundation for later work with vectors.

Then the velocity concept is introduced by discussion. The students use their newly acquired understanding of displacement to develop the concept of velocity. The idea that velocity differs from speed in the use of displacement instead of the distance is pointed out and made clear.



Discussions of acceleration being a change in velocity follow velocity discussions. The students are accustomed to riding in a car, or even driving a car. They know how to change speed with the brake or the gas pedal. An accelerometer is made by filling a pickle jar with water and anchoring a fishing float on a string connected to the lid of the jar. The accelerometer indicates a change in both speed and velocity. The water moves to the "front" of the jar and the fishing bobber moves to the "back" when the speed decreases. The reverse happens when the speed increases.

The water goes to the "back" of the jar and the bobber goes to the front of the jar. The bobber also leans to one side when direction changes. So the accelerometer will register an acceleration by leaning to one side when there is a change in speed or a change in direction. Students satisfy any questions they might have about the accelerometer by performing informal experiments of their own design.

Since the bobber moves in the direction of the speed change it is easy to discuss the direction of the change in speed to constitute a change in velocity. The idea of a direction being *one* of the differences between displacement and distance traveled is carried over to explain that a velocity is a speed with a direction.

The discussion now shifts to examine the acceleration definition more closely. So if the acceleration is the change in velocity, then there must be a number of ways to accelerate. After a few minutes of discussion the students come to an agreement that acceleration occurs when the speed increases, when the speed decreases, or when the moving object changes the travel direction. The students now experiment with the accelerometer to prove to themselves that the bobber leans forward when speed increases. They also verify that it leans backwards when speed decreases. They are surprised to learn that it leans in toward the direction of a turn when the student travels around a corner. With a few more minutes of discussion, the students readily accept the fact that acceleration is a change in velocity. They also seem to understand that acceleration can occur by increasing or decreasing the velocity and changing the direction of the velocity.

Verifying these ideas in the laboratory with standard equipment can be a challenge. The pickle jar accelerometer seems to "make sense" to the students.

Measuring time is easy to do but can be expensive. Prices for high quality, analog stopwatches cost start at $75.00 a piece. In a class of 35 students, the ideal arrangement is for two students to share one watch, this can cost in excess of a thousand dollars to supply stopwatches for a class.

Digital watches that can be turned off and prevent a drain on the batteries cost approximately the same as analog stop watches. Less expensive digital watches cost $10 to $20. These watches run constantly, and cannot be easily opened to change the batteries. In less than two years, the batteries are dead and the cost of battery replacement equals the cost of a new watch.

My solution to this problem was two pronged. I donated 15 Apple IIe computers that I found at a college clearance sale for $3.00 a piece. The fifteen computers came complete with a monitor, CPU, and dual floppy diskette drives. These computers were in poor condition. I was able to produce six working computers with complete keyboards by disassembling the 15 computers. The damaged and inoperable parts were discarded and working parts were then reassembled in the best computer cases. This provided one computer for each of six tables in my laboratory. The unused good parts are stored against a future need to repair the computers. A forty-dollar timing program and site license converted the rebuilt Apple IIe computers to very large stopwatches that are not drain on the budget because they do not use batteries.

The second prong of the solution was to purchase six $25 digital stopwatches that could easily be opened to change batteries. This provides the students with a dozen timers in the lab for experiments. The hand-held digital stopwatches give the students the ability to measure time to two decimal places outside the physics lab.

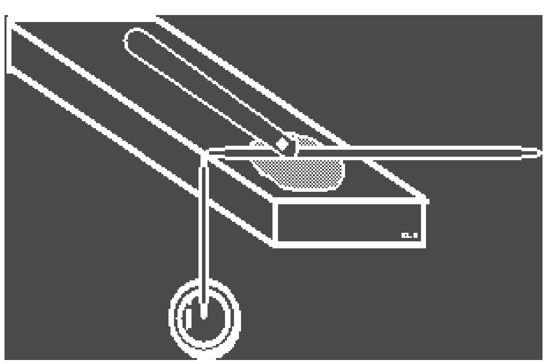
Classroom Activities

Average Velocity Lab

This activity uses approximately three class periods. The first lab gives the student an opportunity to practice their understanding of the concepts speed and velocity. The complete activity is shown in the appendix. It is titled, "Average Velocity Lab". The instructions are self-explanatory on what is to be measured and how the measurements are to be made and recorded. The Data tables also indicate a need to calculate the Average Velocity in each time interval. The teacher is constantly moving among the students to answer their questions and help with difficulties and correct any misunderstandings. By the conclusion of the lab, the students understand the difference between average speed and average velocity. This occurs because the students become inventive about their displacements. Some travel in a straight line from the starting line to the ending line. Others invent unique methods to undergo the required displacement. The inventiveness is encouraged by the fact that the students will vote for the most unique displacement. The students inventing the most unique displacement, as determined by the students, earn extra credit on the lab. The experiment is designed to be conducted indoors or outdoors.

The lab experiment is followed by a discussion of their results and where errors could be introduced either through mistakes or difficulty in operating the stopwatches. This always results in a discussion of who is the fastest person to turn a stopwatch on and off. The students compete to see who is the fastest with only one hand working the buttons. They are surprised to learn that the shortest on-off time for the stopwatch depends upon the student operating the watch and the type of watch used for the trial. This is further emphasized when the students discover the amount of time needed to manually start and stop the Apple IIe stopwatch.

Lesson Two Ticker Tape - Period of a Tape Timer



At this point I show the class an electrically operated ticker tape timer. I demonstrate how a moving object pulling a paper tape through the timer results in a paper strip with a trial of dots placed at equal time intervals on the tape. The ticker tape timer (shown at right) can be purchased in two flavors, battery operated (a drain on future budgets), and an AC electric version.(more expensive initially but less expensive in the long run without the cost of replacing the batteries each year.). The students always ask how much time exists between the dots. This leads into the second experiment shown in the Appendix, "Ticker Tape- Period of Tape Time".

The laboratory instructions are easily understood and gives each student the opportunity to operate the switch on the ticker tape time, use a stopwatch, and also pull a paper tape. Each student pulls three tapes while other members of the lab group measure the time the tape is pulled through the tape timer and another student acts as the data recorder. This gives everyone a chance to do all parts of the experiment. The purpose of the experiment is to measure the period (time between dots) of tape timer.

The point is driven home that no matter how fast the tape moves, the time between dots is always the same. This is a little difficult for some of the students to understand at first. If the tape speeds up as it is pulled, the dots will have a progressively larger spacing between each dot. Some students need further discussion to understand that the increased spacing does not mean increased time.

Some of the students attempt to pull the paper tape very quickly to get the fewest number of dots to count. This results in a set of lines instead of a set of dots. The lines are a result of the tape dragging along the "clapper" while is against the tape. This is a difficult situation to time. The tape moves so fast that some students have difficulty turning the stopwatch on and off while dots are put on the tape by the tape timer.

Some of the students pull very slowly so that they have an easier time using the stopwatch to measure the time the dots are put on the tape. This ends up producing a tape with thousands of dots that are so close together that they are difficult to count.Eventually the students find a good compromise between the ease of counting dots and the ability to turn the stopwatch on and off..

Keep in mind that some students are measuring time with hand held stop watches. These watches measure time to 0.01 seconds. Other students are using the stopwatch mode of the Apple IIe computer. This measures to 0.0001 seconds. This doesn't matter. After each student determines the period for the tape timer, they are required to record their three values on the chalk board. Immediately, the students notice that some of the values are not near most of the other values. A discussion usually develops about who is correct. Another discussion usually develops around how we are going to determine who is correct. This leads to the data analysis portion of the lab.

A computerized spreadsheet could be used to analyze the data. I prefer the students do this analysis in a class discussion format so that they learn how to handle the data. When data analysis is done by a commercial spreadsheet program this knowledge does not usually develop.

The data is plotted by hand on rectangular coordinate paper to produce a histogram of the data distribution. Some data is measured to four decimal places and other data is measured to two decimal places. The students quickly decide that they cannot add decimal places to the values measured to two decimal places because they would have to invent the information. They then decide that it is better to round off all the data given to two decimal places. Data having two decimal places is easier to plot on one sheet of paper.

The students determine the range of the periods so that they can set up the time on the x-axis. They determine the number of occurrences of each time value so that they can set up a frequency value on the y-axis. The raw data for the period of the times usually ranges from 0.01 seconds to 0.70 seconds. When the frequency of value occurrence is plotted against the value of the period, the students quickly see that most values for the period fall between 0.01 seconds and 0.04 seconds. Further examination of the frequency graph shows most period values are 0.02 seconds. This is taken to be the period of the tape timer because it occurs most often in the histogram.

This experimental value is an excellent approximation of the ideal period of the AC electric tape time that vibrates due to the variation in the AC line current. In the United States, the alternating current varies 60 times a second. This makes the period of the timer 1/60 of a second if the timer was an ideal machine with no mass or friction. When the fraction 1/60 is converted to a decimal, the equivalent value is 0.0167777777777 seconds. This rounds off to 0.02 seconds.

The total time for this experiment dealing with data analysis and significant figures is approximately 4 class periods. If the cost of the tape timers is ignored, the annual expense for 100 students is approximately $6 for paper tape and carbon disks.

Lesson Three - Acceleration of a Falling Object

The discussion in the class now switches to the fact that the tape timer can be used to measure times that are smaller than the time required for a human to start and stop a stopwatch.

The students are asked to recall who could start and stop the stopwatch in the shortest time. That person is then asked to measure the time it takes for a 100 gram mass to fall to the floor form the top of the lab table. The students are given time to experiment with this in an informal way. The class quickly comes to an agreement that it is not possible to accurately measure a time interval of such short duration with a hand operated stopwatch. They agree that it doesn't matter what type of stopwatch is used.

The students are asked what happens to the velocity of a falling object. They agree its velocity increases as it falls to the floor. When asked how they know the falling object increases its velocity, they answer that is seems reasonable that a falling object should speed up.

It is at this point that the discussion of a way to verify the acceleration takes place. This is done by timing the fall of a 100 gram mass using the tape timer. The tape supplies information on displacement, travel time, and average velocity over very short time periods. The finished tape should have dots that are spaced at increasingly larger intervals.

This information is graphed and the graph is analyzed to develop the equations for accelerated motion:

$V_f = v_o + at$

$S = v_o t + 0.5at^2$

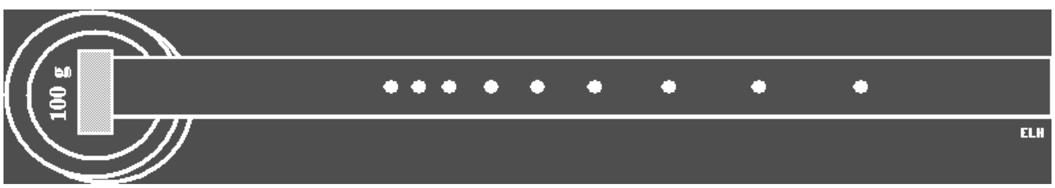
$2as = v_f^2 - v_0^2$

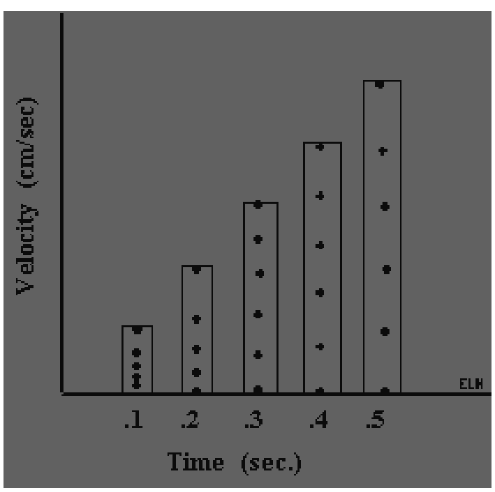
The students have already determined the approximate period of the ticker tape timer by analysis of the data obtained in the last lab activity. The class has accepted the period value to be 0.02 seconds.

The students are asked to verify a falling body accelerates by performing the next lab activity. The instruction sheet for the lab is found in the appendix. It is titled, "Acceleration of a Falling Object".

The instructions direct the student to connect a ticker tape to a 100 gram mass, load the paper in the ticker tape timer, turn on the timer and drop the 100 gram mass.

The ticker tape timer makes marks on the tape at 0.02 second intervals. The finished tape looks like the diagram below





The students are asked to tear the tape at every fifth dot and then tape the piece of paper tape to a rectangular coordinate graph, as shown in diagram at the left. Each 5 dot interval is a total time of 0.1 seconds. (0.02 seconds per dot times 5 dots gives 0.1 seconds.) If the length of the strip (displacement of the falling object) is divided by the time to fall (0.1 seconds), then the length of the strip is proportional to the average velocity of the falling body for that 0.1 seconds. So the y-axis is labeled (velocity cm/sec).

The students should arrange the paper strips so that they are equally spaced along the x-axis. As long as the falling mass did not hit something during its trip to the floor, the length of each strip should increase by the same amount. By drawing a straight line through the top of each paper strip, we can represent the velocity of the falling mass as a straight line.

The slope of the straight line is (the change in velocity)/(the change in time). The slope is the acceleration of the falling mass. If there is no friction there should be an acceleration of approximately 980 cm/sec^2. This acceleration is equivalent to 9.8 m/sec^2, the acceleration due to gravity. There are two sources of friction. The first source is the friction between the edge of the table and the moving tape, the second source is the friction between the tape timer and the moving tape.

Friction causes a reduced acceleration for the falling mass. The change in the acceleration can be used to point out that we can alter what we are measuring by the process of measuring. A discussion of how the acceleration changed because we attempted to measure it is necessary for better understanding of the effect of friction on moving objects.

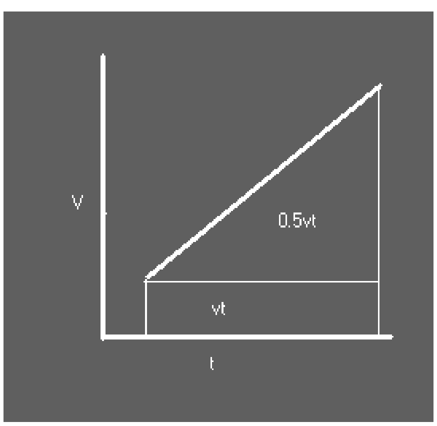
Most students, depending upon how carefully they cut the tape and placed it on the graph, will calculate the slope to be in the neighborhood of 500 cm/sec^2 to 700 cm/sec^2. This part of the assignment should take one class period.

The next class examines two things. First, the definition of acceleration and how the equation can be rearranged to solve for velocity. Second, we look at the area under the line that represents the acceleration of the falling mass.

On a velocity vs time graph the slope of the straight line is $(v_f-v_o)/t$ This is the definition of acceleration. So $a = (v_f-v_o)/t$

The equation $a = (v_f-v_o)/t$ can be solved for v_f. This gives $v_f = v_o + at$

The area under the straight line can be divided into two areas, a rectangle and a triangle. The area of the rectangle is vt. The area of the triangle is one half the time times the velocity. By rearranging the definition of acceleration, and solving for velocity. The velocity becomes at. This is multiplied by t/2 to give $0.5(a)(t^2)$ as the area of the triangle. (see the diagram at the left)



The resulting area under the velocity plot is $vt + 0.5at^{2.}$ Using doing dimensional analysis on each term we find the units for the area under the line to have the units of displacement.

The first equation, $v_f = v_o + at$, is solved for t.

$$v_f = v_o + at$$

$v_f - v_o = at$

$(v_f - v_o)/a= t$

Substituting the above equation into the equation $S = v_o t + 0.5at^2$ and simplfying results in the equation

$$2as = v_f^2 - v_0^2$$

As a result of the this class discussion, the slope of the line, the derivative of the velocity plot produces the equation $v_f = v_o + at$. The area under the graph, the integral of the velocity plot, produces the equation $S = v_o t + 0.5at^2$. So without going through strange algebraic contortions, and using the integral and the derivative, the three lab activities have led to the three equations used to analyze accelerated motion.

In the process of this in class discussion, the students have unknowingly found the derivative and the integral of the velocity plot. This discussion occurs in all first year physics classes. I never tell the students they have just done the basics of calculus. This is a seed that is planted for the math teacher to nurture in the calculus class. When the students encounter these concepts in their calculus classes, they are pleased to realize that they suddenly understand what the mathematics instructor is doing. Because of these three labs, the students more easily make the connection between calculus and the accelerated motion equations they learned in physics.

Appendices

Student lab Assignments

NAME____ Average Velocity Lab

Average Speed is defined as $\dfrac{\text{distance traveled}}{\text{time}}$.

Average Velocity is defined in a number of ways

• $\dfrac{\text{displacement}}{\text{time}}$

• $\dfrac{Vf - Vo}{2}$

Speed is the rate at which a distance is traveled or the rate at which position changes.

Speed depends upon the path traveled.

Velocity is a speed with a direction attached. It does NOT depend upon the distance traveled. It depends upon the change in position (displacement).

You will calculate the average velocity (along a straight line)of each of your two lab partners and yourself.

INSTRUCTIONS:

Each lab partner must walk or stroll OR MOVE IN SOME MANNER along the top concrete walk in front of school (if it is raining or some other problem exists, make your measurements in the hallway.) Your displacements must be at least the length of 4 concrete slabs.

While you undergo a displacement, your lab partners must measure the time needed for the walk.

You must do this at least three times each.

Record the distance walked and the walk time for each trial.

Calculate the average velocity of each lab partner.

Enter the data in the tables below.

NAME ______________________

Trial	disp. (yds.)	Time (sec.)	Avg. Velo. yds/sec
		Average	

NAME ______________________

Trial	disp. (yds.)	Time (sec.)	Avg. Velo. yds/sec
		Average	

NAME ______________________

Trial	disp. (yds.)	Time (sec.)	Avg. Velo. yds/sec
		Average	

*NAME*____ *Period*___________________

Ticker Tape -- Period of Tape Timer

CONSTANT SPEED means that the speed doesn't change.

You will use a stop watch and the constant speed of a paper tape to calculate the period (time between dots) of the tape timer.

DIRECTIONS:

- Attach the Tape timer to the lab table.
- Assign one student to operate the stop watch.
- Assign one student to turn the timer on and off.
- Assign one student to pull the tape through the tape timer at a CONSTANT SPEED.
- The student pulling the tape must pull three tapes through the timer at a constant speed while one of the students uses the stop watch to measure the time that the tape moves in the timer.
- Record the time on the tapes along with their numbers.
- Record this information in the data table also.
- Switch assignments repeatedly until all students get to pull three tapes.

PERIOD = TIME DIVIDED BY NUMBER OF DOTS

$$\text{PERIOD} = \frac{\text{TIME}}{\text{NUMBER OF DOTS}}$$

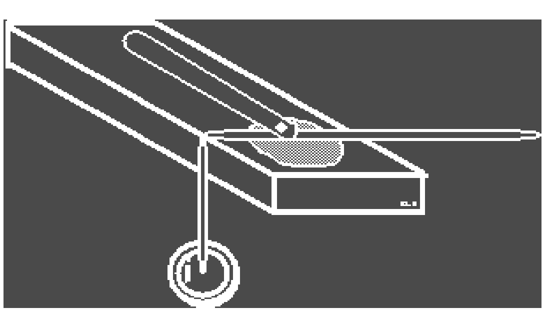
TAPE NUMBER	TIME FOR TAPE IN TIMER (SEC)	NUMBER OF DOTS ON THE TAPE	PERIOD (SEC)
1			
2			
3			

Name ______________________________________ Period _______________

Instructions:

Acceleration of a Falling Object

Set up the equipment as shown in diagram #1



Each student must make his or her own tape.

With the tape loaded and connected to a weight, turn on the tape timer and allow the weight to pull the tape through the tape timer.

diagram #1

mark every fifth dot.

tear off the first five dots and tape them to a piece of graph paper

tear off the next five dots and tape them to the graph paper beside the previous five dot strip of paper.

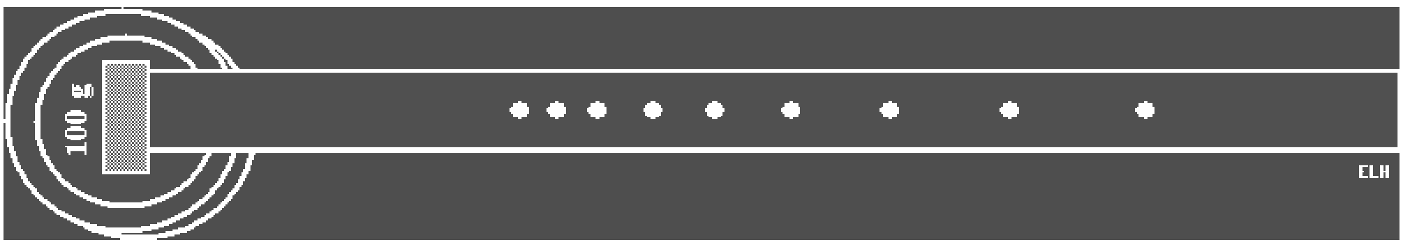
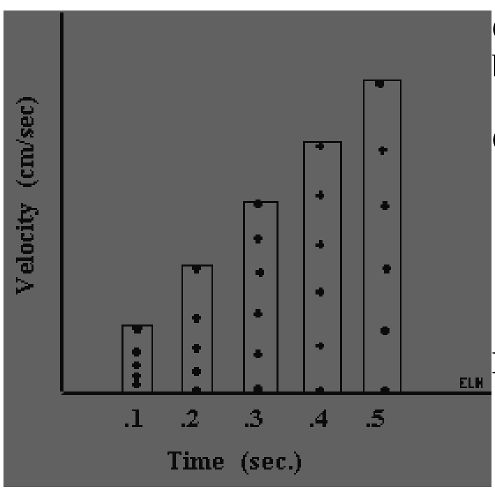


diagram #2 - Tape after it is pulled through the timer by the falling weight



Continue this process until all of the paper strips have been taped to a sheet of graph paper.(See diagram #3)

Clean up the desk and sit down.

Diagram #3

Standards

The Pittsburgh Public School Standards met by this unit are:

1. All students explain how scientific principles of chemical, physical, and biological phenomena have developed and relate them to real-world situations.

2. All students demonstrate knowledge of basic concepts and principles of physical, chemical, biological and earth sciences.

4. All students explain the relationships among science, technology and society.

5. All students construct and evaluate scientific and technological systems using models to explain or predict results.

6. All students develop and apply skills of observation, data collection, analysis, pattern recognition, prediction and scientific reasoning in designing and conducting experiments and solving technological problems

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