A Unit in Geometry That Really is an Earth Measure

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Overview

This unit is to be used during the last nine weeks of a high school geometry course. It involves practical geometry in that it uses computations of lengths, angles, areas, and volumes. Although the course and unit involve written logical arguments, they contain no formal proofs. The general strategy of the unit is that the students will compute the theoretical size of real objects and then attempt to physically measure these sizes to verify the calculations.

Specifically, the students will begin with small objects that can be held, the dimensions of which can be measured with a ruler. Later, the students will find the size of larger ones, such as buildings, trees, and mountains. The unit will end with the calculation of the size of the Earth using a modified version of the method of Eratosthenes (300 B.C.).

Stories about ancient Greek and Egyptian mathematicians play an important role in the unit. The students will be told stories about these ancient mathematicians and are to role-play by choosing mock Egyptian or Greek names for themselves, while they work on the unit.

Rationale

The ultimate purpose of this unit is to make geometry more interesting to high school geometry students. Interested students will essentially teach themselves and will continue to learn the subject after the course is over. The least that can be expected is that students will believe that the geometrical calculations they do represent something that actually exists.

The unit attempts to accomplish this by focusing on geometry's ability to allow us to calculate the size of real objects in the physical world. Once calculated, it is believed that it will be quite gratifying to physically measure the object's size. For example, in the beginning of the unit, the students will compute the volume of various geometrical shapes by taking measurements of edges with a ruler. The students will then attempt to measure the object's volume using displacement methods.

To enhance interest, these skills and concepts will be learned in the context of stories about ancient mathematicians such as Archimedes', who shouted "Eureka" once he discovered that he could measure an object's volume by displacing it in water. (He actually discovered the concept of density and this story will be told later).

It is also believed that the ability to compute dimensions and sizes of existing objects and to understand and be able to explain the formulas behind them is a more marketable skill to the workforce of today than the ability to "do" proofs.

Until about fifteen years ago, high school geometry had placed its emphasis on the theoretical. Much of the course had been spent doing proofs, such as proving that two triangles are congruent. While these are good exercises in logic and thinking, most students do not quite "get it" today and proofs do not lend themselves to a view of the relation between geometry and the physical world. Proofs make geometry appear artificial to most students, as if it is anything but a representation of something that is real.

It is believed that the majority of students leave geometry not knowing or caring about it. It has lost its purpose and meaning in the classroom. Students do not see it as relevant to their lives, or worthy of further study. It rarely leaves the textbook or SAT preparation. We are living in an age when nearly everyone has the opportunity to learn it, yet so few care to do so. Ask anyone if they liked and remember geometry, as I have done to many myself. A common response is that no they did not like the course and will say something as, "the only thing I remember is that $a^2 + b^2 = c^2$."

This unit will take a different approach in that it will leave the student with a delight in the awesome power that geometry has enabled us with. The ability to allow us to calculate the dimensions and size of anything encountered in the physical world, even the size of the planet on which we live, is amazing.

The unit is to take place during the last nine weeks of a yearlong geometry course. By this time, the students have learned much of the concepts involved in what I call practical geometry. They have learned in depth the Pythagorean Theorem, area, and volume and have learned some basic trigonometry, such as finding missing lengths and angles of right triangles. The students can compute the area or volume of any shape given measurements of edges and angles. They are sophisticated enough, for example, to find the area of a regular pentagon given the length of a side. The solution to this problem as well as others can be found by referring to the Appendix.

The students can not only solve the problem, but can write an explanation giving the process, and can come up with general formulas for computing the area. Although not a formal proof it is a logical argument, which is essentially what a proof is.

Incidentally, the symbol for the Pythagorean Secret Society is the five-point star embedded in a pentagon, often times ad infinitum with smaller and smaller pentagrams inside the pentagons. Legend has it that The Pythagoreans had this symbol tattooed on their hand and wore it on their clothing.

Many historians consider The Pythagoreans creators of the foundation of modern mathematics, in that it was they, ironically, that were the first to require proof. They are also the first to recognize irrational numbers and their relation to geometry, The Pythagorean Theorem itself. Numbers and their relation to geometry were part of The Pythagoreans religion. Pythagoras preached that to understand numbers was to understand life itself. A fact that is useful toward increasing all students' interest is to note that The Pythagoreans were one of the first organizations of its kind that allowed women as members.

Stories of ancient mathematicians such as this add to the interest. Many have heard the theory that the development of geometry was greatly advanced in Ancient Egypt, because of the annual flooding of the Nile. The flooding changed the landscape and accurate surveying was necessary to determine how much land a farmer had lost or gained. Others believe that the Egyptians, as the Greeks, had a deep love for geometry and studied it for its intrinsic properties. We don't know why the Egyptians built massive pyramids with incredible accuracy, but they did. In the spirit of discovery and love for geometry, the students

ws, and distances from bases, some of which are quite large. In addition to this, they must learn to measure angles of elevation with clinometers, which they will build in the unit.

Many of these skills are not developed here but have been created earlier, as the students measured lengths and angles of their constructions throughout the year. The difference here is that the students are going to be measuring lengths of real objects such as a pyramid they hold in their hand, and later shadows of large objects with a tape measure. This is, of course, different than measuring the length of a line segment on paper. Students will also have to be able to create general formulas for computing the surface area and volume of the objects, given only the measurements of the edges. They will also understand that the volume of these objects equals its displacement in a liquid and be able to create instruments for performing this task. They will have to understand proportions and their relation to similar triangles.

Soon after this, they will have to be able to understand how to calculate an object's height, given the angle of elevation and distance from the base. Later, they are expected to come up with a mathematical method to calculate the object's height without the distance from the base being known.

In addition to this, the students must understand the principals of basic Euclidian

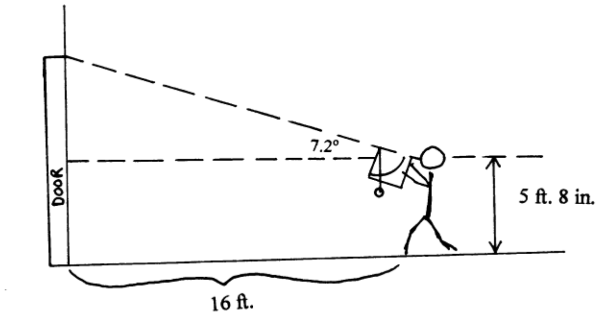
Geometry and be able to write an account of their calculation of the size of the Earth. If there is time there are extra topics and activities that can be inserted or added on to these.

One of these that involves itself with the question: "now that we know the size of our planet, how can we explain precisely where we are on it?" In order for the student to do this they must learn the x, y, z rectangular coordinate system and be able to apply it using some spherical trigonometry. .

In conclusion, the primary objective here is that the students leave with a profound interest in the subject, which is believed the nature of the questions and work on them will produce.

Strategies

Any of the concepts and skills acquired by the students are first attempted by presenting the student with a problem and then letting the students come up with
(as Discovering Geometry would say, "discover") a method themselves. A particular example will be presented here which exemplifies this learning method. When the students are learning how to use a clinometer to calculate an unknown object's height, they will be presented with the problem of calculating the height of the doorway in the room by measuring the angle of elevation with their clinometers and measuring the distance from the base of the door, which is easy if your floor is 1 ft. x 1 ft. tiles (see diagram)



Before the students are told anything, they are to struggle with this problem. It is hoped that they will make the mistake of forgetting to add the height to the person's eyes, forgetting that the angle of elevation is really from this point. In the clasby the teacher in the Socratic method of asking questions, "What are we forgetting?" This same problem is developed during the Activity section next.

When students are actually physically measuring distances that they had calculated, they are more likely to believe and understand the concepts. They can see it right in front of their faces.

The students are to come up with their own conclusions and be able to write it all down on paper in such a way that other students with a knowledge of geometry would be able to perform the same calculations themselves. This example displays the general method of learning and guidance toward the students meeting the objectives and standards. A case-by-case strategy for meeting each of the particular objectives will take place in each activity in the next section.

Activities

An Egyptian Method of Creating a Right Angle

There are several activities, which should be done before the unit. Ideas of ones that are not suggested can be fì

ment, which succinctly describes the size of a solid. It is the one number that describes the amount of space an object takes up.

Once the students believe this, they are ready to compute the volumes of the solids by taking measurements of the edges and heights of some faces. Discuss the difficulty in physically measuring the heights of the pyramids. After this point, do not allow the students to use their physical measurements of the heights of the pyramids, but only their measurements of the edges and perhaps heights of the faces. This forces them to use the Pythagorean Theorem or similar methods to find the height.

After the students take their measurements they are to compute the volume of their solid. The beginning of the computation begins with a drawing or construction of their solid and then the computation. Examples of solutions to the square pyramid and regular tetrahedron can be reached by referring to the Appendix.

Whenever you provide the templates for the students, such as photocopies, make sure that you do the computations and measurements of each student's solid before the class, which can be done quickly and easily by measuring segments on the templates themselves. This makes you intimately familiar with the solids and methods and enables you to quickly check the student's work.

The student's exploration of the volume of these solids should be followed with methods of measuring the object's volume. Given that the solids are made of paper they are not easy to measure by methods such as the displacement of a liquid used later. Instead of measuring these solids, the students will be given intuitive evidence by being given cubic centimeter blocks. This will be done by making stacks of the blocks that loosely represent the solids. It will then be decided if the calculated volumes make sense.

Day 4 and 5

Next, the students are given a can of soda and asked to find its volume by taking measurements. After careful measurements of the diameter and height, the students will compute an approximate volume. The students are to draw pictures of the can along with measurements and explain how they came up with their solution. An example of the teacher's measurements and calculations are shown below. The punch line of the lesson is that a good approximation to the can's volume is written on the can, for it is marked as having 355 mL of liquid and 1 mL = 1 cm^3.

D = 6.4 cm

H = 12.5 cm

During this class the teacher will ask the students for their computed volumes. After discussing high and low values, the reasons for each and difficulties encountered, the teacher can present their measurements and calculations from above. A good question to ask is why does the above calculation produce a result that is higher than the stated volume of 355 mL in the can?

Incidentally, students are not discouraged from using inches as measurements. Students at this stage of the year should be able to convert between the two and know that: 1 in = 2.54 cm, 1 $in^2 = (2.54\ cm)^2$ and 1 $in^3 = (2.54\ cm)^3$.

During the same class as the soda pop the students will be given a plastic cup (truncated cone). The cup is not to be used to drink the soda, but as another object's volume to compute. Given that the students have solved similar problems several possible results are expected. Once again be reminded that the students are to find a method and solve the problem themselves. Let them struggle with the problem and come up with something before any leading questions are asked. Even if the teacher highly suspects their students will not solve the problem, the students should still be encouraged to solve it. This thinking process is necessary for the student to fully get the idea. It also provides a more interesting base for learning. The students should be lead to discover perhaps that measuring the diameter of the top and bottom of the cup as well as the perpendicular length along the side makes the most sense.

Finding the volume of the truncated cone is a hard problem for many students. A variety of solutions as well as the solution to the general formula for a truncated cone can be reached by referring to the Appendix.

After the students have made their calculations of the volume of their cups it is now time to verify with a physical test. The teacher will at this time produce a large graduated cylinder, which was "borrowed" from a science teacher, and a large bucket of tap water. Each student or several of them will fill their cup to the brim and pour it into the graduated cylinder. The student can do this or the other way around. The teacher can fill up the graduated cylinder with the students' "calculated" volume and then pour it into the cup. This of course should be done over a bucket in case the student overestimated the volume. Remembering that 1 mL = 1 cm^3, the volumes can be verified.

Day 6

The next day present student responses. The students' written responses to involved questions should have been thoroughly examined and discussed throughout the year. Hopefully, examples of good responses will be the student's. These are examined and discussed with the class by photocopying the work anonymously onto an overhead and presented. Discuss the different solutions such as the ones done here. Ask questions such as: Which are better approximations to the solid's actual volume?

As this session on solids has progressed the student should be encouraged to come up with general formulas for the volumes of the solid. They should come up with a formula for the volume of a general truncated cone (cup) from above (reached in Appendix).

Day 7

The practice of creating general equations is exemplified in the final problem of the session, that of finding the volume of an actual stone carved into the shape of the following 14 faced polyhedrons (referred to as truncated cubes) :

As can be seen the solids are not perfect. The students are expected to either measure the edges, (e) or the diagonals (x). They are then to come up with a general equation for the volume given their choice of measurement. For example, using the diagonals (x) as a measurement the following equation can be created (the solution of which can be reached in the Appendix):

$$\frac{5x^3}{6}$$

x V =

e

where x = the length of the

diagonal of a square

face

What the students do with the measurements is up to them at first. How they deal with the imperfections is also up to them. It is hoped that they will do something, such as the following: measure all of the diagonals (x) of the square faces and find an average diagonal. Then use this length in the general formula from above. For example, measurements of the diagonals of the faces of truncated cube #3 are listed by faces below. Incidentally the faces are marked in permanent pen F1 through F6.

Truncated Cube #3

Length of diagonal (x) cm

F1 - 5 37
5.25

F2 - 5.3
5.29

F3 - 5.31
5.23
F4 - 5.1
5.28

F5 - 5.32
5.36

F6 - 5.1
5.22

Incidentally the solids are made of onyx, which is a type of quartz, and the square faces are marked in permanent marker, F1 through F6. The student is once again encouraged to come up with the volume given only a ruler and a calculator. They are also expected to explain in writing what the have done. If they are not successful the first attempt (one class period) the teacher may offer some advice and lead the students to "discover" a method. The teacher can choose any type solid for this activity. The truncated cube is suggested because it forces the student to come up with a formula that involves some complexity. It is also a shape, which lends itself to a variety of solutions for its volume. If the teacher does not have such truncated cubes, any solid, which can be measured by displacement, can be used.

Day 8

This class begins with the story of Archimedes' discovery of displacement (actually density). The students can either read the story themselves or have it told to them.

One such version of the story is as follows. A king came to Archimedes one day with a problem. He had had a crown made from gold coins, but suspected that he had been cheated by the craftsman, who had supposedly replaced some of the gold with silver. Archimedes asked the king if the crown weighed the same as the original gold coins and the king replied "yes." Later when Archimedes was taking a bath he shouted "Eureka, Eureka!" ("I have found it.")

Silver, which takes up more space per unit volume than gold, would displace more volume than would gold of the same weight. He had essentially discovered the concept of density, which will play a role in the boat in the swimming pool activity later in the unit.

The concept the students are to get from this story is that of displacement. The amount of water displaced by an object in a liquid will equal its volume. This can easily be verified by submerging rectangular solids of "known" volume and measuring the displacement.

The students are going to attempt to build such an apparatus that measures volume. It consists of a plastic container, a plastic tube, and some sort of caulking substance that is non-toxic. By filling the container with so much water that no more will go out a spout, a solid can be submerged with a fishing line. The displaced water can be collected and measured in a graduated cylinder. If this is not successful the students can be given such an apparatus. They will then attempt to measure the volume of the solids, using the displacement principal.

Hopefully the activity will generate discussion about what the best answer is. Which one more closely represents the solid’s actual volume? How good does a solution have to be to receive a score of a 5? That is a good question and the answer of which is up to the teacher.

What is important here is that the student believes that their mathematical representation if made well enough could lead to solutions that were closer and closer to the solid’s actual volume. They could continuously improve their solutions. How do we really know which solution is the most accurate?

Before moving on to the next session, let it be noted that a discussion on accuracy of measurements, and significant figures would be worthwhile. Having the students compute what an error of say .1 cm would do when used to compute a volume would be very meaningful. Note that most of my solutions are rounded to the hundredth place, even though to some scientists this would not make sense because of significant figures. Many math teachers, as myself, are often guilty of ignoring the concept of them.

A topic related to accuracy of measurement is that of the accuracy of the calculations, which does not imply mistakes but how lengths that turn out to be irrational numbers are dealt with. Whenever students are calculating unknown distances, they will often encounter irrational numbers, for example. What if the height of a pyramid is

cm. When we use this value should we use 1.4 cm, 1.41 cm or 1.414 cm? It depends on the significant figures in the original measurements. If they had three significant figures then the height with three significant figures (1.41 cm) would be most appropriate. Earlier in the year the students will be exposed to an Egyptian/Greek method of proving the existence of irrational numbers. Since the unit is to contain these stories and the existence of irrational numbers is central to its study it will be presented here.

y

x

Briefly the Egyptians noted that the area of the large square is twice that of the smaller square. Egyptians and the Greeks, namely Pythagoras and Thales asked themselves the following question. What if X were an odd number? Y is certainly even. Can the square of an odd number ever be even? Following this logic it is seen that X and Y are non-commensurable. A more detailed version of the proof can be found in *A Mathematical Mystery Tour* by A.K. Dewdney.

Day 9

The session begins with another ancient story. It is a story of Thales in one of his many trips to Egypt. Upon reaching the great pyramid of Giza (Khufu), he asked the guide what its height was. While the Guide was asking some scribes, because he did not know the answer, Thales calculated the volume of the pyramid. He had measured the pyramids shadow as well as his own. Then by knowing his own height he was able to calculate the height of the pyramid. When telling this story give the students a mock sample of the problem Thales had to solve.

Thales, who is 1.6 meters, casts a shadow that is 2.3 meters at the same time of day that the Pyramid of Giza casts a 210-meter shadow. What is the height of the pyramid?

$$\frac{H}{1.6} = \frac{210}{2.3} = H \approx 146.08 \text{ meters}$$

After the students solve this problem discuss the validity of this story. Would Thales really be able to measure the distance from the edge of the pyramids shadow to the point where the height was perpendicular to the base, which is inside the pyramid? If the story is true perhaps Thales did more math than was stated. Incidentally many of the stories are different when found in other sources and this should be shown to the students.

Day 10

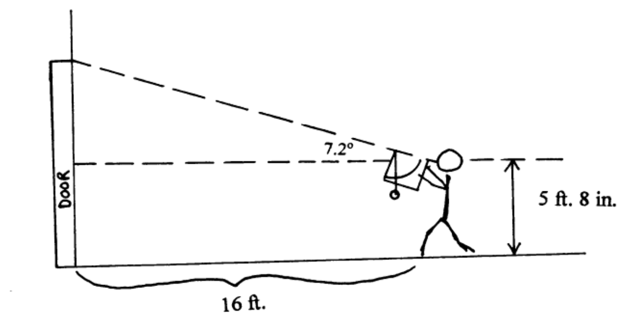
The students are to go outside and attempt to calculate the height of objects using Thales' method. The students will begin with a small object, such as a ruler, with a known height to verify that the method works. At my school the students will then be doing the flagpole, a tree and the lights near the football field. If the day is not sunny then there are many alternatives: The experiment can be done with a strong light in the classroom. Or if a sunny day is the wish, then a standards problem from the Appendix can be given, problems of which the students should be doing all year long. A last alternative is to do an activity that follows no scheduled time in the unit, as the boat in the swimming pool problem.

Another alternative, which can be done during some other student collections of real data in the unit, is for the teacher to "make up" the data and tell the students that you yourself collected the data on a sunny day. Much preparation must be put into these "measured" distances for the data must come close to the objects' actual heights for the students will soon be measuring and calculating their heights again using more sophisticated methods. It is suggested that the teacher use these more sophisticated methods to come up the shadow lengths.

Day 11

The day after students compute the height of the objects using Thales' method they are ready to develop another method for finding the heights of these objects. They are to make clinometers out of the materials described below. An enlargement of the quarter circle can be used with thick card stock paper, dental floss, tape, plastic drinking straw and washer.

During the first day with the clinometer, students will do the activity mentioned in the Strategy section of this unit. They will calculate the height of the doorway using the angle of elevation of the clinometer and the distance from the base of the door. In many of the classrooms of today, we are lucky enough to have 1 ft. x 1 ft. square tiles on the floor. The student can count the squares from the base to get its length. As was noted in the Strategy section of this unit, the students are to be allowed to make the mistake of forgetting to add the height to the eyes, which is where the angle of elevation is taken.



$$\text{TAN } 7.5 = \frac{H}{16}$$

H = 16 TAN 7.5

H = 2.1 ft.

Once the students arrive at this result for the "height" of the door, ask them what is wrong. The door is certainly higher than this. Eventually they will "discover" that the eyesight height must be added. This is when the teacher pulls out the tape measure(s) and the students attempt to measure their eyesight heights. Suppose the student above measures their eyesight height as 5 ft. 8 in. The student’s final computation of the doorway would be:

Door ≈ 5ft. in. +2.1 ft. ≈ 7.76 ft ≈ 7 ft. 9 in.

It is now time to physically measure the door to verify the calculations. The interest can be increased if it is made into a little contest to see whose calculation will come the closest to the measured distance. Select student volunteers to measure the door with a tape measure. A "good" measure of the door with a tape measure is 7 ft. 8 in. Ask the students why the calculated measures of the door are not exactly the same as the measured length? Which one is a better one?

It should be noted that the doorway is an easy distance to check by measuring with a tape measure. The students are to be told that the purpose of the activity is to see that it works and can be easily verified. Explain to the students that a verification will not be so easy when dealing with trees, buildings and mountains. How can we measure the height of a tree with a tape measure? Better yet how can we measure the height of a mountain. We cannot go into the mountain to physically measure the distance to the perpendicular from the vertex to the base, as in our noted problem in Thales’ shadow of the pyramid story. When we calculate the height of large objects using these same methods they are going to be harder to verify. But if we believe the methods work then we can believe the results. The next day we will make this step into that of larger objects, whose heights are harder to physically measure, yet we will do so one last time.

Day 10

The clinometer will be used to measure the angle of elevation from a users eye to the top of the

object as in the diagram below.

The student knowing their eyesight height and the distance from the base of the object, which will be measured with a tape measure, can calculate the height of the object.

On this day the students are to collect data and calculate the height of a tree, flagpole and building. When the students do their actual recording of the measurements as well as the calculations, they are to do so by creating a chart, which can be modeled on the board. They are to collect the angles of elevation from three different distances from the object. The students are then expected to come up with a height by averaging the calculations from the three data points and perhaps using final calculations of others. They are to neatly fill in the chart, and briefly explain what they have done to calculate the height of the object. Included of course are the students' calculations, which are also neat and easy to follow.

Also noted here is that students should compare their new calculated heights to their old ones using the shadow length method of Thales. Which ones are better approximations?

Day 11

On this day the students are to collect data on the stadium lights using a slightly different method. The students will be forced to collect the following type of data:

Stadium

Lights

31º 40º

20 m X

The difference between this type of data and the previous one is that the distance from the base is not known. The purpose for this is to develop the mathematics necessary for calculating the heights of large landmasses and other objects in which it is not practical or possible to measure the distance to the point where the height is perpendicular to the ground. Examples of such objects are mountains and Thales' pyramid. The math involved with this calculation is shown below. Once again make sure that the student solves this problem. Do not lead them toward a solution, unless it is necessary.

TAN 40º = TAN 31º =

$H = X$ TAN40º $H = (X + 20)$TAN 31º

Since both expressions to the right are equal to the height (H), by substitution:

X TAN40º = (X + 20)TAN31º

X TAN40º = X TAN31º + 20 TAN31º

X TAN40 - X TAN31º = 20 TAN31º

X(TAN40º - TAN 31º) = 20 TAN31º

X =

Day 12

What is needed next is the physical measurement of the height of the stadium lights in a manner that the students will believe actually measures the height. This will be the last time a height will be physically measured using a tape measure method.

A volunteer, Mike Varlotta, a teacher at a rival high school on the North Side, Perry, who used to teach at my high school, Oliver, has agreed to climb the stadium lights. This is if it is permissible of course. Mike is also a professional rock climber. He will climb the pole carrying a climbing rope with him. Once he reaches the top, students will mark the point on the rope where it touches the ground. Mike will then come down and we will measure it.

Field Trip

The next activity will be a field trip down to the Allegheny River directly below the North Side. The students will collect data similar to their last data collection of the stadium lights using a clinometer, as below.

North Side

Bridge

θ

The two angles of elevation will be taken from the north and south sides of the river. In conjunction with methods used by the ancient Egyptians and Greeks, the distance across the bridge will be approximated using pacers. With this data and some assumptions the height, surface area and volume of the landmass of the North Side will be calculated. The students will then examine topographical maps of the region to get verification of their work.

Size of the Earth Project

Days 13,14 and 15

Once this is completed to whatever extent the teacher wishes or has time for the culminating project will begin. The project involves the question: how big is the Earth ?

The session begins with the story of Eratosthenes(300 B.C.), the Greek librarian that calculated the circumference of the Earth. The students are told the story and given a mock version of the mathematical problem Eratosthenes had to solve. The story is as follows: Eratosthenes knew of a certain well, which at noon during the equinox had no shadow. This is to say that the sun's rays went directly to the bottom of the well. Using professional pacers he found that 500 miles north of this place the sun's rays formed a 7.2º angle with a stake perpendicular to the ground. Assuming that the suns rays are parallel and that the earth is in the shape of a sphere he was able to make the following calculation:

sun's rays

500 miles

7.2 sun's rays

well

Of course the students should solve this problem themselves given Eratosthenes' data. This is a simplification of the problem to get the idea across. Note that the students will have to deduce that the central angle is 7.2º because of the alternate interior angles of the "parallel" sunrays. Incidentally Eratosthenes' actual calculation was only 80 kilometers off of our modern value.

Point out the amazement in the fact that someone was able to figure this out with the knowledge and technology available at the time (≈300 B.C.).

Once the students get the general idea behind this concept they are better prepared to use it in their computation using their own measurements. In collaboration with Erindale High School in Toronto, approximately 225 miles North of Pittsburgh, the students will collect data on the length of shadows of stakes that are perpendicular to the ground. The students in Toronto will be collecting their data at the same time of day. The student is to find the angle that the sun's rays make with the stake. This will be done with trigonometry, as is done below.

Sun's

Rays

Stake

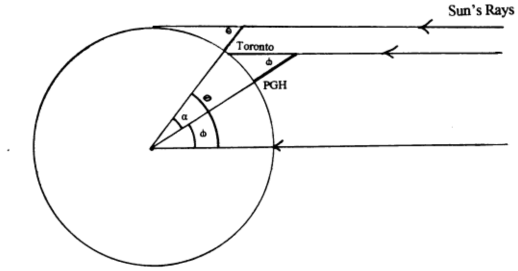
(18 cm)

Shadow

(20.6 cm)

Incidentally the method of placing the stick so that it is perpendicular to the ground should be discussed. What are some good methods at making sure that the stake is perpendicular to the ground?

Using average angles formed from both sets the circumference of the earth can be calculated using the method below. It is assumed that the angle formed in Pittsburgh ( ) is 48.85º and in Toronto (θ) is 52.1º



Once this is accomplished the radius, diameter, surface area and volume of the earth are easily computed. The students are going to need the radius of the earth in order to do the next activity. Since C = 2pr

2pr 24,923.08

r

r 3966.63 miles

In our calculations in the next session we will use a radius of 3960 miles

Where on the Earth are We?

Days 16,17,18

Now that the size of the Earth is "known" the next question to answer is where on Earth are we. The students will answer this question by producing the x,y and z coordinates of Pittsburgh, given the information that it is 79.95º west longitude and 40.45º north latitude.

To develop this the students will come up with general formulas using a sphere of radius r. The problems are hard to visualize and draw. To help, the student should be given plastic blow up globes that can be written on with non-permanent markers. To begin they should be given problems such as the one below.

Find the x,y,z coordinates of a point that is 30º west longitude and 60º north latitude on a sphere of radius r. To solve this the students should be lead to first deal with the circle on the x,y plane(the equator) below:

Next the students should deal with the 60º-north latitude:

So the final coordinates are:

x = r cos 30ºcos60º

y = r sin 30ºcos 60º

z = r sin 60º

Using θ as degrees east-west longitude and ϕ as degrees north-south latitude and r as the radius the general formulas are:

$x = r \cos\theta \cos\phi$

$y = r \sin\theta \cos\phi$

$z = r \sin\phi$

Students should experiment with the plastic globes or other spheres to see if they believe this. Earlier in the year when the students are learning about p they do an activity where they are given circular objects and asked to measure the diameter as well as the circumference, using string and a ruler. They are then asked to divide the circumference by the diameter. After collecting the entire classes measurements and calculations on a chart they are to discover that what they should get is the number p itself. Using a similar method a sphere's diameter can be estimated. The student can measure its circumference by string and using: $\pi d = C$ find the

diameter, d = and hence radius of the sphere. The students then have more information about whether they believe these equations by experimenting with points on a real sphere or globe. Good questions to ask to initiate the students to begin working with the plastic globes is to ask them to point out points, such as the 30ºW and 60º N one.

After the students have experimented and believe that the equations work they are to find the x,y,z coordinates of Pittsburgh giving its longitude and latitude. (79.95º W, 40.45º N)

This should be practiced with other cities, the coordinates of which by longitude and latitude can be found in an Almanac. Note that the coordinates in an Almanac will be in degrees and minutes. The students can be given the data as such or it can be converted to decimals in degrees, as the data is here. The students practice of these problems of finding the x,y,z coordinates of other locations on earth can be used to develop new questions.

What does it really mean to us to be located at this point on the Earth? There are a multitude of interesting and meaningful questions that can be asked: How far does the rotation of the Earth take us in Pittsburgh during one day? How fast is it spinning us? Given two cities coordinates what is the distance between them? In particular given that Pittsburgh is located at 79.95º west longitude and 40.45º north latitude and that Toronto is located at 79.4º west longitude and 43.66º north latitude, what is the distance from Pittsburgh to Toronto on the Earth? What is the distance through the Earth (the shortest distance)? How could we ever physically measure such a distance?

Once again the students responses to such questions have coherent math work (steps) and are explained in writing. Examples of solutions can be reached in appendix.

Boat in Swimming Pool Problem

Any day

The following is a problem that can be incorporated into the unit at any time, such as a cloudy day during Thales' method. There are more problems that can be used before, during or after the unit and can be reached by following directions in the Appendix. This problem was chosen here because it lends itself so well to the practice of taking measurements, performing calculations and then physically measuring to verify the result. It exemplifies the process of learning suggested in other activities in the unit and it is interesting.

The problem is a classic engineering question and one that Archimedes may have been interested in. You are in a boat in a swimming pool with a 50 lb lead block in your hand. You throw the block into the water. Does the water level go up, go down or remain the same? Why?

The answer to the question is that the water level goes down. The amount of water that is displaced with a 50 lb. force (50 lbs of water) is more than the volume of the lead block, which is much more dense than water.

A classroom activity can be done, which not only verifies the answer to the question, but that of a calculation of how much it actually will go down. Use an aquarium, a smaller plastic container and twenty 100-gram cylindrical weights. An example of the problem is shown below:

The aquarium is 36 cm long and 21 cm wide. The height (h) is irrelevant to the problem.

h

21 cm

36 cm

The cylindrical weights have a diameter of 2.5 cm

and height of 3 cm.

2.5 cm

3 cm

Twenty 100-gram weights have a mass of 2000 grams so they displace 2000 grams (or 2000 cm^3) of water. Once they are removed and placed into the water, the 2000 grams of water is no longer displaced, but only the volume of the cylinders (93.75π cm^2) is displaced. So the water level will go down

$2000 - 93.75\pi \approx 1705.48$ cm^3.

The aquarium is a rectangular prism, with V = LWH. We need to find the height of a prism with the dimensions of the aquarium, so that its volume is 1705.48 cm^2.

LWH = V

(36)(21)H = 1705.48

$H \approx 2.25$ cm

So the water level will go down 2.25 cm. Next it should actually be done by marking the two levels on the aquarium with non-permanent marker and then measuring the difference. When this activity is done in class, students should be the ones taking all of the measurements as well as doing the calculations. As was noted before, the teacher should have done the experiment themselves before the students, but should allow them to use their own measurements and be prepared to revise their own calculations.

This is the end of the activity section of the unit. It is highly suggested that the teacher read The Suggested Readings in the Annotated Bibliography below and follow the instructions in the appendix section to receive the solutions to problems mentioned in the unit as well as a sample of good standards type problems to be used at anytime.

Annotated Bibliography

Resources

Books

Amir D. Aczel: *Fermat's Last Theorem,* NY: Dell Publishing, 1996

Amir D. Aczel: *The Mystery of the Aleph: Mathematics, the Kabbalah, and the search for Infinity,* NY: Four Walls Eight Windows, 2000

A.K. Dewdney: *A Mathematical Mystery Tour,* US: John Wiley & Sons, Inc., 1999

Mark Lehner: *The Complete Pyramids, Solving the Ancient Mysteries,* London: Thames and Hudson Ltd., 1997

L. Sprague de Camp: *The Ancient Engineers,* US: Doubleday & Co., 1960

Peter Thompkins: *Secrets of the Great Pyramid,* NY: HarperCollins, 1995

Luetta Reimer, Wilbert Reimer: *Mathematicians are People, Too*, Palo Alto, Ca: Dale Seymour Publications, 1990

E.T. Bell: *Men of Mathematics,* NY: Simon & Schuster, Inc., 1937

Halliday,Resnick: *Fundamentals of Physics*, US: John Wiley & Sons, Inc., 1988

Michael Serra: *Discovering Geometry*, US: Key Curriculum Press, 1989

David Macaulay: *Pyramid*, US: Houghton Mifflin Company, 1975

Piotr O. Scholz: *Ancient Egypt: An Illustrated Historical Overview*, US: Barron's Educational Series, 1997

Time Almanac 2001, US: Time Inc., 2000

Suggested Reading List (for Teachers)

Amir D. Aczel: *Fermat's Last Theorem,* NY: Dell Publishing, 1996

Amir D. Aczel: *The Mystery of the Aleph: Mathematics, the Kabbalah, and the search for Infinity,* NY: Four Walls Eight Windows, 2000

Luetta Reimer, Wilbert Reimer: *Mathematicians are People, Too*, Palo Alto, Ca: Dale Seymour Publications, 1990

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A.K. Dewdney: *A Mathematical Mystery Tour,* US: John Wiley & Sons, Inc., 1999

E.T. Bell: *Men of Mathematics,* NY: Simon & Schuster, Inc., 1937

Appendices

The appendices of this unit have been removed because of page constraints. It consists of two parts. The first are well worked out solutions to all problems mentioned in the unit. The second is sample standards problems. Anyone can receive these by sending an e-mail request toprenne1@pghboe.net. I will send them back as an attachment, which is at least 20 pages long, anytime after 9/1/01.

If you are interested in the unit it is highly suggested that you do this. Many of the problems have multiple solutions and some of these are addressed in an easy to follow format with many diagrams. The standards problems are excellent and would interest anyone teaching the subject and can be used anytime.

Standards

All seven of the math standards used by my school district, Pittsburgh Public Schools, are addressed in the unit and are listed below. The use of these are demonstrated throughout the unit. In particular it incorporates Standards 2,4,5 and 6 by the nature of the questions asked and the answers and work expected. Students are to take measurements, perform calculations and come up with theoretical predictions, which are later verified by physically testing. They are expected to not only solve the problems but also thoroughly explain them in writing. They often do this by collecting several data points and sometimes use collections of data from others. The standards are:

1. All students use numbers, number systems, and equivalent forms (including numbers, words, objects and graphics) to represent theoretical and practical situations.

2. All students compute, measure, and estimate to solve theoretical and practical problems, using appropriate tools, including modern technology, such as calculators and computers.

3. All students apply the concepts of patterns, functions and relations to solve theoretical and practical problems.

4. All students formulate and solve problems and communicate the mathematical processes used and the reasons for using them.

5. All students understand and apply basic concepts of algebra, geometry, probability and statistics to solve theoretical and practical problems.

6. All students evaluate, infer and draw appropriate conclusions from charts, tables and graphs, showing the relationship between data and real world situations.

7. All students make decisions and predictions based upon the collection, organization, analysis and interpretation of statistical data and the application of probability.