The Algebra of Motion

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Overview

The purpose of this unit is to give students an experience to show that algebra can represent the motion of objects. The students will accomplish this by collecting and algebraically analyzing data from actual moving objects. The CBR/CBL motion detector machines make this collection extremely accurate. The students will begin with such data collecting to introduce the physics equations of motion and show how algebra nearly perfectly represents the object's placement through time.

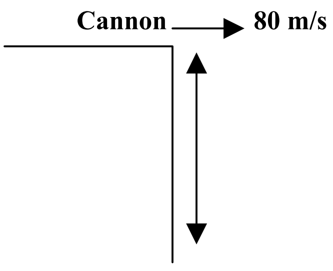
Soon after students will perform motion experiments modeled after those of Galileo. To make the study more interesting and to introduce the history of the subject the students will learn of Galileo(1564-1642) and his actions, which were considered rebellious toward the church and its Aristotelian views. Mock arguments will be conducted by actors(students) playing his three characters in his *Dialogues*, Salviati, Sagredo and Simplicio.

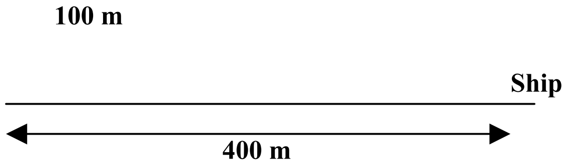
The algebraic work of this unit is to create curves of best fit including data obtained from real moving objects and recorded on graphs and tables. The culminating activity is to create a curve of best fit to a graph and table values of the vertical motion of a bouncing ball. In doing such work the students will discover the acceleration rate of freely falling objects on earth. The students' rigorous work on the data will give them further practice of algebra and is a perfect review to wrap up the course.

Rationale

Students solve many supposedly real problems in math and physics classes. For example here is a typical problem given in an introductory physics course:

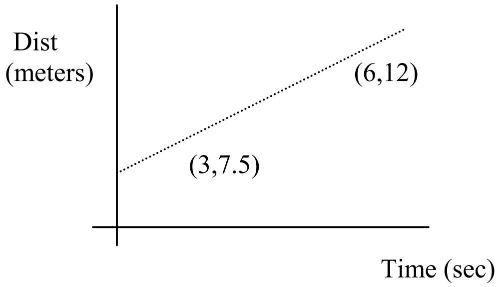
A cannon is to be fired horizontally off a cliff that is 100 meters high. The ship is 400 meters away. If the initial velocity of the cannon ball is 80 m/s does the cannonball have any chance of hitting the ship?





Students solve these problems, but do they really believe that the outcomes are real. Since this is intended for an algebra class it should be asked, do students believe that algebra can represent something that actually exists and occurs? Usually they do not.

The purpose of this unit is to give the students experience where they will see that algebra can and does represent the motion of real objects. The unit will begin with the motion of the students themselves, walking at constant velocities in front of the CBR/CBL motion sensor. For example on Day 1 a student will walk away from the motion sensor at a constant speed. This will produce a straight line on the graph with a positive slope. By tracing the graph, different points on the line can be generated. Below is an example of such a graph:



The students will find the line of best fit of this data by computing the slope and y-intercept, using two points. It can be seen that the equation of this line is $y = 1.5x + 3$. The difference between this algebra and that of the past is that the students can actually see first hand that it is true and correct without a doubt.

Many of the classrooms today have 1ft by 1ft square tiles on the floor. Using these or a tape measure the students will actually see that this walker was 3 ft (the y-intercept) from the motion sensor when it began to graph them and that the person covered 1 ½ ft.(the slope) each second. Students will see and therefore believe that the graph, table and equation of this person's walk really does represent the time and distance covered during the walk.

Next the students will do motion experiments, modeled after those of Galileo. To make it more interesting and introduce history the students will learn of Galileo. In 1632 Galileo published his greatest work, *Dialogue concerning the two Chief Systems of the World, the Ptolemaic and the Copernican*. He was shortly after sentenced to detention in his home in Florence (house arrest) for the remainder of his life. His writings were dialogues in that they were discussions between three men, Salviati, Sagredo and

Simplicio. Salviati and Sagredo represent Galileo's and the Copernician views and Simplicio(literally translated as simpleton) the Church's Aristotelian views.

The students will be told of these three actors and will hold two mock discussions with three students each as one of these characters. I have done similar activities in my classes before and the students love it. Many will remember this experience more than any other.

While the people of Europe were fooled about motion, believing in the Aristotelian views for over a thousand years, it will be good for the students to experience how some of the scientists of the day studied it. It is much more difficult to see and believe these propositions about motion, using the crude methods of 16th and 17th century scientists, compared with modern technology. Recall that later the students will be using more sophisticated methods.

The unit ends with the students using the CBR/CBL motion machines to collect data on the vertical motion of a bouncing ball. After recording "good" bounce data in the form of graphs and tables the students will do rigorous algebraic work on this data to produce a curve of best fit. In the process the students will learn the acceleration rate on earth. As was noted, often students do problems involving concepts such as the earth's acceleration rate, but do they really believe the results?

The equations of this accelerated motion are quadratic or of degree two. Algebra 1 classes in The Pittsburgh City School District usually only touch upon quadratics. In addition the algebraic work involved is quite sophisticated relative to the average Algebra 1 student. Therefore the unit may be more appropriate for Algebra 2, Elementary Functions or a good or scholars Algebra 1 class.

An important note to make to those physics teachers reading this unit is that it is intended for algebra. Therefore much of the notation of physics is omitted and that of algebra is used.

A note to the math teachers reading the unit: You may wonder why more effective methods at finding a curve of best fit, such as the method of least squares or matrices, are not being used in the unit. The method of least squares is not used because in order to understand the process knowledge of calculus is required. Matrices are not used because most algebra classes in high school do not cover the topic. There are books and instructions in the bibliography as to where these methods can be learned, if the teacher wishes to do so. This will be briefly mentioned again when examples of suggested methods the students might do are presented in the Activities section of the unit.

A final note to make is that this unit is presented with the assumption that the teacher is familiar with using the CBR/CBL motion detector machines. For those that are not there is information to refer to in the Bibliography and Appendix of the unit.

Objectives

There are two objectives of this unit. The first of which is to demonstrate that algebra can represent real entities and events, being the motion of balls with time(x) and position(y). By physically collecting data on a ball's position and time, modeling this with tables and graphs and then creating equations to represent the data, the student will see that algebra really does represent the ball's motion.

The second is that the student will be expected to do rigorous algebra work with their collected data. They will learn much in the process and will gain experience in algebra. The unit is to be done at the end of the course and can be thought of as a review of what has been learned throughout the year.

Strategies

By what the students have learned throughout the year, they will easily be able to create tables and graphs of their data. The hard part will be in creating equations that represent this data. One of the class periods during the unit will be an activity which will remind the students of concepts and techniques they have learned that will enable them to algebraically calculate the a curve for a quadratic given the values of points. Before students are to take on this task they will learn methods at computing the equation of a quadratic given three of its points, by playing a game I call The Quadratic Equation Game.

.Another day early in the unit will be of lecture form(the only one) in which the students will be introduced to physic's equations dealing with motion, omitting some of the physics notation however. This will enable them to more fully understand the meaning behind the equations they create later to represent their collected data on motion.

The unit addresses all seven math standards adopted by the Pittsburgh City School District and these are listed in the Standards at the end of the unit.

Classroom Activities

Day 1

The first activity is to create excitement and to introduce algebra as a model of motion. A CBR or CBL motion detector will be set up to graph motion. The graph of which is displayed on the overhead for all to see. A volunteer will be asked to walk away from the motion sensor at a constant speed. A partial list of data from such a walk is listed below.

Time (sec)	Dist (meters)

0	1.668	**3.0**	**3.416**
.2	1.777	4.0	4.025
.4	1.925	**5.0**	**4.631**
.6	2.026	6.0	5.326
.8	2.153	7.0	5.869
1.0	2.260	8.0	6.339
2.0	2.826	9.0	6.737

Note that the graph would not form a perfect line. Ask the students why? The walker changed speed slightly. Perhaps the walker slowed down because he or she was near the wall. Give the students the idea of a line of best fit. This is a line which comes as close as possible to as many points as possible. Using the two highlighted points the students can compute a line of fit as below:

$(5, 4.631) \qquad (3, 3.416) \qquad\qquad y = mx + b$

$$m = \frac{4.631 - 3.416}{5 - 3} \qquad\qquad 4.631 = .608(5) + b$$

$$m = .608 \qquad\qquad b = 1.591$$

So the computed equation between these two points is y = .608x + 1.591. Place this equation on the calculator graph on the overhead so that the students can see how good a fit it is. If it is not good, modify the equation by picking better points or guessing and checking until everyone is satisfied that we have found a line of best fit.

Note that our method here is ambiguous and subjective. A more detailed numerical analysis can be done to find an unambiguous line of best fit, such as the method of least squares, however, as was mentioned, to understand the method a knowledge of calculus is required. It has been decided that although the students would be able to perform such a task, they would gain little through the experience. The subjectivity of their work should be mentioned to them and those teachers and students interested in a more objective method, which uses more or all data points, should be referred to the two books mentioned on the subject in the bibliography.

Ask the students what the 1.591 means in terms of the walk. The expected answer is that the motion sensor began to graph the walker when they were 1.591 meters from it. It is the walkers starting point. With a tape measure or meter stick it can be verified that the walker was 1.6 meters away from it when they began. Incidentally the CBR/CBL machines are programmed to allow for data to be collected in meters or feet. During this activity feet may be a good choice, since most tape measures are in feet and most classrooms have 1 ft. by 1 ft. square tiles on the floor that can be used to measure. Both the metric system and the British system should be used in these experiments and the students should be able to convert between the two by remembering that 1 inch = 2.54 cm. Also note that the actual starting point of the walker was 1.669 meters. Discuss these problems that arise when computing lines of best fit with your students.

Next ask the students what the .608 before the x means in terms of the walk. It is of course the walker's speed or velocity away from the motion sensor. By a careful trace of the x and y values on the graph or by examining the table it can be seen that this walker did indeed increase their distance by approximately .6 meters each second. The tape measure, meter stick or floor tiles can be used to further verify that this walker really did walk .6 m/s.

Next have another volunteer walk at a constant speed by starting away from the motion sensor and walking toward it. This of course will produce a line graph with negative slope and the y-intercept will be much larger than before. The table below is a partial list of data points from such a walk with distance from the motion sensor this time in feet.

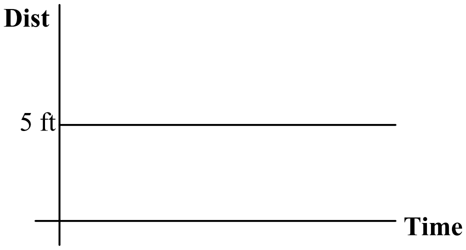
Time (sec)	Dist (feet)
0	19.665
1	16.701
2	13.506
3	10.375
4	7.352
5	3.93

Before the students compute an equation for this line, as was done before, ask them to guess at what they think the equation might be. Place a student's guess onto the graph on the overhead. Invariably many will forget to make the slope negative and the line will go the wrong way.

Have the students compute an equation for the line by choosing two "good" points. A computation using two points from above is $y = -3.18x + 20.029$.

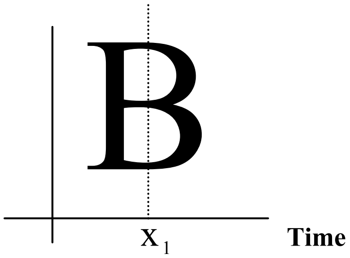
Discuss the negative slope with the students. Make a quick sketch of the table above on the board. (Note the y-intercept of our fit line is a little off from the walker's actual starting point again.) The y values (distance from sensor) are decreasing by approximately 3 feet each second, therefore the slope is negative. Introduce the students to the concept of velocity in physics. A moving object not only has a speed but a direction. In physics we choose one of the directions as positive and the resultant opposite direction as negative.

There are some other interesting activities that can be done during this class, which will further the students' understanding of graphing motion. Ask for a volunteer to walk in such a way to produce the graph below. Place a drawing of the graph on the overhead or sketch it on the board.



The student of course should stand 5 ft. from the motion sensor and not move at all. Time is going by and the "walker" is neither increasing nor decreasing their distance from the sensor.

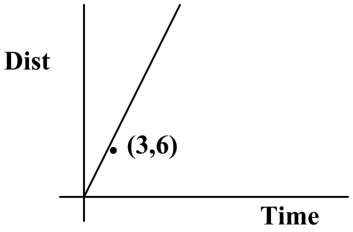
Next have volunteers come up and walk in such ways as to produce capital letters of the alphabet. A capital M is easy, but what about a B? In order to produce a capital B the walker would have to go back in time and be at 3 different places at the same time (x_1). This is of course impossible, but the thought experiment will increase the students' knowledge of graphs and functions.



Day 2

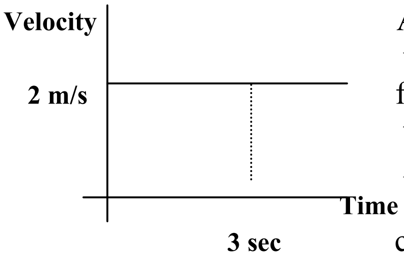
Today will be a teacher lecture to introduce equations of motion. The most important of which will be that $\text{Distance} = \frac{1}{2}(\text{Acceleration})(\text{Time})^2$. In order for the students to understand their equations of the curves representing the ball rolling and bouncing data later, they will have to know this.

Have the students sketch a graph of a person walking away from the motion sensor at 2 ft/sec.



Ask them how they could use the graph to find the walkers' distance from the sensor at any particular point in time, say 3 seconds from the beginning. It is simply the y coordinate of the point where x is 3.

Next have the students sketch the graph of this same walk, but this time have it be the person's velocity versus time. Let the students guess at what the graph may look like. This will keep them interested and actively engaged. The graph would look like the one below:

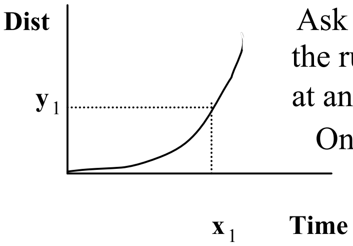


Ask them how we can tell how far the walker traveled by using this graph? How far in 3 seconds again? It is of course the area of the rectangle, $A = (2)(3) = 6$ ft. Algebra students are familiar with the equation, distance = rate x time. By comparing to this show them why $D = 6$ vt.

As was noted this is an algebra course, therefore the notation common to algebra will be used. So the students might use $y = vx$, where y = distance, v = velocity and x = time. In this example, $y = 2x$.

Introduce the concept of acceleration by recalling the walker who walked in such a way that the graph was curved. Ask the students what about the walk made the graph curved? The walker either increased or decreased in speed of course.

Assume that the walker's acceleration is constant. Explain to the students that this means that the walker's velocity increases at a constant rate. For this example, assume that the walker's speed (it would actually have to be a runner in this case) went from 0 to 12 ft/sec in 5 seconds. Show a distance/time graph sketch of this on the board.



Ask the students how they would find the runner's distance from the starting point at any number of seconds between 1 and 5? Once again this would be the y coordinate (y_1) at the point in time (x_1).

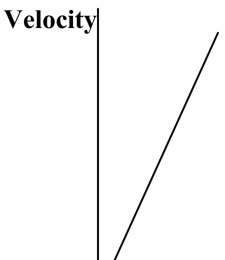
Since the runner's acceleration is constant and it starts at 0, the average acceleration between the starting point and ending point would equal that of the actual acceleration. Note once again that this is an algebra course and the physics notation is omitted.

$$\text{Acceleration} = \text{Av Accel} = \frac{12-0}{5-0} = \frac{12\text{ ft/s}}{5\text{ s}} = 2.4\text{ ft/s}^2$$

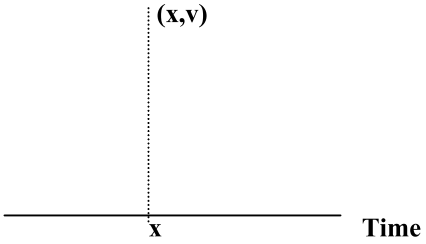
Discuss the units of acceleration with the class, ft/s^2.

Tell the class that the velocity of the runner can be given at any point of time with the equation $v = at$. Verify this with the following table and graph:

Time	Velocity



(sec)	(ft/sec)
0	0
1	2.4
2	4.8
3	7.2
4	9.6
5	12.0



Ask the students how they can use this graph to find the runner's distance traveled , given any number of seconds (x)? As with the velocity/ time graph the distance is the area under the curve, which is a triangle. Recall that v = at.

$$\text{Distance} = \frac{1}{2}\text{bh} = \frac{1}{2}(\text{t})(\text{v}) = \frac{1}{2}(\text{t})(\text{at}) = \frac{1}{2}\text{at}^2$$

$$D = \frac{1}{2}at^2$$

Ask the students what type of equation this is? It is quadratic. Tell them that an object dropped to the earth will accelerate at a constant rate and that the rest of the work they will be doing in the unit will be on this type of acceleration. At the end of the unit they will calculate and discover this acceleration rate on earth.

Day 3

Before this class begins, chose who will be Galileo's three actors in the dialogue concerning objects falling to the earth. It is a good idea to choose a good student, someone everyone knows is a good student, as Simplicio, since he is the one that is to be the fool, or at least wrong. This way no one is humiliated. Also it will require more imagination to come up with arguments that most of us already know to be false. Before the class talk with the three actors and fill them in on each character's beliefs. The arguments should be impromptu, however, the actors should prepare. The teacher could perhaps write out the beginnings of the arguments, as in the examples below, and give these to the actors beforehand.

Tell the class the scenario before the dialogue. Simplicio believes the Aristotelian notion that "heavier" objects will increase in speed and fall faster to the earth than "lighter" ones. Explain to the class that use of the words, "heavier" and "lighter", were the words the scientists of Galileo's day used, but what they sometimes meant to convey by them was the concept of density.

Salviati and Sagredo, however believed that all objects fall to the earth at the same acceleration rate. They note, however, that some "lighter" objects are more heavily influenced by air and hence fall more slowly. Without the air all objects would fall at the same rate. A beginning sample argument could be as follows:

Simplicio- Common sense will tell you that a heavier object will fall to the earth faster than a lighter one. Hold two such objects in your hand. Can you not feel that the heavier one is pulling to the earth much more strongly than the lighter one. Hence it will fall at a faster rate. As a matter of fact it will fall proportional to its weight. An object, which is twice as heavy, will fall twice as fast.

Sagredo-To the experienced this would seem to be the case. Imagine Simplicio three identical objects falling side by side. Would you not agree that they would fall at the same rate?

Simplicio- Undeniably.

Sagredo- What if two of them were really close to each other? As a matter of fact they are so close that they are touching one another. They would still fall at the same rate, but since they were touching one another they could be thought of as one.

Salviati (Holding three pieces of paper, one in one hand and two in the other, crumples what he has in each hand into two balls)-According to your logic Simpilcio this ball of paper weighing twice as much as this ball should travel to the ground at twice the speed. (He drops the two pieces and they hit the ground at roughly the same time) As we have just seen, that is not the case.

While the acting out in class may not go all that smoothly, it is fun and will be remembered by the students. It can be made even more exciting by having the actors wear costumes. The next dialogue that will take place in the class will be purely made up for it will use information and knowledge learned in the unit and aided by technology.

The rest of this class will be spent dropping objects of varying masses. If it is appropriate objects can be dropped out the window. It is hoped that a vacuum machine can be obtained for the ending of this class, where the students can see that a feather will fall just as quickly as a rock in the absence of air.

Day 4

Tell the students that Galileo preached that missiles and projectiles on earth followed the path of a parabola. This is of course those freely moving, as a cannonball or bullet would be. Throw a chalk eraser across the front of the room and tell them that the path of its center (of mass) follows a parabola. Many will ask, “what about a bullet?” Propose the classic physics question: If a man fires a gun perfectly level with the ground on a perfectly flat surface and drops another bullet from exactly the same height at precisely the instant the gun is fired , which bullet would hit the ground first? The answer of course is that the bullets hit the ground at exactly the same time, because the vertical distance each bullet has to fall is the same.

Propose the next question to the students: If a man fires a gun perfectly horizontal with the ground at a height of 1.6 meters and the bullet's initial velocity is 600 meters/sec, how far will the bullet travel before it hits the ground?

Tell the students that after the experiments and work they will do over the next few days they will be able to answer this question.

Day 5

To prepare the students for creating curves of best fits to quadratic equations by using given points on the graph or from a table, the following game will be played. It is called the quadratic equation game.

Have the general form of a quadratic equation on the board, $y = ax^2 + bx + c$. Tell the students that they are playing against you. They are to make up a quadratic equation. You are going to then ask them three questions. What does y equal when x is –1, 0 and 1? After the class gives the answer to the last question, you have 3 minutes to come up with the equation and write it down on the board.

Although inappropriate I make the game more exciting by telling them that if I do not come up with the equation, everyone in the class gets one dollar. I also show them the one dollar bills I have to cover myself if I lose. I also tell them that if they give me the wrong y values, which they should not because they have the whole class to check them, then they all owe me one dollar each. I am of course joking about them giving me money, I however do intend on paying up if I lose. I have played the game for 6 years and I did lose and pay up to the whole class once. I also tell the class that if they want to beat me and make it a challenge they should come up with difficult coefficients, such as in the equation,

$$y = \frac{2}{3}x^2 + \sqrt{5}\,x + 4\pi .$$

I stand outside the room while a student writes the equation on the board. Once everyone has written the equation down and it is erased from the board I enter the room and ask my three questions.

The class is always amazed that I can come up with the equation. I show them how it was done with a simple example. First of all, the three questions amount to three points on the graph of the quadratic. Also note that when $x = -1$ the general form of the equation becomes $y = a - b + c$. For example:

$x = -1$	$x = 0$	$x = 1$
$y = 4$	$y = 5$	$y = 10$

Using either substitution or addition the system can be solved. The following is

$(-1,4)$ $(0,5)$ $(1,10)$ completed with substitution.

$a - b + c = 4$ $c = 5$ $a + b + c = 10$

$a - b + 5 = 4$ $a + b + 5 = 10$

(1) $a - b = -1$

(2) $a + b = 5$

(1) $a - b = -1$

$a = b - 1$

(2) $b - 1 + b = 5$

$2b = 6$

$b = 3$

$a = b - 1$

$a = 2$

So the equation is $y = 2x^2 + 3x + 5$

The students should be given problems to practice as homework. This gives them the necessary experience they need to create equations in the rolling and bouncing ball experiments that will take place during the rest of the unit.

Assignment- Find the quadratic equations through the following sets of points. (The solutions have been provided for the teacher)

a) $(-1,4)$, $(0,5)$, $(1,8)$ ANS. $y = x^2 + 2x + 5$

b) $(-1,4)$, $(0,6)$, $(1,7)$ ANS. $y = 3x^2 - 2x + 6$

c) $(-1,7)$, $(0,10)$, $(1,17)$ ANS. $y = 2x^2 + 5x + 10$

*d) $(0,6)$, $(1,10)$, $(2,16)$ ANS. $y = x^2 + 3x + 6$

*e) $(0,1)$, $(1,2)$, $(2,9)$ ANS. $y = 3x^2 - 2x + 1$

*f) $(1,9)$, $(2,12)$, $(3,19)$ ANS $y = 2x^2 - 3x + 10$

Note that d),e) and f) do not follow the standard procedure. The student is going to really have to think to solve these. This represents the type of work the students are going to have to do if they are going to find a curve of best fit to represent their data in the rest of the unit. Below is the solution to f):

$x = 1$ $x = 2$ $x = 3$

$y = 9$ $y = 12$ $y = 19$

(1) $a + b + c = 9$

(2) $4a + 2b + c = 12$

(3) $9a + 3b + c = 19$

(3) $9a + 3b + c = 19$

- (1) $\underline{-a - b - c = -9}$

- (1) $-a - b - c = -9$

(2) $\underline{4a + 2b + c = 12}$

(4) $3a + b = 3$

-2(4) $-6a - 2b = -6$

(5) $\underline{8a + 2b = 10}$

(5) $8a + 2b = 10$

$2a = 4$
$a = 2$

(4) $3(2) + b = 3$
$b = -3$

(1) $2 + (-3) + c = 9$
$c = 10$

So the quadratic equation is

$$y = 2x^2 - 3x + 10$$

Days 5,6,7

Explain to the students how Galileo studied the acceleration of objects to the earth by slowing this motion down and rolling balls down ramps. While he used heart beats as a measure of time, the students will use digital stop watches with time to the hundredth of a second. Model for the students what they are about to do by rolling several balls down ramps. By using several students together as partners, they will collect data on the ball's position at different points in time. It seems that the students have a choice about making the time data easy to work with or making the distance data easy to work with. For example one student can release the ball while another calls off the seconds, "1 sec, 2 sec, 3 sec" and so on. While this is going on at least 5 other students will record the ball's distance at their designated time. This makes the time data easy to work with. The other option is to have each of the data collectors with stop watches stopping them at designated distances. It is suggested that long ramps (15 to 20 feet) be used and that they are inclined only slightly (12° or less). This is because the motion is so fast that it makes data collecting too difficult with smaller boards of steeper inclinations. Because of this the classroom may not be large enough for this activity and it may have to be done outside or in a larger room.

After each student has collected good data from at least 5 rolls, they are to make graphs and tables of this data. Once completed, they are to create curves of best fit to represent the data. Below is an example of data collected and a sample of work at creating a curve of best fit. Note that 7 people were involved in collecting this data. The first method mentioned above was used and 5 people recorded the distances. The board was inclined at 12° and was 20 feet long. It was collected by volunteers in my scholars' geometry class after school and there were many trials before such good data were collected. The distances have been converted to decimal form (rounded to the hundredth place)

Time (sec)	Distance (feet)	
0	0	
1	1.42	(1ft 5 in.)
2	3.25	(3 ft. 3 in)
3	6.88	(6ft 10½in.)
4	11.92	(11 ft. 11 in.)
5	18.17	(18 ft. 2 in.)

Choosing two points and assuming that c = 0 (at first) which of course makes sense for the ball was 0 ft. in 0 sec, we come up with,

(2, 3.25) (3,6.88)

(1) $4a + 2b = 3.25$ (2) $9a + 3b= 6.88$

(1) $4a + 2b = 3.25$
(2) $9a + 3b = 6.88$

-3 (1) $-12a - 6b = -9.75$
2 (2) $\underline{18a + 6b = 13.75}$

$6a = 4.01$
$a = .668$

(1) $4(.668) + 2b = 3.25$
$2b = .578$
$b = .289$

Our equation so far is

$y = .67x^2 + .29x$

Although it makes little sense to find c, in the spirit of finding a curve of best fit, lets do so with the point (4,11.92).

$16a + 4b + c = 11.92$
$16(.67)+4(.29)+c = 11.92$
$c = .04$

As was expected c is almost 0. Therefore our equation of best fit is $y = .67x^2 + .29x + .04$. Let us check our equation with the 5th point by plugging 5 in for x. If the equation is good we should obtain 18.17 for y.

$y = .67(5)^2 + .29(5) + .04$
$y = 18.24$
Not too bad.

While this equation should be able to be formed with just an **a** coefficient, at least a **b** coefficient is needed to find a curve of best fit. Students and teachers who do not believe this conjecture can try to come up with a better curve of the form $y = ax^2$. It cannot be done. Incidentally using the method of least squares with all six points leads to

$$y = .66x^2 + .3x + .14,$$

which is quite close to ours, using only two points.

Incidentally collecting data using the other method suggested, where the distances are easier to work with leads to data of the type:

Time (sec)	Dist (feet)
0	0
1.47	2
2.20	4
2.77	6
3.65	10
4.90	18

The algebraic work involved with such data is much more difficult to work with, but the students should be required to do so.

Day 9

Data on the rolling balls will be collected using the more sophisticated and accurate data collecting method of the CBR/CBL motion sensor machines. A good program to use for collecting this data is called MoRamp. Instructions on how to obtain this program, as well as the program itself, are provided in the appendix of this unit. Data can be collected using steeper inclines than when it was collected manually. For example the data collected below represents that of a 24 ft. ramp inclined at 20°.

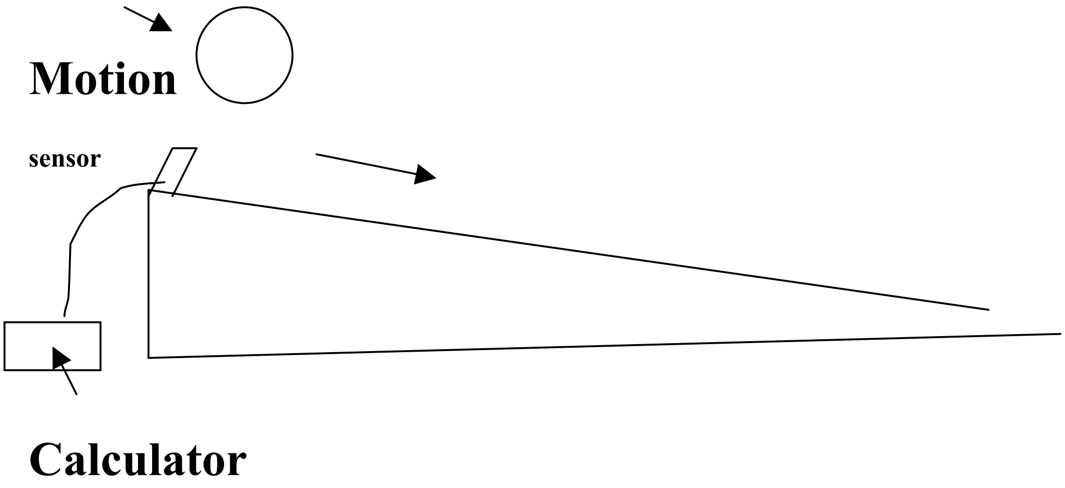
Time (sec)	Dist (feet)
0	1.8
.25	3.11
.5	4.73
.75	6.66
1	8.9
1.25	11.45
1.5	14.31
1.75	17.48
2	20.96

The students are once again to make tables and graphs of their data. Then they are to come up with a curve of best fit, which will be much better because the data is more accurate. The curve of the data above certainly has b and c coefficients, because the motion sensor has not begun to graph this ball until it is already in motion. A good curve for this data is

$$y = 2.48x^2 + 4.62x + 1.8.$$

Ask the students what the **a** and **c** coefficients mean in this equation?

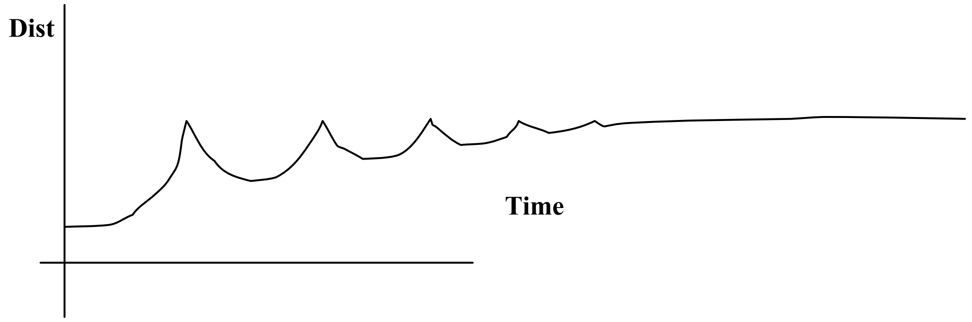
In order for the class to collect this data at least six motion sensors and graphics calculators should be available. The students should tape the motion sensor to the top of the ramp. One student should set to the program and press enter while another releases the ball.



Day 10

Today is the culminating activity of the unit. It is the day when the students will collect data that with their curves of fit, they will find the acceleration rate of earth. This will be done, by creating a curve of best fit for data involving vertical motion. What they are going to do is hold the motion sensor up in the air and release a bouncy(elastic) ball under it. A super pumped up basketball works well.

An interesting activity to do first is to model what is going to be done first to the students and then to ask them if they can make a sketch of what the graph might look like. Recall that the ball is first held under the motion sensor at an elevated height. Once enter is pressed the program begins to graph the ball's motion and then it is released and allowed to bounce under the motion sensor. Incidentally, several trials are often necessary, for if the ball bounces away from the motion sensor rather than under it the data are lost. Here is what a collection of good data will look like.



The motion sensor program is graphing how far the ball is from it. Therefore before dropping the ball it is close to the sensor. After dropping the ball it accelerates until it hits the floor and reverses direction until it comes to a stop, which is a distance not as close as that it was dropped from. It continues this process until it eventually comes to a stop.

The students are to choose a "nice" bounce in their data and create a table and graph. They are then to find the curve of best fit. Below are the values representing "good" data from a little bounce.

Time (sec)	Distance (meters)
3.4	1.278
3.5	.984
3.6	.788
3.7	.69
3.8	.69
4.0	.789
4.1	.985
4.2	1.281

Using (4,.789) , (3.5,1.278) and (4.2,1.281) the following equations can be set up:

(1) $16a + 4b + c = .789$
(2) $12.25a+3.5b+c = 1.278$
(3) $17.64a+4.2b+c = 1.281$

(1) $16a + 4b + c = .789$
-(2) $-12.25a-3.5b-c = -1.278$

$3.75a + .5b = -.489$
$.5b = -3.75a - .489$
$b = -7.5a - .978$

(3) $17.64a + 4.2(-7.5a - .978) +c = 1.281$
$17.64a – 31.5a – 4.1076 + c = 1.281$
$-13.86a + c = 5.3886$

$c = 13.86a + 5.3886 \longrightarrow$ (1) $16a + 4(-7.5a - .978) + 13.86a + 5.3886 = .789$

$16a - 30a - 3.012 + 13.86a + 5.3886 = .789$

$-.14a = -.6876$

$a = 4.911 \longrightarrow$

$b = -7.5(4.9) - .978$

$b = -37.728 \longrightarrow$ (1) $16(4.9) + 4(-37.73) + c = .789$

$78.4 - 150.92 + c = .789$

$c = 73.309$

Therefore the curve of best fit is

$$y = 4.9x^2 - 37.73x + 73.308$$

Using other points to check our equation it is seen that it is quite good. By doing further algebraic work to our equation, which the students should be instructed to do, it can be modified to:

$$y = 4.9x^2 - 37.73x + 73.308$$

$$y = 4.9(x^2 - 7.7x + 14.96)$$

Recall $(-\frac{b}{2a})^2$

$$y = 4.9(x^2 - 7.7x + 14.8225 + 14.96 - 14.8225)$$

$$y = 4.9((x - 3.85)^2 + .1375)$$

$(\frac{7.7}{2})^2$

$$\mathbf{y = 4.9(x - 3.85)^2 + .674}$$

14.8225

The students should understand what each of the coefficients mean in the form of the equation above. By examining the table of values of the graph it can be seen that the ball was .674 meters from the motion sensor at its highest point in the bounce (the vertex of the parabola) and that this occurred in 3.85 seconds.

What does the 4.9 represent in the equation? It is the acceleration rate of the ball of course. When good data of a bouncing ball is collected the coefficient of the x^2 term is very close to 4.9.

The students should be reminded that an objects distance with constant acceleration can be computed with the equation $y = \frac{1}{2}at^2$. With this in mind they can be shown that $\frac{1}{2}a = 4.9$ so that $a = 9.8$ m/s^2, the acceleration of freely falling objects on earth.

The students should now be able to solve the bullet problem given on Day 4 and should be given this as homework. They should not only solve the problem, but give a written explanation of the process they used.

Day 11

The last day will be a celebration of discovering the acceleration rate on earth. Before this celebration is to take place, though, the final dialogue between Salviati, Sagredo and Simplicio will take place. This one will be really make- believe because the proponents of Galileo's views will use knowledge and data obtained from the work in the unit.

Simplicio believes that a freely falling object's speed increases in proportion to the distance traveled. Salviati and Sagredo provide the new knowledge from the class to disprove this conjecture. Although this argument is far from authentic it is noted that Galileo would have used such information, if he had had the knowledge and technology. This was a very hard concept for him to convey. Below is a beginning to the mock argument:

Simplicio- An object that falls twice as far as another will be traveling twice as fast. One that falls four times as far will be traveling four times as fast. This is common sense and has been known for over a thousand years.

Salviati- Do you believe that the motion sensor machine that you have seen perfectly graphs a walker's distance through time.

Simplicio- Yes

Salviati – Then you would also believe that it graphs a bouncing ball's vertical motion through time and after careful analysis we have discovered that the acceleration rate of these bouncing balls is 9.8 m/s^2 .

(Sagredo then displays the following chart and explains why Simplicios conjecture is false.)

Time (sec)	Dist (meters)	Velocity (m/s)	Acceleration (m/s^2)
0	0	0	9.8
1	4.9	9.8	9.8
2	19.6	19.6	9.8
3	44.1	29.4	9.8

4	78.4	39.2	9.8

Sagredo – Under this assumption a falling body's time, distance, velocity and acceleration would be as such. Looking at the distance and velocity values from 1 to 3 seconds, the distance has increased nine times, but the velocity has not increased nine times, but 9.8 m/s each second. The velocity, therefore has increased 3 times its former value.

Annotated Bibliography

The first list of books in this section are ones that could be used by teachers or students that want to learn more about Galileo and his *Dialogue* involving the three characters Salviati, Sagredo and Simplicio:

Cooper, L., *Aristotle, Galileo, and the Tower of Pisa* ,Ithaca, N.Y.:Cornell Univ. Press, 1935

Drake, S *Discoveries and Opinions of Galileo*, New York: Doubleday, 1957

Galilei, Galileo, *Dialogue on the Two Chief World Systems*, trans by S. Drake, Berkeley: Univ. of California, 1953

Santillana, G. de, *The Crime of Galileo,* Chicago: Univ. of Chicago, 1955

Shamos, M. , *Great Experiments in Physics*, Toronto: General Publishing Company, 1959

The following list of books are ones that I have read and referred to while writing the unit. The first of which is my physics book used at The University of Pittsburgh for my first three physics courses. The second book on the list is good for students to read, for it is an easy read and covers all of the basic concepts covered in university physics courses 1,2 and 3.

Halliday,Resnick, *Fundamentals of Physics,* US: John Wiley and Sons, Inc., 1988

Gonick, Huffman, *The Cartoon Guide to Physics,* N.Y.: HarperCollins, 1990

The two books listed below are ones in which a teacher can use as a reference if they want to use the method of least squares to find the exact curve of best fit for a given set of data. The page numbers, where this information can be obtained is also included.

Beckwith,Marangoni, Lienhard, *Mechanical Measurements (5th Ed.),* US: Addison-Wesley, 1993 (pgs 108 – 115)

Gerald, Wheatley, *Applied Numerical Analysis (5th Ed.)*, US: Addison-Wesley, 1994 (pgs 260 – 271)

The next book on this list is one in which a teacher can refer to if they want to solve the systems of equations in the unit by using matrices instead of the algebraic methods presented.

Pettrofrezzo, *Matrices and Transformations,* New York: Dover, 1966

The last book to refer to on this list is one that teachers can use when writing and using programs in the TI-82 or TI-83 calculators. It is the reference manual for the TI-83 plus calculator and comes with the calculator when one is bought.

Annotated List of Materials

The only items, which are essential to the unit are:

1) **TI-82 or TI-83 Graphics Calculators**
   -Preferably every student should have one of these. They not only need them to collect data with the CBR/CBL motion machines, but use of the calculators will aide them in the rigorous algebraic work that the student will be required to do with data which is non standard.Regardless of this fact it would be possible to get by with at least six of these calculators, so that the students would be able to collect data in small groups.

2) **At least six CBR or CBL motion detector machines**
3) **A view screen** which can be connected to a TI-82 or TI-83 calculator will be needed so that tables, and graphs produced by the programs in the calculators can be displayed to the entire class at the same time.
4) **Long boards as ramps**-Preferably these ramps should have grooves in them so that the rolling balls do not roll off of them.
5) **Perfectly round balls**, including those that are elastic- The term “perfectly round ball” is one that Galileo used himself. In actuality soccer balls were used in the ball rolling experiments and pumped up basketballs were used in the bouncing one.

Standards

All seven of the math standards used by The Pittsburgh Public Schools are addressed in this unit and are listed below:

1. All students use numbers, number systems, and equivalent forms (including numbers, words, objects and graphics) to represent theoretical and practical situations.

2. All students compute, measure, and estimate to solve theoretical and practical problems, using appropriate tools, including modern technology, such as calculators and computers.

3. All students apply the concepts of patterns, functions and relations to solve theoretical and practical problems.

4. All students formulate and solve problems and communicate the mathematical processes used and the reasons for them.

5. All students understand and apply basic concepts of algebra, geometry, probability and statistics to solve theoretical and practical problems.

6. All students evaluate, infer and draw appropriate conclusions from charts, tables and graphs, showing the relationship between data and real world situations.

7. All students make decisions and predictions based upon the collection, organization, analysis and interpretation of statistical data and the application of probability.

Appendix

As was mentioned in the unit some of the programs used in the unit have to be obtained before the CBR/CBL motion detectors will work. The TI-83 Plus calculator has programs already in the calculator that can be used with the CBR or CBL machines. The TI-82 and regular TI-83 calculators have no such programs. These must be either written by the teacher, some of with are printed in the manual that comes with the machines, or they can be obtained by linking two calculators, one of which has the programs. The programs can also be downloaded directly off of the internet, however a special machine must be first purchased from Texas Instruments to connect the calculator to the computer.

It is a good idea for teachers to write their own programs, however it is time consuming. By writing your own program or even copying one from print into the calculator, you learn about the calculator and program. It enables you to modify the program if you wish. For example if you want the motion sensor to collect data every eight of a second instead of every fourth of a second you can do so.

If you are using this unit and are having problems with your programs you can e-mail me at prenne@pghboe.edu. If you are a PGH Public School Employee I can get you just about any program you want.

For those interested in writing a program here is the MoRamp program used in the ball rolling activities in this unit. If you have never written any of these programs before refer to your TI calculator manual.

MORAMP

```
:ClrList L1, L2, L3, L4, L5, L6
:PlotsOff
:FnOff
:AxesOn
:ClrHome
:Disp “APRX DISTANCE”
:Disp “IN FEET”
:Input Y
:Disp “APRX SECONDS”
:Disp “TO TEST”
:Input X
:0–>Ymin
:Y–>Ymax
:.5–>Yscl
:0–>Xmax
:X–>Xmax
:.5–>Xscl
:ClrHome
```

```
        :{1,0}–>L1
        :Send(L1)
        :{1,11,3}–>L1
        :Send(L1)
        :(X/.25)–>dim(L4
:(X/.25)–>dim(L2
:Disp"HIT ENTER TO"
:Disp"START"
:Pause
:Disp "WORKING…"
:{3,.25,X/.25,0,0,0,0,1,0}–>L1
:Send (L1)
:For (N,0,X,.5)
:Get (L4)
:End
:seq(I,I,0,X–.25,.25)–>L2
:Plot1(Scatter,L2,L4,□)
:DispGraph
:Text(1,Y,"D(FT)")
:Text(51,81,"T(S)")
:
:Stop
```

continued on the next page