

Operations and Everyday Mathematics

Vita Nemirovsky

Index

Overview
Rationale
Objectives
Discussion
Conclusion
Resources

Overview

Basic mathematical operations of addition, subtraction, multiplication, and division of whole numbers are the foundation of mathematics. Without having a strong and automatic grasp of the above concepts other areas in mathematics are not only difficult, but almost impossible to understand. The focus of this project is to analyze Everyday Mathematics' position and philosophy on operations with whole numbers in the area of addition, subtraction, multiplication, and division in grade levels kindergarten through fifth grade. Through this project I will describe and analyze the types of activities that are presented at each grade level in relation to operations of whole numbers. This project will connect how each grade level supports, facilitates, and progresses the learning of mathematical operations among the grade levels above and below them.

Rationale

The purpose of this project is to understand Everyday Mathematics' approach and philosophy towards whole number operations. This project will discuss and analyze Everyday Mathematics' approach to giving students a solid foundation in mathematical basic skills, so that students can use their skills and apply it to other areas of mathematics.

Everyday Mathematics' philosophy about learning the basics of mathematics is that it should relate to real world concepts and should be taught in a more exciting way than the traditional version of drill. Everyday Mathematics' belief is that without the above components (of real world connection and excitement in learning basic skills) in a mathematics curriculum, students will quickly learn that mathematics is boring and that it does not relate to them and their daily lives.

The Pittsburgh Public School District utilizes the Everyday Mathematics curriculum in grades kindergarten through fifth grade. Everyday Mathematics is used to prepare students for the future as it relates to mathematics and the real world problems that are connected to it.

As a teacher in the Pittsburgh Public School District working in the elementary school setting, I have had the opportunity to teach and observe Everyday Mathematics instruction. Over the past several years I have had some questions such as: how does the curriculum relate to students' knowledge of basic facts and is there generalization between learning mathematics in a more exciting way (games) to knowing basic facts.

Due to the fact that Everyday Mathematics is already an adopted curriculum and therefore has the qualifications of working with the Pittsburgh Public School Mathematics Standards and has been recommended by the National Council of Teachers of Mathematics Principles and Standards, standards as they relate to Everyday Mathematics will not be addressed.

Objective

It has been my experience that Everyday Mathematics puts so much emphasis on the real world concepts that the basic skills needed to solve them seem to be forgotten. Through this project I will study the Everyday Mathematics curriculum as it relates to whole number operations in grades K-5. My goal is to learn if operations with whole numbers are as strongly stressed in the curriculum, as are the real world concepts that rely on the knowledge of basic facts in order to solve them.

The objective of this paper is to define Everyday Mathematics' perspectives about operations with whole numbers and then to discuss how these definitions are related to the foundations and basics of mathematics. In order to fully understand Everyday Mathematics' one must understand its perspective and its philosophy about basic facts, operations, mental arithmetic, algorithms, and calculators. In this section the above subjects will be discussed as they pertain to the Everyday Mathematics' curriculum.

Operations

According to the Everyday Mathematics Teacher's Reference Manual Grades K-3, operation is defined as an *action performed on one or two numbers producing a single number result* p. 236. Everyday Mathematics sees its curriculum as going far beyond the four basic operations traditionally associated with elementary mathematics and applying the basic knowledge of operations into daily life.

Everyday Mathematics approaches the basic operations of arithmetic indirectly, by looking at how they are used (Everyday Mathematics Teacher's Reference Manual Grades K-3, p.73).

Operation sense is essential. Students should understand and be able to explain what the four operations mean. They should be able to explain why multiplication of whole numbers results in products larger than the factors... (Everyday Mathematics Fourth grade Teacher's Manual & lesson Guide, Volume A, introduction, xiv).

Basic Facts

It is the perspective of Everyday Mathematics that learning basic facts is a key factor in being successful in mathematics. Having automatic basic knowledge in complicated mathematical tasks is important in order to achieve success; therefore knowing basic facts is the cornerstone of mathematics (Everyday Mathematics Teacher's Reference Manual Grades K-3 p 89-91).

In Everyday Mathematics basic facts are known as *Fact Power*. The greatest concentration of Fact Power is in the first grade between lessons that are taught primarily in February and March as in accordance with the first grade scope and sequence chart. By the end of second grade Everyday Mathematics expects that students are secure in their basic facts for both addition and subtraction. By third grade, multiplication and division become the emphasis and by fourth grade mastery of multiplication and division facts are expected (Everyday Mathematics Teacher's Reference Manual Grades K-3, p.90).

In order to achieve basic knowledge of facts, Everyday Mathematics relies heavily on games as part of the curriculum. Games are seen as an integral part of the curriculum beginning kindergarten and are not seen as an add on or time filler. It is Everyday Mathematics perspective that by involving students in games versus skill sheets, students will continue to be interested in their learning and will then therefore learn their basic facts.

Choral drills begin in first grade and are made up of many short drills that teachers are encouraged to facilitate during breaks and or down time. Fact Triangles are Everyday Mathematics' version of flash cards that can be used as part of whole group instruction, but is suggested for use in cooperative group settings. However, with Fact Triangles there are more skills that are used to complete each one. Each Fact Triangle focuses on a fact family using addition/subtraction by the using *turn around rule* or what traditionally is known as the commutative property of addition/multiplication. Fact Triangles have two addends and a sum; a multiplication/division fact triangle has two factors and a product Double Nine Dominoes is also a technique used by Everyday Mathematics to teach basic facts. By using dominoes students can visualize facts from 1-18 and use the dominoes for both addition and subtraction (Everyday Mathematics Teacher's Reference Manual Grades K-3, p91).

Algorithms

An algorithm is defined as: *a well-defined procedure or set of rules used to solve a problem* (Everyday Mathematics Teacher's Reference Manual Grades K-3, p94). In Everyday Mathematics algorithms are presented in various ways. Everyday Mathematics does not teach the traditional set of algorithms, but has introduced new ways of solving the same problems. This curriculum also gives strong backing to students' discovery of their own personal algorithms that should be shared with classmates.

Calculators

Calculators are seen as a tool to help students complete work at a faster pace. Students are introduced to calculators in kindergarten and use them throughout the curriculum. Students use the calculator to help them complete basic facts. First students are taught Fact Power, but after a baseline for basic facts is established calculators are encouraged in the curriculum. The game beat The Calculator pits students' basic fact knowledge against the calculator.

Mental Arithmetic

Mental arithmetic is defined as *Computations done by people "in their heads," either in whole or in part. In Everyday Mathematics, children develop a variety of strategies for doing arithmetic- calculating mentally, using paper and pencil, drawing pictures, counting jumps on a number grid, and so on* (Everyday Mathematics Teacher's Reference Manual K-3 p. 232).

In order to develop mental arithmetic students need to have strong basic facts skills or fact power. Fact power is the foundation of mental arithmetic. Techniques such as games, fact triangles, and choral drills are used in the curriculum. The Minute Math + books in the curriculum are a good source to use in increasing students' ability to be proficient with their basic skills. Everyday Mathematics goal for mental arithmetic is to introduce students to many different strategies to solve problems. By having many ways to solve problems students will eventually become comfortable with one or two of the ways presented to them and will then use those ways in a quick and easy manner. Some of Everyday Mathematics' common strategies can be found in Everyday Mathematics Teacher's reference Manual K-3 pp.112-113, some are described below:

Round Techniques include rounding to the nearest 10, hundred, thousand, and so on, and computing with rounded numbers. Ex. $647+284$ is approximately $600+300=900$ or $650+280=930$

Adjust the Numbers A sum is unchanged if one addend is increased and another is decreased by the same amount. Ex. $86+37$ is equal to $(86+4)+(37-4)=90+33$, which is equal to $(90+10)=(33-10)=100+23$. A difference is unchanged if the same number is added to or subtracted from the original numbers-the minuend and the subtrahend. Ex. $54-37$ is equal to $(54+3)-(37+3)=57-40$.

Look For Easy Combinations In $17+25+3+15+8$, add $3+17$, and add 15 to 25.

The topics described above: operations, basic facts, algorithms, calculators, and mental arithmetic are areas that will be returned to in the discussion portion of the paper.

Discussion

Everyday Mathematics' view of daily mathematical operations is that it should be an integral part of students' lives. It stresses the importance of having mathematics relate to daily activities such as counting the days in the school year, adding and subtracting the amount of students present and or absent for attendance, and calculating the difference in day light hours to night hours through the sunrise and sunset chart. In fact it stresses connection to daily life so much that it at times overlooks or does not put enough emphasis on the importance of learning basic mathematical facts to help students increase their ability to complete and understand higher order mathematical concepts. Basic facts are introduced to students in the first grade as Fact Power. It is stressed in the curriculum that Fact Power is very important in order to be successful in mathematics. Such a perspective of basic facts is obviously a very important one to have as basic skills are being developed. However, the presentation and time spent on Fact Power i.e. basic facts seems to be very limited. As each grades' learning objectives and lessons are introduced limited amount of time is spent on basic facts skills. A lesson may have a set of 4 Fact Triangles to complete and will then move on to time and or money. Students spend time practicing their basic fact skills via games, but short amounts of time is dedicated to the mastery of basic facts in any systematic way past the use of fact triangles and What's My Rule activities. With "bursts" of exposure in small amounts in the above areas, it seems difficult for students who fall in the middle of the learning curve to get enough practice to feel as if they are proficient at any of the basic skills.

Everyday Mathematics also seems to contradict itself as it presents its philosophy of knowing basic facts; in the Second Grade Teacher's Manual & Lesson Guide volume A unit 4, Everyday Mathematics states that:

Mental arithmetic, as defined in Everyday Mathematics, does not require all computation to be done in ones head ...What is most important is that children devise their own solution strategies. Instead children should be encouraged to experiment with various approaches to solving computational problems and to talk to one another about their work...The work in this unit assumes that children have near-instant recall of the addition facts, facility with the subtraction facts, and experience with counting on techniques for subtraction.

By stating that basic facts should be available for near instant recall and then stating that it is not necessary at this time to have complete knowledge of basic facts, Everyday Mathematics is sending a confusing message of its expectations.

Also students are encouraged throughout the curriculum to first explore mathematical topics and come up with their own avenues for finding the answers to problems before the teacher gives them a particular way of solving the equation. This type of learning seems to be backwards in its presentation of information. How are students able to figure out the answer if they are not first given specific guidance?

Everyday Mathematics puts great emphasis on basic facts in the middle of first grade through games and short choral drills. Everyday mathematics puts the drill

part of learning basic facts into games; this in and of itself seems like a very positive concept for making mathematics more interesting and fun for students. However, before students can get to learning the basic facts, much time and energy is spent on learning how to play the game instead of the mathematics itself. Students may also begin to see these activities as games only and may not recognize that what they are learning in the games should be transferred information to other areas of mathematics such as story problems.

As the curriculum spirals ahead, review of addition and subtraction topics of whole numbers becomes limited. This is not to say that the repetition of basic skills concepts disappears, but that the time that is spent on it does not seem to be enough for those who did not “get it” the first time. More time needs to be spent on the practice of basic skills, so that students can easily apply their knowledge of basic skills to problem solving.

Everyday Mathematics would argue that, this is all right, due to the fact that the curriculum spirals and that eventually the student will “get it” because the topic will be continued as part of the curriculum. With this I find there to be a problem, because if a student did not get it the first time, and the concept is reintroduced in the spiral a week or a month later, how will the student be able to remember any part of what was taught earlier to develop the skills that he is still weak in?

Everyday Mathematics would also state that basic skills and operations are being constantly reviewed via Fact Triangles that not only review basic facts but also make students think about turn around facts i.e. inverse operations. However, it has been my experience to see students be proficient with completing Fact Triangles, but still having difficulty with basic facts or not being able to use basic facts in number stories and or number models. The ability of completing Fact Triangles but also having difficulty with basic facts may stem from seeing the basic triangle as part of routine and or rote memorization and not internalizing the concept. Students may know how to work out the fact triangle but not what it really means mathematically.

In addition to having some students miss out on the spiral effect, the teacher must first spend valuable instructional time teaching the routine of Fact Triangles before they can be used as a learning tool. Here time is lost on the “language “ of Everyday Mathematics versus mathematics. Students not only have to understand the mathematics of $2+2=4$ and $4-2=2$, but first know how to apply it in a Fact Triangle. Why not teach the basics first $2+2=4$ and $4-2=2$ (Wu, 1999) and then show students how to apply this into a Fact Triangle, this way they will not only have a higher chance of success with their basic fact, but will also learn how to connect these facts to higher thinking skills.

As stated before, games are also an important part of the curriculum. Games are not in the curriculum as time fillers between activities but are there to help students learn their basic addition, subtraction, multiplication, and division facts. It seems that having games in the curriculum in such a way would only increase students’ knowledge. However, there is a problem in having games be the main part

of learning basic facts. First if teachers are constricted by time they may skip the game for the lesson, thinking that ‘oh, its o.k. it’s a game”.

An example of this becomes clear when in the Fourth Grade Teacher’s Manual & Lesson Guide Volume A unit 2, Lesson 10’s overview of the following is stated: *If Lesson 10 is running long, you may want to postpone playing Name That Number.* By giving the option of postponing the game, it seems that the game is less important and may very easily not be played at all. This quickly makes it very difficult to have consistency with what students are being taught. Also if games are being played routinely as Everyday Mathematics states, then the following problem may occur: if students play a game in cooperative groups and do not have a strong grasp on their basic skills then they may routinely get wrong answers without the teacher realizing it because the teacher may be floating around the room and can not be with each student at all times. Also students may begin to think of math as a time to play games, this type of thinking is not a problem unless students have difficulty generalizing what they have learned through the games into everyday math situations. For example students may play a place value game very well but may have trouble answering questions about place value. When it comes time to apply the skills that students should have learned through the games they may not be able to transfer and generalize that knowledge.

According to Bloom’s Taxonomy of educational objectives, there are six major areas in the cognitive development process that includes: knowledge, comprehension, application, analysis, synthesis, and evaluation. Knowledge is defined as remembering previously learned material, from specific facts to complete theories. Comprehension is defined as the ability to grasp the meaning of material. Application is defined as the ability to use learned material in and concrete situations. This may include the application of such things as rules, methods, and concepts. Analysis is defined as the ability to break down material into component parts so that its organizational structure may be understood. Synthesis is defined as the ability to put parts together to form a new whole. Evaluation is defined as the ability to judge the value of materials for a given purpose (Linn and Gronlund, 1995 p.534).

With algorithms Everyday Mathematics stresses students’ ability to create their own algorithm or to pick an algorithm that they feel comfortable using. This type of learning takes into account application and analysis, but leaves out its basic level of cognitive development: knowledge. For students to be able to apply, analyze, and synthesize mathematical information they first need to be able to know and comprehend the information that they are given. It would be difficult for students to create their own algorithms if they are not proficient in their basic skills. It would be more beneficial if students were first given direct instruction with algorithms and after they have acquired enough skill and confidence in their mathematical skills, should they be given the flexibility of choosing a method that works for them. If students do not have one method that they are secure in how are they expected to come up with their own methods and solve equations correctly?

Calculators are also a strong component of Everyday Mathematics they are introduced at the kindergarten level and are seen as tools to help students progress in mathematics. Everyday Mathematics even describes a situation where in the real world an employer gets angry at his employee when he does not use a calculator to save time, while a classroom teacher is angry with her students for using a calculator because then her students will finish the work too quickly (Everyday Mathematics Teacher's Reference Manual Grades K-3, p96).

Everyday Mathematics perspective is understandable of wanting students to be up to date with the latest technology and compete in the real world at a rapid pace. However, if students are introduced at such an early age to calculators they may become dependent on them and may disregard their teacher's instructions when they are told they are not to use the calculator. Students may also get a false sense of security that all they need is the calculator for basic operations.

With mental arithmetic Everyday Mathematics stresses many different ways that students can show their work and or work out the problem. This type of approach seems to be helpful to a student who is a non-typical learner. The question goes back to what if the student did not have enough time to learn at least one specific way to complete an operation? How can he then use the higher attributes of Blooms Taxonomy to show his knowledge?

As stated above Everyday Mathematics uses the spiral effect throughout its curriculum. It introduces concepts and ideas in many areas of the curriculum across grade levels and its success is dependent on the spiral's building blocks of each strand and or topic from lesson to lesson, unit to unit, and grade level to grade level to teach knowledge and understanding of mathematical concepts. As teachers plan their lessons they are given background information about what information students had been taught and or exposed to in previous units or subsequent years. Though prior knowledge can be found at the start of each unit, the information is limited. In tables 1-5.1, teachers will find easy access to the operations strand of the curriculum, lesson-by-lesson grade level to grade level. The purpose of tables 1-5.1 is to give teachers quick and easy access to what each of their students should already know as well as what each student is expected to know in the upcoming grade. The information below is a combination of lessons and units dealing with operations, from the *Scope and Sequence Chart by Strand and Month Kindergarten- Fifth Grade, Everyday Mathematics, The University of Chicago School Mathematics Project.*

| | Kindergarten | First Grade |
|------|--|---|
| Sept | | (Lessons 1-15) Estimates with attendance data Penny-Dice Game to show “as many as” Number stories with penny and animal contexts: Represent with simple pictures, share solution strategies |
| Oct | Subtract by ones p.106 | (Lessons 16-27) A total as the sum of two parts using Two Fisted Pennies Game Comparisons, differences, and sums using penny game Number stories: verbal concrete, pictorial, , and number model representations, sharing of solution strategies using a school store poster |
| Nov | Explore “counting” on the number line. Pp. 130-131 Tell number stories. pp. 138-139 Play a game using (+) and (-) signs. Pp.144-145. | (Lessons 28-39) Add and subtract by counting back on a number line Differences using a number grid Number combinations on dominoes; two parts and a total Use total number of dots on dominoes for Top-It game Readiness for addition subtraction facts by recording splits of numbers in Two Fisted Pennies Games |
| Dec | Tell number stories with the calculators p. 176. | (Lessons 40-49) Use total number of dots to sort dominoes |
| Jan | N/A | (Lessons 50-65) Number Stories involving relations, addition, subtraction Estimation routines: two step problems to compare sums of weights Number stories for parts-total, change to, and comparison Number models for stories; horizontal and vertical formats for number models Establish routines for using posters to generate number stories |

Table 1.0

Table 1.1

| | Kindergarten | First Grade |
|-------------|---|--|
| Feb. | <p>Tell addition number stories p. 277</p> <p>Tell subtraction number stories p. 228-229</p> <p>Play a game (Go Forward and Back Up Game) using (+) and (-) signs p. 231</p> <p>Divide even groups in half. P. 232</p> <p>Add and subtract objects; use symbolic language. P. 238</p> <p>Play a game (Plus or Minus Game) using (+) and (-) signs. p. 244</p> <p>use different names for the same number (equivalents) p. 245</p> | <p>Review facts with Fact Power Game and Addition Top It game</p> <p>Illustrate Equivalence with pan balance; review with Facts Table</p> <p>Review facts with Beat the Calculator and Go Forward, Go Back games</p> <p>Inverse relationships between addition and subtraction facts</p> <p>Fact families with dominoes, Fact Triangles, addition/ subtraction Facts Table</p> <p>Review facts with subtraction Top It game</p> <p>Missing part of total with Two Fisted Pennies game</p> <p>Domino sums, Domino Top It, platter facts, Egg Carton Digits game</p> <p>Add and subtract 0 and 1 to/from 2 and 3 digit numbers</p> <p>Addition fact problems with ZIP codes digits</p> |
| Mar | <p>Introduce the concept of equivalent numbers as “number families” p. 255</p> <p>Continue exploration with equivalent numbers; record equivalencies. P256</p> <p>Add dots on dice; write numbers; use for ongoing assessment; conduct product assessment. P. 275</p> | <p>Review fact Families</p> <p>Strategies for adding or subtracting tens and hundreds (fact extensions)</p> <p>Act out sharing one dollar (100 pennies among everyone in class)</p> <p>Number stories and mental arithmetic interactions School Store 2 poster</p> <p>Number stories about class weights</p> |
| Apr. | <p>NA</p> | <p>Mentally add or subtract tens and multiples of tens to/from any number</p> <p>Facts and fact extensions with Two-Fisted Dimes game</p> <p>Number stories for adding and subtracting tens using data about animals</p> |
| May June | <p>(May) Play a game (High Roller); count on; add. P. 314</p> <p>Use symbols (+), (-), and (=) to tell number stories; act out number stories. Pp. 326-327</p> <p>Find missing addends with craft sticks. P.329</p> <p>(June) Identify a number using addition and subtraction clues; use for ongoing assessment. P. 359</p> <p>Write number stories using the (-), (+), and (=) signs. P. 362</p> | <p>Compare year end weights and birth weight, Determine changes</p> <p>Add three digit numbers using Tic-Tac-Toe Addition game</p> <p>Find half of a number</p> <p>Review facts and fact extensions with Fact Triangles</p> <p>Determine height and weight changes during the year</p> <p>Estimate answers to number stories</p> <p>Write number models for number stories</p> <p>Mental arithmetic for adding 2 digit numbers</p> |

| | Second Grade | | Second Grade |
|-------|---|------|---|
| Sept. | (Lessons 1-12) Practice addition facts with Addition Top It Review relation words and symbols | Feb. | (Lessons 59-73) Use memory keys on the calculator Add three or more addends Practice making multiples of ten on the calculator Find differences between 2 digit numbers and next higher multiples of ten Drill addition and subtraction facts and extend fact families Explore ways to compare quantities; differences and ratios Solve comparison stories about temperatures Make and solve fraction stories Write and solve number stories and number models using information given in diagrams |
| Oct. | (Lessons 13-26) Use unit boxes for all number work Facts practice through games, mazes, dominoes, Fact Triangles, magic squares, and Tic-Tac-Toe puzzles Solve addition problems that are near doubles Relate addition and subtraction facts Differences as distances on the number line Multiplication concepts using rectangles on geoboards Add and subtract 2 digit numbers using coins Addition number stories using unit boxes and writing number models Subtraction “change to less” and “compare quantities” number stories Share mental strategies: addition/subtraction stories | Mar. | (Lessons 74-88) Add multidigit addends Subtract 2 digit numbers from 2 digit numbers Compare fractions; and practice +, - facts with games Develop and share addition and subtraction algorithms Estimate distances by converting strides to feet Convert data given in feet to yards Solve road map stories about distances between cities Draw a line segment twice as long or half as long as another line segment |
| Nov. | (Lessons 27-37) Multiply with pattern block arrays Add and subtract multiples of 10 Add any 1 digit number to a multiple of 10 Number models and mental arithmetic: make change Find arithmetic mean Estimate and project volume/weight of classroom and school rash for a week/year School Supply Store Poster: Estimate costs Use thermometer readings to find temperature differences | Apr. | (Lessons 89-104) Add multiples of 10 Parentheses in number models Introduce terms factor, product, quotient, and remainder Change a number by adding or subtracting on the calculator Draw arrays for multiplication stories Share strategies and solve multistep sharing stories Make up and solve 2 step number stories Set up and solve problems involving multiples of equal groups Set up and solve equal sharing stories Represent number stories with diagrams and number models |
| Dec. | (Lessons 38-45) Add a multiple of 10 and ant 2 digit number Figure the cost of two items and any change due Games and drills: +, -, *, and / facts Multiplication and division concepts: Use rectangle arrays of cm cubes Estimate id difference between a day’s high and low temperatures will be <n F Solve problems about temperature differences | May | (Lessons 105-120) Write number models for multiplication facts Relate “10 times” and “1/10 of” to each other Build arrays and record products of numbers multiplied by themselves Review +/- facts and extensions Write = and- number models in vertical format Facts tables: Practice and generate * and / facts Use multiplication stories to make up related division stories Compare precipitation data –differences and ratios Solve number stories for data on temperature and precipitation chart Figure restaurant bill and estimate tip amount Relate number of decades to number of years Compare human and animal speeds |
| Jan. | (Lessons 46-58) Practice + and * with multiplication Mental arithmetic routines Strategies for solving addition problems with three or four addends Increase and decrease quantities: “change” diagrams and number models for stories Make up and solve number stories about equal groups of objects Multiplication concepts: x by y arrays Division concepts with “equal sharing” and “equal grouping” | June | NA |

Table 2.0

| | Third Grade | | | Third Grade |
|------|--|------------|--|-------------|
| Sept | (Lessons 1-14) Compute a sum, difference, product, and quotient using a calculator Name equivalent fractions using all operations Use $>$, $<$, and $=$ to record comparisons Review missing addends and related subtraction problems Use Fact Triangles and the Addition/subtraction Facts Table to practice addition and subtraction facts Extend the use of fact families to number families for any triplet of numbers Review the inverse relationship between addition and subtraction Find equivalent names for multiples of 10 and 100 Find the difference between a number and a multiple of 10 using a calculator | Feb | (Lessons 65-78) Study square number facts 10×10 Recognize products of a given factor as counts by that factor Survey multiplication facts using the turn around rule Use parenthesis in number models Extend $*$ and $/$ with number triangles and families Multiply pairs of multiples of 10, 100, and 1000 Estimate order of magnitude: How many digits will the product have? Use fraction in number stories about equal parts of almost anything | |
| Oct | (Lessons 15-29) Practice addition subtraction facts using "What's My Rule?" tables Develop "ownership" of multidigit addition and subtraction algorithms Find the sum of three or more numbers Identify equivalent names of numbers Use parts and total diagrams to solve missing addend problems Use compare diagrams to solve problems involving differences Use Change Diagrams to increase a given quantity Write number models for part-part-whole; missing addend; comparison; And change stories Check reasonableness of sums and differences with ballpark estimates | Mar | (Lessons 79-92) Name fractions for quantities greater than one Given a fraction of a set, determine the total number Find equivalent fractions Draw items to represent a fraction of a collection Fold rectangles to represent a fraction of a fraction Find products using base 10 blocks Calculate products using the partial product algorithm Identify factors of a number; recognize 1 as a factor of a number Use the North American Weights Poster to solve multiplication and division number stories involving multiples of 10, 100, and 1000 | |
| Nov | (Lessons 30-52) Double, halve, and triple numbers Use the Array Exhibit: pictures showing items arranged in arrays Read division notation as "how many x 's in y " or " x goes into y how many times?" or " y divided by x " Use the turn around short cut to recall unknown facts from known Solve number stories involving equal rows of things Use rate diagrams to solve problems with multiples of equal groups Use the Variety Store Poster to write, illustrate, and solve rate multiplication stories Explore square numbers Solve division stories using number models Notation for division number models with remainders: $19/5 = 3 R4$ | Apr | (Lessons 93-106) Use equal sharing stories to explore strategies for division using the calculator Use lattice multiplication to find products of two multidigit numbers Find products of 2-3 digit numbers multiplied by 1 digit numbers Express division problems in various ways Multiply 2 digit numbers by multiples of 10 Solve division stories with remainders expressed as fractions Explore solving division problems having multidigit quotients Estimate answers to multiplication problems with 3 digit numbers Solve multiplication / division calculator puzzles | |
| Dec | (Lessons 43-52) Compare 5 digit numbers Solve 3 and 4 digit addition and subtraction problems; estimate to check answers Use $<$ and $>$ to compare | 132 May | (Lessons 107-120) Find sums and differences between 4-5 digit numbers Multiply 4 digit numbers by 1 digit numbers Use a partial products | |

| | | | |
|------------|---|-------------|-----|
| Jan | (Lessons 53-64) Place Value Book: Show numbers 10, 1, 0, 1, 0.01, or 0.001 more (or less) than a given number Compare pairs of decimal numbers using $<$, $>$ or $=$; order of decimals Solve multiplication/ division calculator puzzles Practice finding sums and differences of 4 digit numbers without using a calculator Review multiplication and division arrays Make up and solve multiplication and division stories Solve addition and subtraction stories including a multiplication system | June | N/A |
|------------|---|-------------|-----|

Table 3.0

| | Fourth Grade: Mental Arithmetic and Number Systems | | Fourth Grade: Mental Arithmetic and Number Systems |
|------|--|------|---|
| Sept | (Lessons 1-18) Mastery of addition/subtraction facts (7) | Feb | (Lessons 76-90) Estimate products (77) Add positive and negative numbers; Credits/ Debits Games (83) Round numbers to a given place (86) Meaning of percent (ratio) 986) Percents as fractions (denominator of 100), and as decimals (86) Equivalent fractions and percents (86,87,89,90) Mental arithmetic: facts; multiplication (2 digit by 1 digit); multiples of powers of ten; fraction of a number (87) Coins as percents of a dollar (87) "Easy" fractions as decimals, percents; Fraction/ Percent Concentration (87,88,89) |
| Oct | (Lessons 19-30) Practice: Beat the Calculator, Baseball Multiplication, Multiplication Top-It, Fact Triangles, 50- facts test (19-22,30) Multiplication and division symbols (19) Factors and products (19) Memorization shortcuts: square numbers; turn around facts; times 0, 1, or 10; doubles (19) How many x's are in y? (22) Dividends, divisors, quotients, remainders (22) Fact Families: multiplication/division relationship (22) Addition/subtraction diagrams (25) Mental-arithmetic: addition and subtraction (26) | Mar | (Lessons 91-104) Mental arithmetic: multiples of powers of ten; fraction of a number; extended division facts (91,92) Multiplication and division; ratio, rate, fraction, percent (91) Equivalent fractions, decimals, percents (92) Extended division facts (94) Ratio and difference comparisons (95, 96) Interval estimates (97) Divisor, quotient, remainder (97) Multiplication/division diagrams (97,98) Division Dash; use the square root key (99) Actual lengths and scale drawings (103) |
| Nov | (Lessons 31-45) Multiplication/ division facts (41,42) Extended multiplication facts (41, 42) | Apr | (Lessons 105-120) Power of a number (106) Calculator 10,000 9106, 108) Compare quantities with subtraction (how many more?) and with ratios (how many times as much/) (113) Mental arithmetic: the four operations: fraction/percent of a number (120) |
| Dec | (Lessons 46-60) Mental arithmetic: multiplication/division facts and extended facts (48,49,50,52,58) Mental arithmetic: subtraction facts (54) | May | (Lessons 121-135) Mental arithmetic: rate problems (121) Unit price of oft drink containers (1210) Estimate products (125) Subtraction of positive and negative numbers; Advanced Credits/Debits (128) Multi-step problems involving rates (129) |
| Jan | (Lesson 61-75) World Tour: English Channel crossings (61) Mental arithmetic with extended multiplication and division facts (66) Exponential notation for powers of ten (67) Scientific notation (67) Round numbers to a given place (68) Estimate and find sums of distance and time measurements (71) Mental multiplication with multiples of powers of ten and one digit numbers (71) Missing digit problems (71) Estimate products (73) | June | N/A |
| Nov | (Lessons 31-45) Multiplication/ division facts (41,42) Extended multiplication facts (41, 42) | Apr | (Lessons 105-120) Power of a number (106) Calculator 10,000 9106, 108) Compare quantities with subtraction (how many more?) and with ratios (how many times as much/) (113) Mental arithmetic: the four operations: fraction/percent of a number (120) |
| Dec | (Lessons 46-60) Mental arithmetic: multiplication/division facts and extended facts (48,49,50,52,58) Mental arithmetic: subtraction facts (54) | May | (Lessons 121-135) Mental arithmetic: rate problems (121) Unit price of oft drink containers (1210) Estimate products (125) Subtraction of positive and negative numbers; Advanced Credits/Debits (128) Multi-step problems involving rates (129) |

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|------------|---|-------------|-----|
| Jan | (Lesson 61-75) World Tour: English Channel crossings (61) Mental arithmetic with extended multiplication and division facts(66) Exponential notation for powers of ten (67) Scientific notation (67) Round numbers to a given place (68) Estimate and find sums of distance and time measurements (71) Mental multiplication with multiples of powers of ten and one digit numbers (71) Missing digit problems (71) Estimate products (73) | June | N/A |
|------------|---|-------------|-----|

Table 4.0

| | Fifth Grade: Mental Arithmetic and Number Systems | | Fifth Grade: Mental Arithmetic and Number Systems |
|------|---|------|--|
| Sept | (Lessons 1-13) Factors, products, factor pairs (92) Divisibility (3); divisibility tests: by 2,3,5,6,9, and 10 (94) Factor Captor (factors whole numbers) (3) Factor rainbows (4,7) Exponents and exponential notation (6) Square roots of square numbers (7) Prime factorization (using factor strings) (8) Buzz; Bizz-Buzz 9 multiples and common multiples (100) Mental arithmetic *and / facts; doubles; fractions of a number (11) Multiplication fact extensions 912,13,14) Subtraction using a thermometer (13) Subtraction Pole Vault 9 subtraction) (13) | Feb | (Lessons 67-81) Doubling and halving to mentally multiply numbers (67) Estimates with money in number stories (68) Four-4's Problem (69) Area in an irregular shape (69); of playing surfaces for sports and "footprints" of buildings (74) Computation in number stories (71) Fraction of a number (73) Addition/subtraction of positive and negative numbers using counters (77,79) |
| Oct | (Lessons 14-28) Estimation and number stories (15,17,18,22) Magnitude estimates (16,17) Multiplication Bull's-eye (estimate products of two digit numbers) (16) Estimate sums, differences, products (17) Mental arithmetic: fact extensions (16): successive calculations (18); multiply a number with 9 in the ones place (920); multiply multiples of 10 (22); fraction of a number (22) | Mar | (Lessons 82-97) Number riddles (84) Rate tables (86,88); rate problems (88) Addition/subtraction of decimals (89) Addition /subtraction of decimals (93, 94, 95,) Fraction number stories (93) Fraction Action (96) Prime factorization, greatest common factors, and simplifying fractions (97) |
| Nov | (Lessons 29-44) +/- with whole numbers, decimals, and fractions in number stories (31) Fractional part of a whole (32,35) How many of a unit fraction in a mixed number? (34) Add with fraction stick pieces (34) Round decimals: to nearest place, always up, always down (36) Estimate percent of a population data as urban or rural (43) Mental arithmetic: divide multiples of 10 (31,42); multiply multiples of 10 (40); fraction/percent of (40): easy fractions to decimals and percents; unit fractions to decimals and percents; unit fractions to find thirds, fourths, and fifths (43) | Apr | (Lessons 98-111) Fraction number stories (98) Missing parts of equivalent ratios (99, 108) Part to whole ratio number stories (99) Mental arithmetic: division fact extensions; fraction of a number; percent of a number (1000; mixed operations (107) Addition /subtraction of fractions (102) Estimates sums/differences of fractions (107) |
| Dec | (Lessons 45-52) Estimates in problem solving (46) Mental arithmetic: extended division facts (46); fraction of a number (50) Addition/subtraction number stories (51) | May | (Lessons 112-125) Round decimal to nearest 0.5 (112) Rate problems (112) Fraction of a number with number line models (114) Fraction multiplication (114) Mental arithmetic: multiplication fact stories (115) missing factors (122) Area model for products of whole numbers and fractions (116) Multiplication of whole numbers, whole numbers, mixed numbers (115-1170) Find a percent of a whole number (118) Unit percent and unit fraction to find the whole (119) Percent one number is of another number (120) Calculate ratio for bicycle data (123) |
| Jan | (Lessons 53-66) Division facts and fact extensions (53) Uses of division in number stories (53) Division Dash (divide by 1 digit numbers) (53) Multiply by 1 digit numbers (54) Mental math: divide with powers of 10 (54); successive calculations (54) Magnitude (10 times) (57) Exponential notation; Exponential Ball (58) Powers of 10 and 0.1 in exponential notation (61) Scientific notation (62) Estimates with percents (64) Ratio in number stories (64) | June | N/A |

| | Fifth Grade: Algorithms and Procedures | | Fifth Grade: Algorithms and Procedures |
|------|--|------|--|
| Sept | (Lessons 1-13) Squaring Key on a calculator Unsquare numbers by successive approximations (7) Square root key on a calculator (7) Addition algorithms: opposite change and partial sums methods Whole numbers and decimals (12) Subtraction algorithms: same change and partial differences Methods for whole numbers and decimals (13) | Feb | (Lessons 67-81) Rules to add and subtract positive and negative numbers Divide by one digit divisors by breaking dividends into friendly numbers(78, 81) Add/subtract positive and negative numbers with a calculator; change of sign key (81) |
| Oct | (Lessons 14-28) Multiplication algorithms: partial products method; lattice method Multiplication Wrestling (multiply two digit numbers) (16) Ancient (Egyptian) multiplication algorithm (17) | Mar | (Lessons 82-97) Rule of addition/subtraction of fractions with common denominators (93) Find common denominators: multiplication rule for equivalent fractions (94); fraction sticks (95) multiply denominators (95) Calculator keys for fractions (96) Factor trees (97) Least common multiples and least common denominators (97) |
| Nov | (Lessons 29-44) Rules for generating equivalent fractions (35) Fractions to decimals using a calculator (37, 39) Fractions to percents using a calculator (39) | Apr | (Lessons 98-111) Fractions as percents with a calculator (101) Percent one number is of another number with a calculator 9108) |
| Dec | (Lessons 45-52) multidigit multiplication (46, 50) | May | (Lessons 112-125) Fractions to nearest whole percent with a calculator (114) Fraction multiplication algorithms: Multiply numerators and denominators 9115); multiply fractions and whole numbers (116) multiply mixed numbers (117) |
| Jan | (Lessons 53-66) Mental strategy for division: Break dividend into “friendly” parts (53) Division algorithm: successive estimation (54) One-two digit divisors (55) Interpret remainders in equal sharing problems (56) Powers key on a calculator (58) Scientific notation on a calculator (62) | June | N/A |

Table 5.0

Table 5.1

Conclusion

As the operations strand for whole numbers is followed from kindergarten through fifth grade, it is clear to see that basic facts for addition and subtraction of whole numbers are heavily emphasized in first grade. At the kindergarten level students are exposed to addition and subtraction through number stories in an informal way such as if a teacher asks her students how many pencils will she have in all if she gets 3 pencils from one student and 2 pencils from another. In first grade fact power is introduced as a way of knowing basic facts students are encouraged to practice their skills via games and short choral drills. By mid second grade multiplication is informally introduced through drawing arrays. In third grade students are expected to have solid foundation of basic facts and should be developing and securing addition/ subtraction facts with two digits. By the beginning of fourth grade multiplication should be secure. Double-digit multiplication and division are introduced. In fifth grade students should be proficient in all basic skills for addition, subtraction, multiplication, and division.

The above are the expectations that the Everyday Mathematics has for its students. However, there are many drawbacks that prevent their expectations to be fully reached. They include: the spiral of the curriculum, the presentation of burst or chunks of skills in lessons from many different strands, The inappropriate use and application of Blooms Taxonomy of Education the amount of time that it takes to learn the language of Everyday Mathematics, the possibility of practice being skipped because they are in the form of games, lack of generalization between games and their application to problem solving, and the use of calculators so early on in the curriculum.

The spiral in the curriculum is too wide. The time that it takes to return to a specific skills and or concept provides students with the opportunity to forget what was already learned. Students, who did not catch the concept the first time, have difficulty remembering the base of information that was initially introduced. In order for the spiral effect to work more appropriately, the time between concepts should be shorter.

The set up of each lesson in regards to trying to keep the spiral moving teaches mathematical concepts in limited bursts and or chunks. A lesson may focus on addition, then time, then a number story, and data collection. Due to so many different concepts, students do not have the opportunity to truly practice basic skills and become secure in using them. Instead of having so many different activities per lesson, have the lesson focus on one or two skills, so that students have the opportunity to practice what has been taught and become confident in their abilities.

The inappropriate use and application of Blooms Taxonomy of Education seems to be a major weakness in the curriculum. If according to the Taxonomy, knowledge is the base for all other levels of learning, how is it that students are first encouraged to create their own ways to solving problems? Students are then shown several ways to solve problems. They are then encouraged to choose a way that is

most comfortable to them to help them solve the problem. This puts a big burden on the student to come up with the correct answer with limited guidance. Students should certainly be responsible for their learning, but they should first be given the appropriate guidance. The students should first be given direct and systematic instruction in how to solve equations, and then after they have become secure in their skills should they be given the option and opportunity to experiment with other methods of solving the problem. When someone is asking for directions, they are first given the easiest way to get there, then over time they may discover shortcuts that they are comfortable in using.

The amount of time that it takes to learn the language of Everyday Mathematics also seems to become an issue of time management in the classroom. When teachers must spend valuable instructional time to explain the language of Everyday Mathematics so that students can complete Fact Triangles and what's My Rule? Activities, this is time taken away from students having the opportunity to secure their skills. It would seem that if basic facts were taught first and then the various activities were introduced as a way to practice what was already learned, and then instructional time would be used more efficiently.

The possibility of practice being skipped because they are in the form of games is a critical issue in the curriculum. Everyday Mathematics is built on the understanding that the games must be used throughout the curriculum in order for the curriculum to work. However, the word "game" in and of itself may cause a problem. Teachers may see it as just a game and not an integral part of the curriculum and may therefore skip over the games. Also when students play the games, they may become proficient at the game and the skills that are associated with it, but may still have difficulty applying that knowledge in problem solving. The games should remain as part of the curriculum, but other means of practicing basic skills should also be used.

The use of calculators so early on in the curriculum seems to give students the illusion that knowing their basic skills is not important because they can use the calculator. This gives students a false sense of security because the calculator may not always be around and they will need to rely on their own knowledge to solve the problem. Students should definitely have exposure to the calculator, but not before they have secured their basic skills independently.

This curriculum had a lot to offer when it comes to making mathematics interesting and relevant to students. However it is weak in areas of consistent practice and the foundation of basic skills as they pertain to transfer knowledge to areas such as problem solving. The question becomes, what can be done to incorporate the best attributes of the Everyday Mathematics curriculum with the concerns of its critiques; that are at times classroom teachers?

Resources

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