

Music in Mathematics

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Overview:

Whether society realizes it or not, music is all around us. From the songs of the birds to the music on the radio to the classics by Beethoven and Bach, music fills our lives as well as our ears. Every person on the planet likes some sort of music; some people listen to the soothing jazz of Duke Ellington, others enjoy Elvis' rock and roll, and others still like the hip-hop beats and raps of LL Cool J. What people may not realize, however, is that within each song they listen to is a miniature mathematics lesson. Music involves many of the basic mathematical skills learned in elementary school, such as counting, pattern identification, addition, division, and fractions. A piece of music can be taken apart and analyzed by its time signature, the rhythmic measurements of each note, and the various repeating patterns that those notes make. Such an analysis can in fact be used to teach these skills in a mathematics class. A child who normally finds patterns boring would pay attention if s/he had to look for patterns in their favorite song. Students who think that division is hard might benefit from having an easy to understand visual layout, such as music notes. A class full of ten year olds may get restless if lectured to for a long period of time, but if those same students are clapping or hitting a tambourine while learning about rhythm, they are being both engaged and educated. Children enjoy involvement through active participation, which makes music a prime-learning tool for them. Whether they are listening, clapping, playing, or just simply reading the notes, students are taking a physically and mentally active role in the lesson while at the same time learning skills that, if presented in a traditional mathematics class, they may automatically dismiss as unimportant or uninteresting. Due to the reasons stated above, the mathematics of music is a topic worthy of consideration.

Rationale:

As I think back to my student teaching experience, many memories are evoked. The one memory that stands out the most is experiencing my epiphany in teaching. The epiphany I refer to is the innovation of creating a game out of any lesson. I observed my cooperating teacher do this on a daily basis. He was able to transform a mundane lesson into one that had all the students in the class willing to participate without very little encouragement. Some of the games he used in his lessons, he actually had copyrighted. I found through my observations that you could take any lesson in the curriculum that you are given to teach and transform it into a game. Taking the objectives in the lesson and combining it with a favorite game format put a different spin on your lesson objective. During my student teaching experience, I witnessed students play games throughout their school day. For some of the students this was the one lesson that they eagerly anticipated. The students showed an excitement when it was announced that they were going to participate in a lesson that was in a game format. Games by nature tend to bring out in human beings a time of enjoyment and lower their level of concern. Playing a game puts students in a mindset of relaxation and enjoyment. The traditional lesson now becomes one that attracts their attention, one in which students feel they can participate and actually have an opportunity to succeed. Their level of concern is lowered and learning becomes something that summons their interest. When playing a game, students seem to forget that you need certain skills to successfully play the game. Skills become secondary; they are not the main focus of the lesson to the student. Students appear eager to work on an objective if it is put into the format of a game. The game selection you wish to incorporate into your lesson is where your creativity as the teacher intervenes. You have the ability to reach into your teaching repertoire and pull from your experiences to create your game lesson. By doing this, you capture and engage your audience; you tap into their inner thinking. Students are more inclined to participate in a lesson that evokes their senses. In every lesson you teach, your objective is to get the students actively involved in the lesson. Being a participant in a game is one form in which students show active participation.

In the world of education, there has always been the great challenge of educators versus students. This challenge has intrigued educators for decades. Educators have debated over the best practices for the impartation of knowledge. Traditional versus Non-Traditional Classrooms, Open Classrooms, Charter Schools and Home Schooling are just a few alternatives that are part of the great debate. By this I mean that educators are faced on a daily basis with trying to come up with effective yet creative ways to perfect their craft. Perfecting their craft is more than regurgitating what the textbook manuals have in them. The classroom teacher must take the curriculum and analyze how it will best meet the needs of his/her students. The teacher must also take into consideration the

learning styles of the students. Since all students do not learn in the same manner, lessons have to be created to meet all the students' needs. Following a great deal of research and planning, a lesson evolves. In today's society this is a difficult task. What it actually entails is coming up with lessons that try to compete with the video games that students explore on Playstations, X-Boxes and Nintendo Gamecubes. With the video game technology available to students, it is the educators' greatest challenge to engage the students with meaningful learning opportunities in the classroom. To have active participation from all students is the main objective in any lesson. Games provide students with instant spectator stimulation; that is, a video game gives its player constant colorful visual stimulation while engaging him/her with onscreen action and a place to channel personal focus. What a teacher needs to do is find a way to engage students in such a manner that channels their focus in a similar fashion. Through teaching students music, visual and physical stimulation can be used to gain that focus. Video games make the student a part of a world that does not really exist; actual physical activity makes them a part of the real world while allowing them to have fun at the same time. Teachers in all disciplines are always looking for new and innovative ways to more effectively affect student learning, and music may just be one of those ways.

Many educators believe that music stirs emotion, which has an important effect on memory. Thinking back to my daughter, I was given advice on how to teach her the infamous address and phone number to go to kindergarten. This was a prerequisite for entering school. I was told to select a tune, something that would capture her attention and sing the address and phone number over and over to it. I was quite surprised how quickly my daughter was able to memorize both her address and phone number. She enjoyed singing it and did so frequently. Revisiting that memory made me think of how I could incorporate that advice into my mathematics curriculum. Having students sing songs about counting numbers, pattern identification, addition, division and fractions helps students with their internal vocabulary and mathematical concepts as well. Songs can be an ideal way to get students involved and interested in daily mathematical routines in the classroom. They can also help to solidify new mathematical skills through repetition. Pairing directions with familiar song melodies adds an element of fun, but also provides structure to the daily schedules.

In the mathematics of music I would like to explore teaching several mathematical concepts through music (i.e. songs, dance, and instruments). In music sound is measured just as the clock measures time. Where the clock is based on divisions of 1's and multiples of 5 until you reach 60, music can be measured in all types of divisions most commonly in groups of twos and threes. At the beginning of every song a fraction is written on the staff. This indicates how the measures are being divided through the song. The top number of the fraction (Time Signature) indicates how many beats are in a measure; 4/4 means

four beats per measure, $3/2$ means 3 beats in each measure. The bottom number indicates which note value will receive a beat. A whole note is worth 1, a quarter note is 4, an eighth note is 8, and a sixteenth note is 16. So, $4/4$ means that each measure will have the equivalent of 4-quarter notes. So 4 quarter notes **or** 8 eighth notes or 2 half notes or 1 whole note will work. So $4/4$ means there will be four beats. Each beat will be represented by a quarter note (four/quarter notes). Another example, $3/2$ means there will be three beats, with each beat represented by a half note (three/half notes). The note names are based on the most common division of time, which is $4/4$. Although there are three half notes, each half note still receives two quarters, so the division remains consistent. The beats can be divided into smaller divisions/measurements, as long as the divisions total the needed beats indicated in the TIME SIGNATURE $4/4$, $3/4$ $6/8$, $3/2$, After students have learned about note values in music, this would now give them the opportunity to transfer this knowledge into fractions. In teaching fractions word choices are very important for the repetition of concepts. Calling out the fraction two fourths ($2/4$) is more easily introduced as “two of four”, which is in alignment with Everyday Math’s concept of division. The reverse would be two divided by 4, giving you a transition of changing fractions to decimals to percents, $2/4 = 0.50 = 50\%$. Having students build from music divisions where four is equal to the whole, enables them to transfer this knowledge to their mathematics class.

In the Everyday Mathematics Fractions Unit, your anticipatory set could be to use a piece of sheet music. At the beginning of every song a fraction is written on the staff. The teacher explains to the students that the fraction indicates how the measures are being divided through the song. This leads to the introduction of fractions and their components. Each fraction has two basic parts, the numerator and the denominator. In a fraction, the number written above the line or to the left of the slash is the numerator. In a part-whole fraction, where the whole is divided into a number of equal parts, the numerator names the number of equal parts being considered. In the fraction $2/4$, 2 is the numerator. The number written below the line or to the right of the slash is the denominator. In a part-whole fraction, the denominator is the number of equal parts into which the whole (or ONE) has been divided. In the fraction $2/4$, 4 is the denominator. Having students read the sheet music during your fraction lesson gives them a musical connection to mathematics.

Another example of infusing music into your mathematics class is to have the students learn their basic facts of addition, subtraction, multiplication and division through the art of song. For some students this is a task that is unreachable through the traditional methods. Rote memorization, a practice that many educators were exposed to during their tenure in school does not work for all students. According to Jere Brophy, “students are more likely to want to learn when they appreciate the value of the classroom activities and when they believe

they will succeed if they apply reasonable effort.” Since students don’t learn in the same way, finding alternatives to meet all students’ needs is a necessity for the teacher. Paper and pencil activities in the math curriculum may not address the learning needs of all students in the classroom. For students who have the main intelligence of music, listening and singing along to basic facts might be more beneficial for them. This method gives these students a more meaningful learning experience. This method can be used as an entire classroom activity or set up in learning stations in the room.

Jere Brophy in his article Synthesis of Research on Strategies for Motivating Students to Learn gave me a great deal of insight on motivation. He states “the simplest way to ensure that students expect success is to make sure they achieve it consistently.” This should be one of the teacher’s main objectives for the students. Teachers must be cognizant that “people do not invest effort on tasks that do not lead to valued outcomes, even if they know they can perform the tasks successfully, and they do not invest effort on even highly valued tasks if they are convinced that they cannot succeed no matter how hard they try.” Brophy continues to states “students prefer tasks that have meaning or integrity in their own right over tasks that are mere subparts of some larger entity. Teachers can portray effort as an investment, which will produce knowledge or skill development and thus empower students, rather than as a risk of failure or embarrassment.”

In Edgar Dale’s Cone of Experience he used a pyramid to show how learning occurs in people. The pyramid was broken down into stages. Each stage was given a percentage value. He looked at different ways of how information was dispersed to people. His research showed that there are multiple ways that people remember information presented to them. He also showed the differentiation of learning outcomes among people. As educators we are always looking for creative ways to get our students to delve into the topics in our classroom. Looking at Edgar Dale’s Cone of Experience lends an opportunity for educators to come up with creative, but also effective methods to incorporate the best learning practices in their classrooms. Without the best learning practices we are doing a grave in-service to our students. According to Dale ‘s Cone:

People generally remember:

10% of what they read (i.e. textbooks, worksheets)

20% of what they hear (i.e. lectures, directions,)

30% of what they see (i.e.: overhead transparencies, movies, videos)

50% of what they hear and see (i.e.: watching a demonstration, fieldtrips, attending exhibits)

70% of what they say or write (i.e.: participate in Hands-On Activity, cooperative groups)

*90% of what they say as they do a thing
(i.e. model a real experience,
design/perform a presentation)*

***People are able to:
(Learning Outcomes)***

10% - 20% These are the people who **read** or **hear** information
They can *define describe list explain*

30% - 50% These are the people who **hear** and **see** information
They can *demonstrate apply practice*

70% - 90% These are the people who **say** , **write** and **do** the
information
They can *analyze design create evaluate*

(see Appendix A)

Incorporating music into mathematics will give all types of learners an opportunity to grasp the information that best meets their learning styles. The utilization of Edgar Dale's Cone of Experience gives the teacher a clearer view into how students process information given to them. As teachers we want the students to remember the information presented to them in such a manner that they will retain a high percentage of it. Through this retention the student is able to transfer it more effectively in their present and future work so that the students learning outcomes are on a level that permits them to analyze and create into their thinking process, a method that brings them positive evaluation. Students don't have to feel frustrated participating in a math lesson incorporating music as they might in a traditional math lesson. A traditional lesson may consist of listening to the teacher, reading directions, observing demonstrations and/or teacher modeling, and ending with pencil and paper activities. This new form of participation now gives them an alternative to receiving the information in the lesson. Their self-esteem is raised and this puts the student in a positive mindset

to receive future information. Promoting active participation is the long-range goal.

My lessons plans will all be based on this principle. Music already has its own math lessons built in, so it makes sense for a teacher to use that to his or her advantage and harness that lesson. All musical activities, even things as simple as tapping your feet to the beat of a song, teach the learner some sort of mathematical skill. Normally, that skill is taken for granted or just considered part of the greater picture. What I hope to do is change the focus around so that instead of that skill being one part of a big picture, it is the entire picture. Clapping a beat would no longer be about working to create a song, but instead it would be about counting and fractions. Through several mini activities, music will be incorporated into math lessons. Students will be able to utilize abstract methods woven into concrete ones to help simplify their understanding of mathematics. This type of teaching style will prove beneficial not only to students who have special talent in music, but also to those who possess skills in spatial and visual, logical thinking, body sense and coordination, interpersonal-emotional, and interpersonal skills. One of the main components of the lessons will be teaching students note values, which involves visual intelligence. Another key part of this unit will be using critical thinking skills involving the basic operations of mathematics, which consist of addition, subtraction, multiplication, and division, based on those notes. Body sense and coordination will be necessary to do things such as clapping out rhythms and dancing, both of which will play some role in a lesson. The cause and effect relationship between two people clapping out two different rhythms or playing two different melodies will also be explored. This relationship deals with how students deal with their own musical/mathematical task (interpersonal intelligence) and how they react and respond to others doing different musical/mathematical tasks (interpersonal-emotional intelligence). While seeming to be only about music, these lessons will in fact be of use to students with a wide variety of intelligences.

As far as the mathematics content standards are concerned, my “Mathematics of Music” unit will incorporate standards one, two, and three. The first standard states “All students use numbers, number systems, and equivalent forms (including numbers, words, objects, and graphics) to represent theoretical and practical situations.” Music, with its use of time signatures, notes, and rests, fulfills this standard perfectly. My lessons will include teaching students the meaning of time signatures, which deal with numbers and notes and rests, which serve as a graphical representation of actual numbers, with each note or rest holding some sort of fractional value. Using these symbols allows a student to apply mathematics to the practical situation of singing, dancing, or playing an instrument. Standard number two states “All students compute, measure, and estimate to solve theoretical and practical problems, using appropriate tools, including modern technology such as calculators and computers.” One part of the

unit will consist of using music theory based computer games to gain a better understanding of note values. Students will also use simple rhythm instruments borrowed from the school's music department, such as bells, triangles, or tambourines. The third standard states "All students apply the concepts of patterns, functions, and relations to solve theoretical and practical problems." Patterns will play an important role in one of the lessons in my unit due to the fact that songs often follow a pattern based on verses and choruses. By teaching students the nature of these patterns, they will be able to better understand songs that they hear or play.

Objective:

Through research we have discovered that all students have Multiple Intelligences. In other words, all students do not learn the same way. Each student has his or her own set of skills, based on specific types of intelligence, that allow him or her to benefit from different teaching methods in different ways. It has been found that students learn best using their different intelligences. According to Howard Gardner's Theory of Multiple Intelligences, "human beings are capable of seven independent means of information processing" (<http://www.bena.com>). Those seven intelligences are language intelligence, spatial and visual intelligence, logical thinking intelligence, body sense and coordination intelligence, interpersonal-emotional intelligence, interpersonal intelligence, and musical and rhythmic intelligence. The main focus of this unit will be the seventh intelligence, musical and rhythmic; however, several of the other intelligences will play a role. I have chosen this particular intelligence because of its natural connection with the field of mathematics. In order for students to have success in music, they must have a mathematical background. Every piece of music played involves numbers and some form of simple mathematical computations; it is as if each piece comes with its own built in math lesson.

Strategies:

Just as music provides many ways to look at mathematics, there are many methods by which to teach musical/mathematical lesson. In my unit, I plan to use several different methods to make the lessons both enjoyable and educational. As I stated earlier, children like it when they are engaged in what they are doing; stimulation will keep their attention and therefore help the information stick in their brains. The strategies that I plan to use are a combination of traditional music theory methods, modern technology, physical activity, and interactive music creation.

First of all, I will teach students about the mathematics of music by teaching them the basics of music theory. As I mentioned in my rationale, basic

theory will be needed in order to understand what role mathematics play in music. If a student does not know what quarter notes and half rests are, they will never be able to see the math behind that. The first strategy that I plan to use is to teach the students using the presentation of notes, rests, time signatures, and the music staff. By presenting them with this background, I hope to build the foundation needed for the rest of the unit.

Next, I will use modern technology to further my lesson. Earlier, I talked about the necessity of game playing in the classroom. If one can turn a lesson into a game, then the chances of a student's becoming interested in the topic go up. Computer games exist that can help students understand things such as music theory. I plan to have the students play some of these games after I present them with an introduction to music theory so that they can have more experience with the material taught while at the same time having fun playing on one of this generation's favorite toys. Another piece of technology that I plan to use is the Compact Disc. Several companies have manufactured CDs that play mathematics themed songs. Through these, students will not only be able to gain whatever math message is presented in each song, but they will also be able to identify various parts of the song, such as note values and patterns.

The third strategy that I want to use is physical activity. Music is one of those things that has a physical activity already associated with it: dancing. Dancing involves counting rhythms and paying attention to patterns and repetition, so it will naturally fit in with my mathematics of music lesson. While dance will help, there are other physical activities that are equally effective. Clapping and/or tapping can be used to help with counting and realization of note values. Not only that, but both of these activities can be used to reinforce critical thinking skills if made into games, which I plan to do. I also want to incorporate general movement into my unit. Simple motions that accompany sounds and note values will give students a more concrete idea of what each of these things mean.

Lastly, I want to make use of music creation. Listening and dancing along with music can definitely help students understand music and the math behind it, but actually creating music brings a whole new level of stimulation and interest. What a student may not get from listening to a CD or even playing a game, he or she may get by banging away at the tambourine or tingling on the triangle. Immersing the student in music creation will bring the lesson to a whole new level, filled with interactive entertainment and the physical realization of what is being explained in the classroom.

Classroom Activities

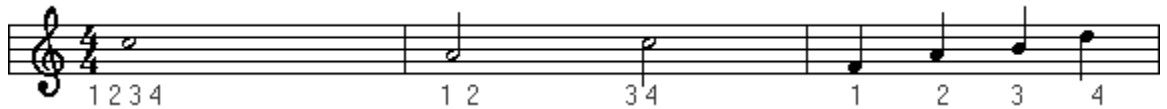
Basic Counting:

One of the most obvious questions is how musicians know when to play. Well, it's easy. They learn to count the beats.

First let us present you with this.

1 whole note = 2 half notes = 4 quarter notes = 8 eighth notes = 16 sixteenth notes.
Keep that in mind while looking at these examples.

Let's start with this example.



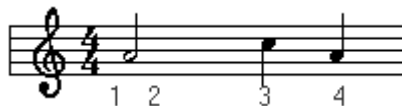
First off, looking at the time signature you know that there are **4 quarter notes per measure**.

In the first measure the whole note gets all the beats (1, 2, 3 and 4) because 1 whole note = 4 quarter notes, and there are a total of 4 quarter notes per measure.

In the second example, each half note gets 2 beats because 2 quarter notes = 1 half note.

In the third example, each quarter note gets its own beat because there are 4 quarter notes per measure (time sig).

Let's intermingle the 2 quarter notes and a half note.



The half note gets the first 2 beats, and each quarter its own beat. This makes sense because the 4/4 time signature means there are **4 quarter notes per measure**. 2 quarter notes + 1 half note (which is

really 2 quarters) = 4 quarter notes, the total number of quarter notes for that measure (time signature).

Let's add in the eighth notes.



In this example there is something new. The + sign. It just means "and". If you said 1 + 2 + ... out loud it would sound like this.

1 and 2 and 3 and 4 and

Each eighth note is 1/2 of a quarter note, therefore it takes 2 eighth notes to equal 1 quarter note.

Think of it like this: the 1 and the "and" are both half of one quarter note and together they form 1 quarter note and from the time signature we know there are 4 quarters per measure.

This may seem a little confusing now, but all of a sudden it will click. You will hit yourself in the head and wonder how you never understood it.

After you understand this, go on to the counting basics 2 page. ;-)

Basic Counting, Part 2:

Let's introduce a mixed example.



The quarter note is obviously beat 1 because from the time signature you know there are 4 quarter notes per measure. You also already know one half note = 2 quarter notes therefore the half note must be beats 2 and 3. Finally, you know that two eighth notes = 1 quarter note so they must be the "4 +".

When many different kinds of notes are intermingled, it starts to become tricky to count. Musicians will sometimes subdivide the notes so the counting flows more easily. Let's use the above example, but this time subdivide it.



Here every note in the measure is subdivided into 8th notes thus making it a lot more "fluid" to count. Its pretty easy to understand too... one quarter note is two 8th notes, so it gets "1 +". The half note is really four eighth notes so it get "2 + 3 +". And the each 8th note get a half so one is "4" and the other is the "and" of 4.

Here would also be a good place to throw in a few examples with rests. These will just show the counting and will not explain them. Just think of the rests in terms of their corresponding notes and you'll have no problem!



Counting the 16th note.



Basically counting 16th notes is similar to 8th notes except that you need to add more things to count with. I was taught using "e" and "a", but feel free to use what you want. Each part, the "1", "e", "+", "a" are all 1/4 of 1 quarter note. Together they add up to 1 beat according to the time sig. (4 sixteenths = 1 quarter)

Different time signatures and different notes.

Here you are at the top of the note hill. Just look at these and the counting section is over!



Remember from this time signature you are counting the 8th notes.

Remember you are counting half notes, and therefore you have to subdivide the eighth notes and quarter notes accordingly.

Music in Mathematics Unit

My unit consists of lessons that can be used in intermediate elementary math classes.

Lesson 1 – Teaching Basic Facts (Addition, Subtraction, Multiplication and Division)

Objective: SWBAT correctly solve basic facts

Motivation: integrating music into mathematics

TSW: actively engage with Music for Teaching Mathematics
<<http://songsforteaching.net/sarajordan/math.htm>>.

TTW: play the tapes to class as a daily, weekly or monthly routine to use with entire class

TSW: use tapes at a Listening Center

Lesson 2 – Multiplication/Division Facts Tic-Tac-Toe

Objective: SWBAT correctly solve multiplication/division facts

Motivation: Integrating music in mathematics

TTW: draw a Tic-Tac-Toe grid on the board
divide the class into X team and O team
review rules of the game

put on light music in the background during the game

TSW: correctly solve basic facts to get 3 marks in a row, column or diagonally or 5 marks at end of game
the first team to win 3 out of 5 games is the winner

TTW: record and post results of each time game is played so students have access to review results

Lesson 3 – 50 Facts (Addition, Subtraction, Multiplication and Division)

Objective: SWBAT correctly solve 50 basic facts in five minutes
SWBAT practice taking timed test

Motivation: integrating music into mathematics

TTW: make a tape with instrumental songs that are 5 minutes long (i.e. Theme from Rocky; Chariots of Fire; Jeopardy)
If you put multiple songs on the tape, you can use the tape the entire school year.

TTW: play the tape while students try to answer 50 facts in five Minutes

TSW: keep their results on a chart in their folders or a classroom chart posted

Lesson 4 – Two-Digit Multiplication Cha-Cha

Objective: SWBAT dance to the steps of multiplication process

Motivation: The teacher will dance in front of the classroom

TSW: actively engage in dancing the Cha-Cha

<<http://www.lessonplanspage.com>>.

Lesson 5 – Music Times Facts

Objective: To memorize multiplication facts 7×3 through 7×8

And 8×3 through 8×8

Motivation: integrating music in mathematics

TSW: actively engage in singing multiplication facts

<<http://www.lessonplanpage.com/mathmusicmultiplicationfacts24.htm>>.

Mathematics Standards

1. All students use numbers, number systems, and equivalent forms (including numbers, words, objects, and graphics) to represent theoretical and practical situations.
2. All students compute measure and estimate to solve theoretical and practical problems, using appropriate tools, including modern technology such as calculators and computers.
3. All students apply the concepts of patterns, functions and relations to solve theoretical and practical problems.

Art and Humanities Standards

1. All students describe the meanings they find in various works from the visual and performing arts and literature on the basis of aesthetic understanding of the art form.
2. All students evaluate and respond critically to works from the visual and performing arts and literature of various individuals and cultures, showing that they understand important features of the works.
3. All students relate various works from the visual and performing arts and literature to the historical and cultural context within which they were created.
4. All students produce, perform, or exhibit their work in the visual arts, music, dance, or theater, and describe the meanings their work has for them.

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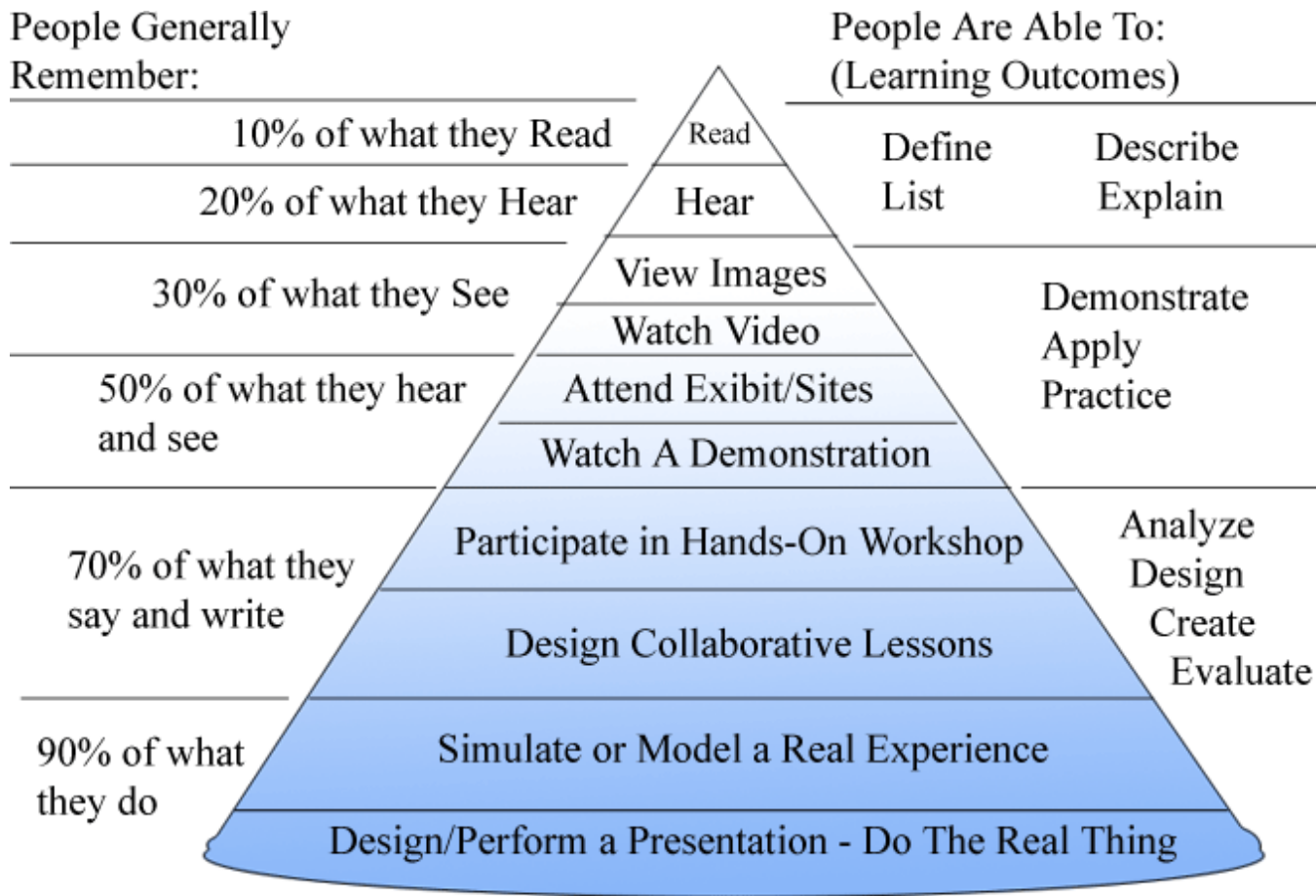
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Appendix A-Dale's Cone of Experience



Dale's Cone of Experience