

Geometry and the Buildings We Live In
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Overview

The unit *Geometry and the Buildings we Live In* can be used along with the regular Pittsburgh Public School 4th and 5th grade Math Curriculum, specifically in conjunction with the geometry units 3 and 11 in 5th grade and 1 and 11 in 4th grade. The activity may be accomplished in a 2 to 3 week period.

Information in this unit provides students opportunity to learn by being an observer, by exploring, deconstructing and building geometric figures within the context of the buildings students live in and around. Lessons include identifying geometric form in the world around us by first observing and then constructing, geometric figures, cubes, tetrahedron, prisms and pyramids. These forms are then explored outside the classroom in design and architecture in the school and community and with the help of the Pittsburgh History and Landmark Foundation, of the greater Pittsburgh area.

The students will complete this unit with an expanded understanding and appreciation of geometry in our world.

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Rationale

Informal geometry is featured throughout the Pittsburgh Public Schools K-3 *Everyday Mathematics* curriculum, and more concretely in 4th and 5th grade.

Students begin to examine properties of basic plane figures using tools such as compasses, rulers, and protractors. They deal explicitly with facts concerning angles, sums of triangles, quadrangles, and polygons in general. Without fail, the students' favorite part of their geometry sampler is always the exploration of tessellations with polygons. They are drawn to the natural order of such design. It may be years until the word Pythagoreans rolls off their tongues, but they understand equilateral, hexagonal, triangular numbers and pentagrams. They recognize properties of polygons and tiling, the art of covering a plane surface with polygons. This mathematics is visual, spatial, tangible and surprisingly to the kids, architectural, historical and even sacred.

Geometry figures are all well-defined sets of points. Geometry deals with the properties of those sets of points and relationships between and among them. From an early age, students are able to make sense of shapes, those that move, those that don't, those that are curved, those that are not. The introduction to geometry involves naming and classifying those characteristics, helping students "notice" what they see and organize their perceptions into a meaningful system.

The presence of geometry in the spaces in which we live is evidence of how we humans interact with and even depend upon the world of geometry. We take patterns and shapes evident in our natural world and use them to create the places in which we live. Buildings, their form and function, are representations of our needs as human beings and landscapes for our visions of beauty and style. Humans don't simply inhabit a space but interact with it.

It is the principles of geometry that dictate how we experience the spaces in which we live. It is the goal of this unit to define the principles of geometry, identify its existence in our everyday world, and interact with form through construction.

Math in Architecture

The word geometry derives from Greek words for "earth" and "measure", which gives a clue about the first geometric activity of humans. The earliest records of geometric thinking, from the Egyptians, Babylonian, and Chinese, confirm that it revolved around solving practical problems – laying out fields, finding areas, volumes, constructing houses and temples, and so on.

The Greeks are credited with formalizing geometry. The approach to studying geometry today has been led by research known as the Van Hiele Levels and it is upon the work of two Dutch researchers in the late 1950s, that the Pittsburgh Public Schools' curriculum is based. They identified five stages in the development of geometric understanding. During the first stage a triangle is called

a triangle because its overall shape is like other objects that are also called triangles. Shapes at this stage are not broken into parts; line segments, vertices etc. Instead, students at this stage grasp the whole figure at once. It is a visualization stage where students benefit from hands on work with geometric solids and real objects from their everyday environment.

In the second stage children begin to notice the individual elements that make up geometric figures. They see that a triangle has three sides and three corners; they see that a square has four sides all the same length etc. It is recommended at this stage that children compare measure, sort and describe geometric figures, identifying names for parts: side, angle, face edge, and so on.

In the next stage students move beyond the analysis of single shapes and start thinking about relationships among different shapes. They can, for example, understand that squares are rectangle since they meet the minimal requirements; four sides and four right angles. This is also a stage for informal proof, which is the highest level in elementary school geometry.

Unit Work

This level of proof is difficult for students. Our whole world is experienced in three dimensions yet we seem limited in describing and defining the world in two dimensions. Where students have some trouble applying the formula for area and volume without a frame of reference, they have zero difficulty building a three-dimensional figure with unit cubes and determining its area.

An interesting exercise, which illustrates this perceptual issue, involves asking students given three toothpicks to construct four triangles. Most students struggle with the toothpicks because they are bound to a single plane. We might help students take a step toward understanding the space around them by encouraging them to step out of the 2-dimensional plane. Holding the 3 sticks at an apex to form a tetrahedron, gives the student three triangles, the tabletop contains the fourth triangle.

If students are asked to add clay to each of the apexes students will feel the strength of this combination of triangles. Students may argue that a square base system is more secure. A construction test could be conducted to test the theory.

Visualizing Shape

Visualization is a skill to be sharpened in this unit's activities because it helps one notice what is around them. But to back up an introductory activity is to remind

students that geometry means, “earth measure.” With two points placed on the board, challenge students to connect the points with the longest line they could. After performing the task it can be noticed that the shortest line between the two points is a straight line and that the straightest line is drawn with a straight edge.

Look at artwork by Saul Steinberg, asking students to view the drawings. The artist made three “visualizations” of what the straight line could become. Invite the students to draw a straight line and see what they come up with. The point of the exercise is to show the power individual ideas and the uniqueness of each visualization and most of all, that the straight line can have different meanings depending on the configuration of the whole picture... the line, the basis of all geometry.

(Saul Steinberg/Norman Shaprio)

Pattern Recognition

For many years, artists have made beautiful designs with geometric shapes. The Islamic religion did not allow artist to show human or animal forms. Islamic artists became very skillful at creating designs with geometric shapes. Many amazing designs can be found in Moorish architecture especially. In geometry, a tiling pattern is called a tessellation. A tessellation is the translation, rotation, and /or reflection of a shape. The pattern completely covers a surface. The shapes in a tessellation do not overlap or gap. Sometimes only one shape is used in a tessellation and this type of tessellation is called a same-tile tessellation. Only some regular polygons will tessellate.

Using the template (fig.1 regular polygon patterns), students can discover whether regular polygons tessellate by cutting a collection of polygons, triangles, squares, pentagon, hexagon and octagons, putting them together and seeing if the patterns meet the criteria. After making a determination, draw examples and designs with tessellating figures (fig. 2). Apply observation skills to the student environment and look for examples of tessellation. Ask students to collect examples in nature and architecture, in every day design and create a tessellation museum.

Is there some pattern to this pattern making or a rule which can be applied to predict if a shape will tessellate? If squares tessellate, will all quadrangles tessellate? We have already proven in our early geometric explorations that the sum of all angles of a triangle equals 180 and the sum of the angles of all rectangles equals 360 degrees, but what about the other tessellating shapes? When we arrange regular polygons that tessellate, what is the sum of those angles? Is there a clue here? In the end most students will see that shapes will not tessellate

if they cannot be arranged around a point so that the sum of their angle measures 360 degrees. Furthermore they will see that when two quadrangles are put next to each other in a tessellation, the longest sides match and the shortest sides match. (fig. 3 – same tile tessellation , tessellation example; no overlap and gaping).

Geometric Solids

Clearly there are 2-dimensional patterns that are identifiable in many tile patterns, but when we move into the 3rd dimension, and look at the structure of things, we can see the geometric solid forms upon which the structure of our spaces are built. Space is the 3-D world of our experience. Like points, lines, and planes, it cannot be defined in simpler terms. Spatial objects have length, width, and depth. Spatial shapes are 3-D objects in space and they come in infinite kinds and sizes.

A person with good spatial sense can mentally manipulate 1, 2, and 3 dimensional objects in space and describe their relationships. Spatial sense is important in constructing 3-D objects, drawing 3-D objects in 2-D. Video games often demand a well developed spatial sense and our kids should be good at that. To have the vocabulary to talk about structural form, working with and building 3-dimensional forms is a good practice. Familiar 3-D mathematical shapes are known as geometric solids. An empty box with a lid/bricks, basketball/baseball, empty ice-cream cone/filled ice-cream cone, empty food can/rolling pin... the first items are hollow the second are filled inside. Geometric solids are the “skin” of a shape and the edges and vertices are the “skeleton.” Points inside the shape, not on any of the faces or surfaces are not considered part of the geometric solid.

A goal in fourth and fifth grade mathematics is to build strong connections between 2-D figures, such as polygons and curves, and corresponding polyhedrons and curved surfaces in 3 dimensions. One way to work toward this goal is to build 3-D models. Students will cut out 2-D patterns of prisms and pyramids and a variety of polyhedrons and fold and tape together. These models will then be used to compare and contrast the properties of vertices, edges, and faces of the solids they represent. The language developing will ideally translate to conversation about architectural structure.

(Fig. 4, geometric forms for construction)

Our Buildings

Students are familiar with the places where they live, so it can be a good starting point for discussion and learning about the evidence or geometry in our world of

buildings. Ask students to think about their homes or apartment buildings while encouraging them to think and remember details and express them in drawing will generate a place to begin looking at architecture.

What shape is their house? Is it attached to another house or does it stand alone? How many windows and doors does it have? Talk about the rooms, spaces and activities that go on inside and outside. Write down the address of the house and list the names of the people who live there. These activities are inspired by the Pittsburgh History and Landmarks Foundation and a partnership through grant money will develop this part of the unit. Students will work on an individual evaluation of their own homes; they will be introduced to architectural styles and time periods and evaluate the shapes of their homes in geometric and architectural terms.

This activity related to our personal spaces will extend in a following lesson to the larger world around us, our city of Pittsburgh. Students will study photographs and artifacts of Landmark buildings in Pittsburgh. Students will compare and contrast the buildings' architectural style in geometric terms and look for trends and style and size as they relates to the architectural time periods of 1817, 1949 and 1994.

Students can walk about our community and find sample patterns. Ornamentation on windows can take the form of square patterns. Tiling on floors can be combinations of squares and hexagons and they exist everywhere. Are there patterns that seem more popular in certain time periods?

Finally, students will do construction of their own, using large 3-d shapes constructed from PVC pipe. These forms will serve as the shells of the structures, which will become buildings of our dreams. They will apply skin to these forms and patterned geometric ornamentation. They will be presented to each other and described in terms of their geometry and architecture and how the structure would function in a community such as ours.

(Fig. 5,6,7,8 – Landmark work pages)

Lesson 1 Perception and Observation and How to Look

Objective: To encourage students to be perceptive, to look.

Process: 1. Begin with a discussion of what it means to look and be observant. 2. A warm up skill would be to look around the room and name/classify all circular, blue, red...etc, objects in the room.

3. A volunteer sits in the front of the room and is studied by the class. They leave and return with something missing... can we identify it?
4. Students look at their non writing hand held in any position. Without looking, at their paper, student draw their hand in 30 seconds attempting to use one line to draw the whole hand. They will see they have been able to capture the basic shape and purpose of a hand with one line.
5. Walk the street in front of school with clipboards and paper. Attempt a similar exercise with objects or buildings.

Lesson 2 Pattern

Objective: Students are introduced to the history and concept of tessellations. They explore which regular polygons will tessellate and which will not and uncover a rule of thumb to test.

Process:

1. Read about and display examples of tessellation (fig.)
2. Review the language of polygons and its properties.
3. Decide which of five regular polygons can be used to create same-tile tessellations using a pattern for multiple cutting of same figure.
4. After determining which figures do and don't tessellate, ask leading questions about what we know about sums of angles with familiar square polygon.... Can we see a pattern develop among those shapes that do tessellate? That being that a polygon will tessellate if their angles can be arranged around a point so that the sum of their angles measures 360 degrees.

Lesson 3 Three Dimension Forms

Objective: Review names and properties of geometric solids and build.

Process:

1. With a whole group use appropriate language to discuss the properties of geometric solids compared to everyday objects such as cans, ice cream cones, balls, etc... focus particularly on prisms and pyramids as their bases may not all look alike. Make sure students understand that a cube is a special kind of rectangular prism. Distinguish between pyramids, prisms and polyhedrons. Explore the variety of characteristics.
2. Using cube pattern and geometric solids patterns, students practice building solids from patterns.

Lesson 4 Drawing Your Home

Objective: Students observe and discuss architectural features in geometric language. They are introduced to building anatomy words and add these to their conversational language.

Process: 1. Open with a discussion about where students live. Ask questions about their house or apartments and encourage them to think about details and express their thoughts in sketches. Does their home attach to another? Stand alone? What are spaces used for? Who lives there? Makes these lists.
2. Using a booklet provided by the Pittsburgh History and Landmark Foundation (grant money pending), help students focus on the details of their homes and draw them.

Lesson 5 Introduction to Pittsburgh Buildings

Objective: Having explored form and structure and our own homes, students will now look at important Pittsburgh buildings with the same eye and conversation.

Process:

1. Display old photographs and poster size pictures of important Pittsburgh architecture and neighborhoods.
2. Discuss the age of, reoccurring motifs of, uses of, structural similarities and differences of... Emphasis in conversation will be not only be on the ideas but to use the language we've become familiar with.
3. Discuss what future needs might be for our community/city. How could we meet those needs in our buildings?
4. Students draw a design they might like to see in a building, with a specific idea for its use and a plan for its geometric ornamentation.

Lesson 6 Building Our Buildings Culminating Activity

Objective: Demonstrate understanding and appreciation for the forms upon which the places we live are built by constructing a community of living, playing, working, or learning space from the ground up.

- Process: 1. Using what we know about the form and function of buildings, use materials provided, construct a building on a designated scale. Work will be done in collaboration with the art teacher (needs to be worked out but roughly student height... 4 to 5 feet.) Construction materials will include PVC pipe and other found objects as gleaned from Construction Junction and donations.
2. Certain specifics must be included and details labeled.

3. Ribbon cutting presentation of the buildings will be to parents and classmates upon completions.

Vocabulary

1. Space Words

Top/bottom

Front/back

Near/far

Leftside/rightside

Infront of /behind

Next to or beside

Above/below

To the right/ to the left

Compass directions: north, northwest, northeast; south, southwest, southeast; east, west location or locus

Geometric field.

2. Measure Words

similar, congruent

units of measure: inch, meter,

percentage, degree

square units of measurement, perimeter, area

polygon numbers: square, and rectangle numbers,

whole

3. Geometry/Art words

Art/design words: element of design: dot, line, shape, direction, movement, color, tone, texture, pattern, dimension, scale

Principles of design: unity, variety, balance, emphasis

Structure: rigid/flexible

Point Words: locus, interest, vertex

Line words: straight line, arc, line segment, ray

Polygon words: triangle, square, pentagon, Hexagon, octagon, regular polygon,

Circle words: center/radius/circumference/diameter/sector/chord

Triangle words: base, hypotenuse, altitude, median Pythagorean Theorem

Polyhedron words: sphere, cube, cone, cylinder, prism

5 platonic solids: tetrahedron, octahedron, cube, dodecahedron

Pattern words: tessellation, unit of repeat, transformations, symmetry, axis

Directional words: vertical/ horizontal, diagonal, rotational

Kinds of geometry: plane geometry, solid geometry

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Standards Met by Unit

NCTM (Mathematics) Standards:

Content Standards

- 1 Number and Operations
- 3 Geometry
- 4 Measurement

Process Standards

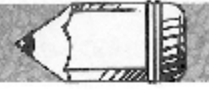
- 6 Problem Solving
- 7 Reasoning and Proof
- 8 Communication
- 9 Connections
- 10 Representation

Name _____

Date _____

Time _____

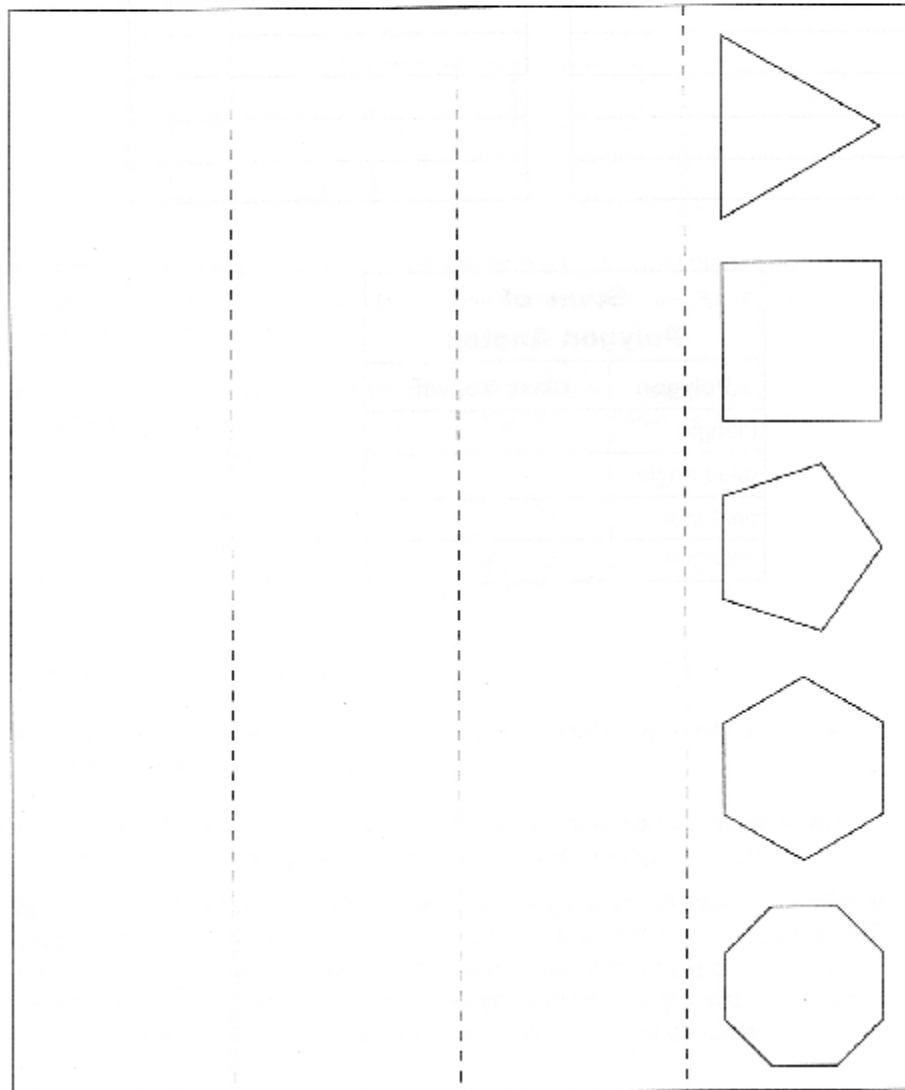
Regular Polygons



Fold the page like this.



Cut out the polygons. You will be cutting out four of each shape at once.



Copyright © S.W. McQueen, 1988

Fig 1a

Date:

Time:

Regular Tessellations



1. A **regular polygon** is a polygon in which all sides are the same length and all angles have the same measure. Circle the regular polygons below.



2. In the table below, write the name of each regular polygon under its picture. Then, using the polygons that you cut out from Activity Sheet 4, decide whether each polygon can be used to create a regular tessellation. Record your answers in the middle column. In the last column, use your Geometry Template to draw examples showing how the polygons tessellate or don't tessellate. Record any gaps or overlaps.






Polygon	Tessellates? (yes or no)	Draw an Example
 _____		
 _____		
 _____		

Fig 1b

Date _____

Time _____

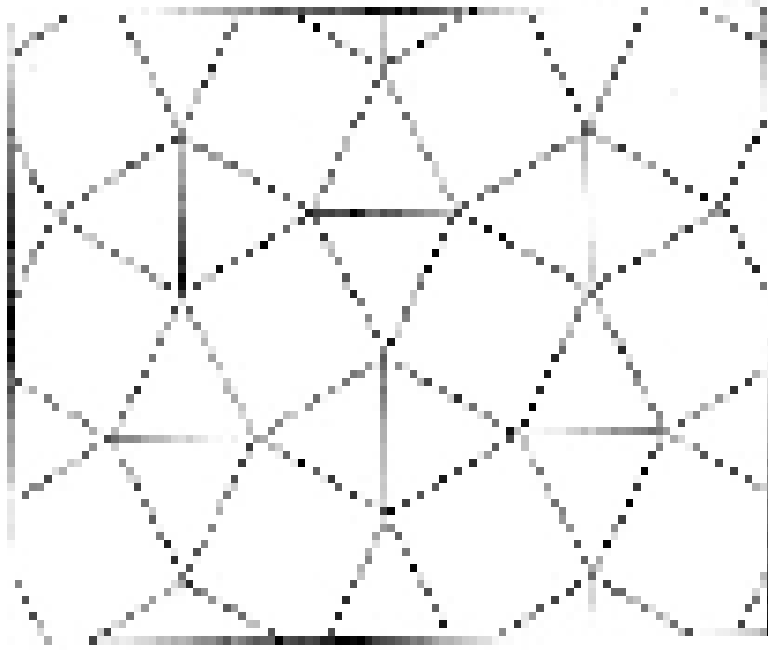
Regular Tessellations (cont.)

Polygon	Tessellates? (yes or no)	Draw an Example
 _____		
 _____		

3. Which of the polygons can be used to create regular tessellations?

4. Explain how you know that these are the only ones. _____

Fig 1c



i tessellation

Fig 2,3

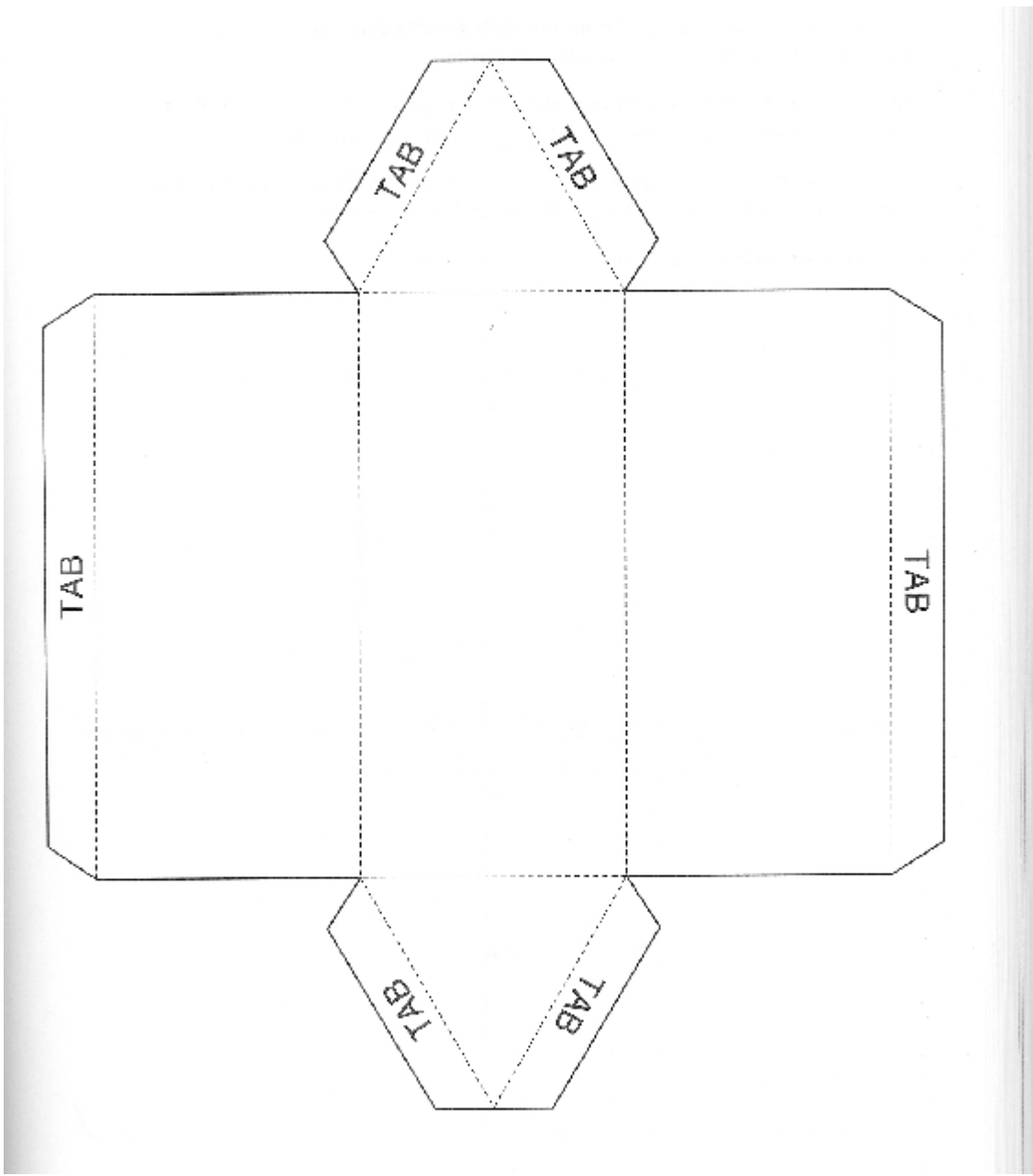


Fig 4a

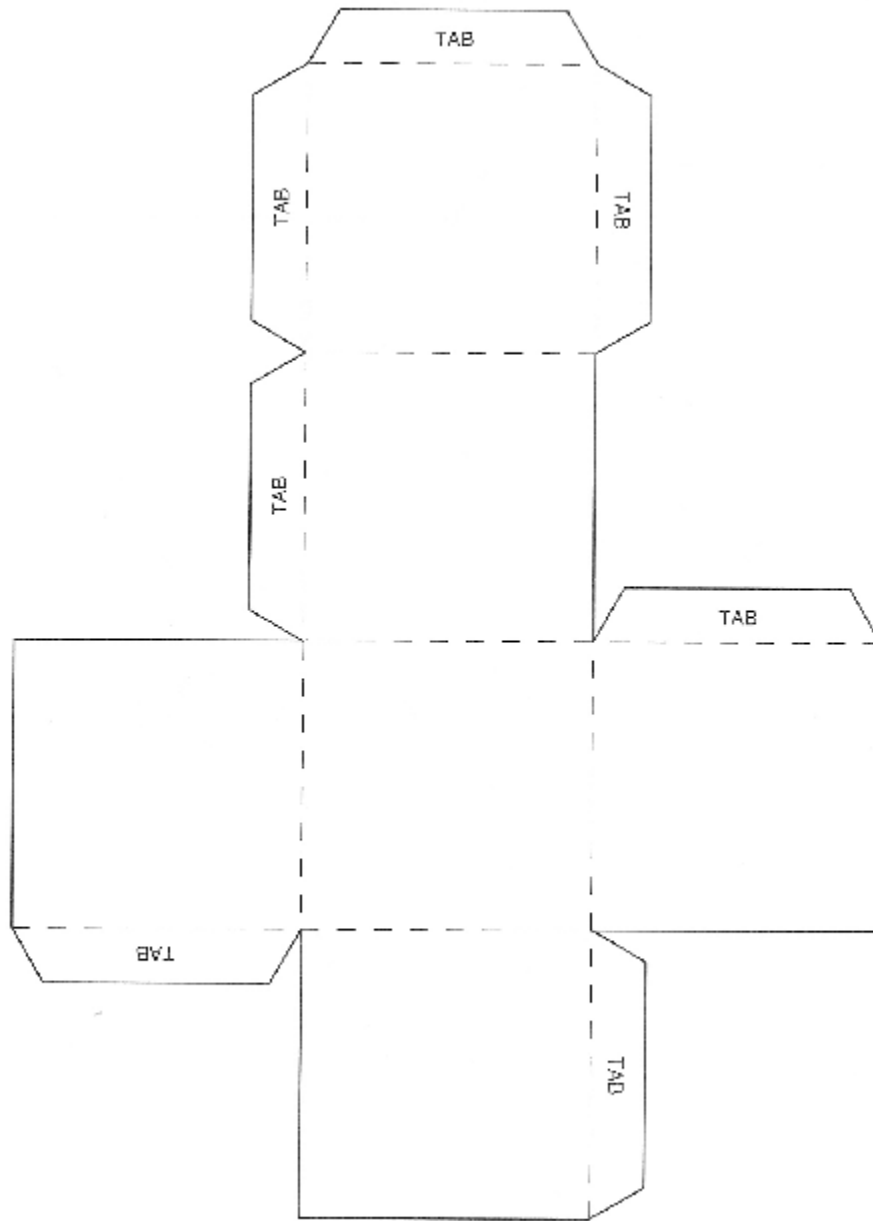


Fig 4b

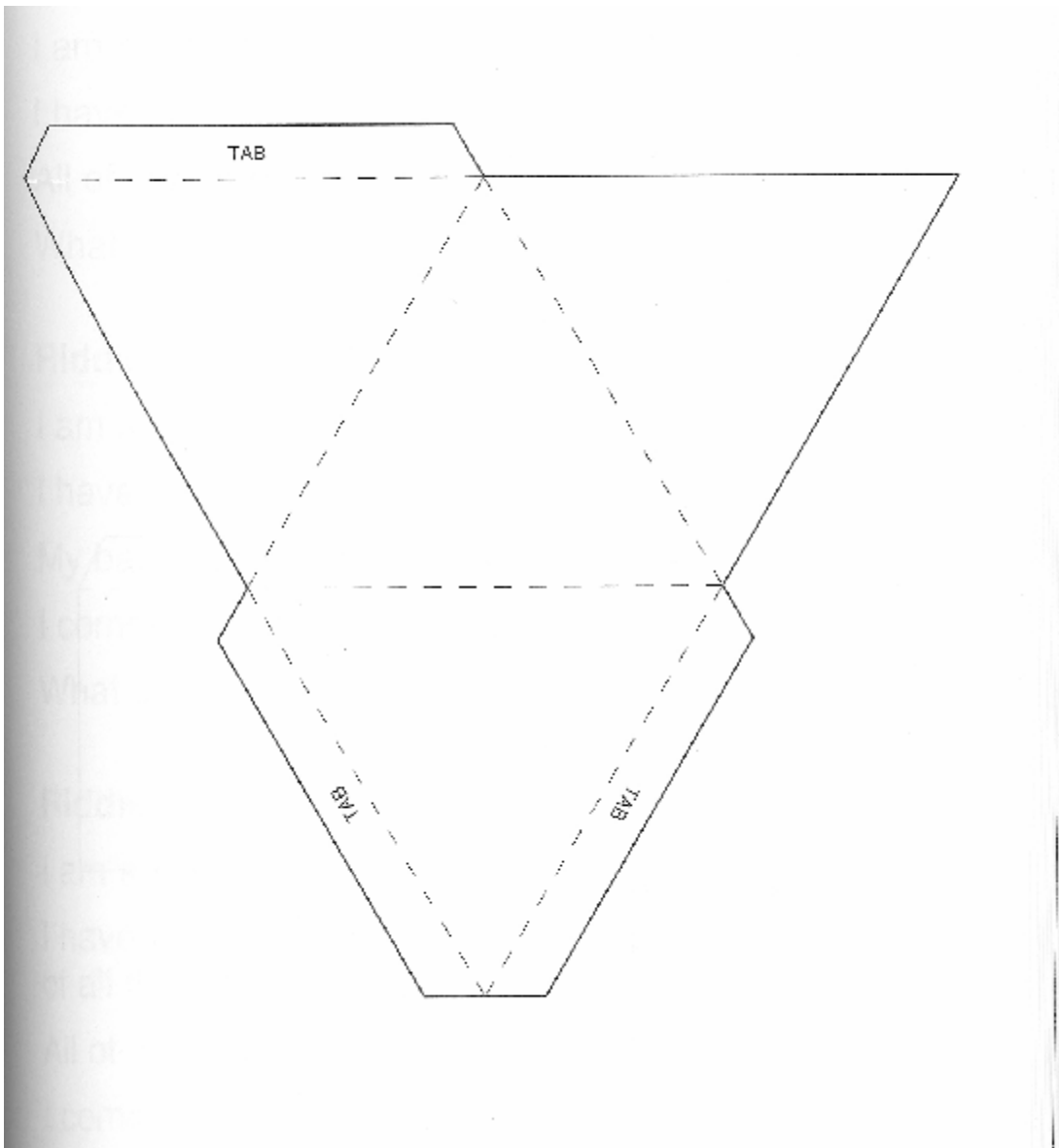


Fig 4c

Warm-up Exercises:

PERCEPTION

Goal: to encourage people to start noticing what is around them

A perception game for students

1. Find a partner and take about 10 seconds to observe how he/she is dressed and how he/she looks.
2. Have one partner (A) turn his/her back on the other partner (B).
3. While partner A is not looking, partner B should change one thing about his/her appearance: tuck in a shirt; unbutton a button; put your glasses on upside down, etc.
4. Tell Partner A to turn around and guess what is different about Partner B's appearance.
5. Switch roles and repeat the process.

Perception games in the classroom

Once a week, or once a month, change something in your classroom and see if your students notice: turn a poster upside down; hang up something new; move something out of the place where it is always supposed to be. This will help keep students alert and remind them that you expect them to be observant in the classroom.

DRAWING

Goal: to help students feel comfortable drawing; and to encourage expression and creativity

Sketching your hand

Each student must have a piece of paper and a pencil.

1. Ask your students to hold the hand they do not write with in front of them, in any position.
2. Tell them to look at that hand during the whole drawing exercise.
3. Without looking at their paper, ask students to draw their hand in 30 seconds. Challenge them to use one line to draw the whole hand.
4. Hold up the drawings so everyone can see.

Usually, students laugh and comment on the drawings. They rarely feel embarrassed because they were not allowed to look at their paper and they were only given 30 seconds. But, point out that in spite of these constraints, the drawings look like hands. Congratulate your students on being able to reveal the basic shape and purpose of a hand with one line, in a limited amount of time.

Repeat this exercise once a week, or once a month.

Quick sketches of objects/people

For variation, you might ask students to draw a particular object in the classroom, or the person seated next to him/her! Remember: students must not look at their paper while drawing and the drawings must be completed within a time limit.

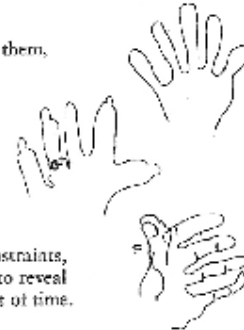


Fig 5a

My House

Name: _____ Date: _____

1. Draw a picture of the house or the building that you live in.

2. Describe the house or building that you drew using as many architectural terms as you can.

Prepared by: HISTORY & LANGUAGE TEACHERS
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Architectural Styles

As the world has changed, so have the styles of buildings. Different styles were popular at different times. In this project, you will learn about the styles of buildings that were popular in the past. You will also learn about the styles of buildings that are popular today. You will also learn about the styles of buildings that are popular in the future.



Colonial Revival (1880-1955)
 This style was popular in the late 19th and early 20th centuries. It was a reaction to the styles of the past, and it was a way to bring back the styles of the past.



Greek Revival (1820-1845)
 This style was popular in the early 19th century. It was a reaction to the styles of the past, and it was a way to bring back the styles of the past.



Palladian (1600-1800)
 This style was popular in the 17th and 18th centuries. It was a reaction to the styles of the past, and it was a way to bring back the styles of the past.



Second Empire (1850-1870)
 This style was popular in the mid-19th century. It was a reaction to the styles of the past, and it was a way to bring back the styles of the past.

Queen Anne (1840-1900)
 This style was popular in the late 19th and early 20th centuries. It was a reaction to the styles of the past, and it was a way to bring back the styles of the past.



Richardsonian Romanesque (1850-1890)
 This style was popular in the mid-19th century. It was a reaction to the styles of the past, and it was a way to bring back the styles of the past.



1. How many years ago did the Victorian Style start?
 2. How many years did the Victorian Style last?

Figure 5B

Downtown Landmarks: Math Facts

NUMBER/NAME OF LANDMARK	HEIGHT IN FEET	DATE
1. Allegheny County Courthouse (tower)	325	1888
2. Allegheny County Jail (stone wall)	30	1886
3. Blockhouse	26	1764
4. Burke's Building	50.5	1836
5. Gulf Building	582	1932
6. The Landmarks Building	75	1991
7. Monongahela Incline	635 (length of track)	1870
8. Smithfield United Church	259	1926
9. PPG Place (tower)	635	1984
10. USX Tower	641	1970

Make a graph showing the approximate height of each landmark.

Height in feet

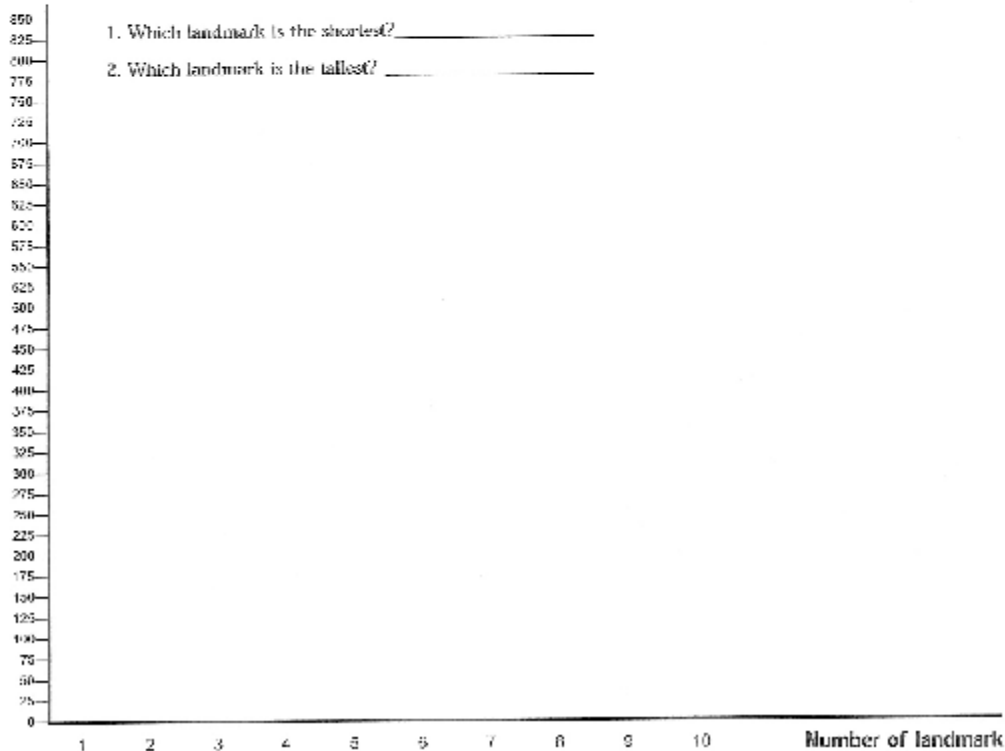


Fig 5d