

Chaos (Mathematical) in Teaching of Ecology and General Science

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Overview

Teaching chaos (mathematics) in Ecology and General Science is an ideal way of collaborating between mathematics and science. The standards in mathematics and science are central to developing a coherent approach to mathematics and science instruction. They set common targets for learning for all students. In April 2006, the National Council of Teachers of Mathematics (NCTM) released its update of the original NCTM standards trilogy. The new document is Principles and Standards of Mathematics. The PA Academic Standards for Mathematics became effective in January 1999, and the PSSA test has been linked to these standards since that time. In 1996 the National Research Council published the National Science Education Standards. In February 2002, the Pennsylvania State Board of Education adopted academic standards for Science and Technology. How best to implement these various sets of standards remains a challenge for teachers and their district leadership. This paper will help bridge the gap to improve the standards for mathematics, science, and technology.

So, one of the goals of teaching Chaos in Ecology and General Science is to improve the partnership between math and science in the classroom. By exploring a deeper understanding of mathematics all students will achieve better in science and other subjects as well. By injecting more math in science all will see the linkage between the two disciplines.

Rationale

National, state and local curriculum requirements and recommendations increasingly emphasize active student involvement in exploratory investigations. Teaching Chaos is one of many ways to help students meet the new standards they face in math and science. Science and math are keystones of life-long learning skills.

The major thrust of this project is to infuse a deeper understanding of mathematics in its relationship to General Science and Ecology. If these courses are more rigorous, adding the theory of Chaos will improve the achievement of students in a district. District leadership (The Superintendent of the Pittsburgh Public Schools in 2006) is calling for more academic rigor in core subjects such as science and mathematics. The National Science Foundation is trying to implement higher standards in the sciences by developing deep conceptual understandings such as the Chaos Theory in curriculum. These ideas emphasize problem solving, reasoning, communicating about mathematics and science, and making connections among topics and between disciplines. They encourage students to have hands on experiences while learning, and incorporate the use of calculators and computers. Programs, such as adding Chaos theory to General Science and Ecology, will challenge students to develop skills through more academic rigor, along with extensive drilling on basic skills, to do more advanced problem solving.

Another reason for incorporating advanced math into mathematics and science courses is to meet the academic standards set forth by the state of Pennsylvania. The Pennsylvania Department of Education is constantly increasing and improving the standards in both science and mathematics. By using Chaos theory it speaks directly to the academic standards concerning numbers, number systems and number relationships. The Chaos theory addresses the standards which deal with prime, whole, irrational, and rational numbers systems. Chaos theory also looks at the standard of computation and estimation. The use of basic operations such as addition, subtraction, multiplication and division are also used in teaching chaos. (Pa., Academic Std)

In fact, almost every Pennsylvania Academic standard for mathematics is met teaching Chaos. Some of these standards are, in addition to the ones already mentioned (1) Measurement and estimation; various types of measurement (e.g., length, time); Units and tools of measurements; computing and comparing measurements: (2) Mathematical reasoning and connections; using inductive and deductive reasoning; Validating argument (e.g., if...the statements, proofs): (3) Mathematical problem solving and communication; problem solving strategies; representing problems in various ways; interpreting results: (4) Statistics and data analysis; collecting and reporting data (e.g., charts, graphs); analyzing data: (5) Probability and predictions; validity of data; calculating probability to make predictions: (6) Algebra and functions; equations; patterns and functions: (7) Geometry ;shapes and their properties; using geometric principles to solve

problems: (9) Trigonometry; right angles; measuring and computing with triangles; using graphing calculators: (10) Concepts of calculus; comparing quantities and values; graphing rates of change; continuing pattern infinitely. (Pa., Academic Std.)

All the Pennsylvania State standards mentioned in the previous paragraph are used in teaching Chaos. These standards describe what students should know and what they should be able to do at eleventh and/or twelfth grade in a Pennsylvania high school. They reflect the increasing complexity and sophistication that students are expected to achieve as they progress through school and Chaos theory can help students meet these standards. (Pa., Academic Std.)

Students who understand Chaos theory will be able to communicate mathematically. Chaos theory is interesting, complex, and somewhat new in its applications in other disciplines. Chaos theory can stand on its own. Like other courses in mathematics, it can be used as a tool to help organize and understand information from other academic disciplines. Because our capacity to deal with all things mathematics is changing rapidly, students must be able to bring the most modern and effective ideas and tools to bear on their learning of mathematical concepts and skills. (Pa., Academic Std.)

Chaos theory will help students meet the Pennsylvania Academic Standards for Science and Technology. Some of the standards are: (1) Biological sciences; Life forms; Inheritance: Theories of Evolution; Cells: (2) Chemistry; Properties; Atomic theory; Thermodynamics: (3) Physics; Matter; Forces; Motion; Energy: (4) Earth Sciences; Land Forms and Processes; Resources; Meteorology; Hydrology and Oceanography; Astronomy: (5) Technological Devices; Tools; Instruments; Computers: (6) Inquiry; Scientific Method; Problem Solving in Technology; Nature of Scientific Knowledge: (7) System; Models; Pattern; Scales: (8) Science, Technology, and Human Endeavors; Impacts; Constraints; Relationships: (Pa., Academic Std.)

Chaos theory helps students meet the Pennsylvania standard in Science and Technology by letting them understand the Nature of Science in a deeper way. Chaos theory uses some unique ways in which scientists and mathematicians search for answers to questions and explanations of observations about the natural world which includes process observing, classifying, inferring, predicting, measuring, hypothesizing, experimenting and interpreting data. Also, Chaos theory can help in unifying themes of science- concepts, generalizations and principles that result from and lead to inquiry. (Pa., Academic Std.)

Chaos theory can lead to pure knowledge—facts, principles, theories and laws verifiable through scientific inquiry by the world community of scientists, which includes physics, chemistry, earth science and biological sciences. It can help students meet the standard of inquiry—a rigorous procedure that includes

universal verification of answers to questions about and explanations for natural objects, events and phenomena. Chaos theory can help students in process skills—how knowledge is acquired and applied in science by observing, recognizing variables, manipulating variables, formulating models, designing models and producing solutions. (Pa., Academic Std.)

Chaos theory can assist students in problem solving—application of concepts to problems of human adaptation to the environment, which often leads to recognition of new problems. Lastly, Chaos theory can help student understand scientific thinking—the disposition to suspend judgment, not make decisions and not take action until results, explanations or answers have been tested and verified and with information. (Pa. Academic Std.)

How can the new mathematics of Chaos apply to the teaching of Environmental Science (Ecology) and General Science? Whereas, Chaos theory is relatively a new discipline, it has not been applied to the teaching of Ecology or General Science at the high school level(s). Also, there are no formal courses of Chaos theory taught in many mathematics departments at the high school level. Chaos theory and fractals may be part of an advanced higher mathematics course. In 2006, subjects such as chaos and fractals are beginning to be taught in many undergraduate and graduate schools. But, with the advent of the personal computer and scientific calculators it is more possible than ever to teach this subject matter.

In order to bring students, teachers, textbooks and ecologists to the forefront of science and technology the new mathematics of Chaos theory must be applied to Environmental Science and General Science. It is comparable to the discovery of DNA in biology. A whole new field of biology and a new perception of biology occurred with the advent of DNA. From the revolution in microbiology, DNA has changed not just biology, but whole aspect of evolution, genetics, and even law. This maybe the way chaos could affect ecology and the teaching of general science.

Recently a few scientists have tried to apply the chaos theory to the problems of how ecosystems function. The chaos theory suggests that ecosystems may be sensitive to very small changes, and the initial state of an ecosystem is crucial to its later development (Dubay 99).

Specifically, population and extinction studies in the past and future can be better understood by chaos theory. The human population growth may be causing more extinctions than any other factor. Habitat destruction, the introduction of foreign species to new ecosystem, and pollution of freshwater are major impacts of human population growth. Chaos theory can help predict the long-term effects of this disruption of the biosphere and its ability to support the human population (Dubay 99).

Again, another application of chaos theory is studying the effects of global warming. This could be considered one of the most compelling topics in Environmental Science. Moreover, to supplement current models, fractals and chaos theory are being used to understanding global warming trends.

The complex visual computer images of fractals are representative of somewhat simple mathematical formulas. These complex images may be a good way to represent complex ecosystem (Dubay 99).

Another specific application of chaos theory is to apply it to old theories of population by Thomas Malthus. What effects do imbalances in nature have on population(s)? Can mathematical models protect us against famine and death (Stewart 263)?

We think we understand certain relationships in Ecology. Take for instance the relationship between predator and prey over long periods. The lynxes and the hare in Canada, shows a very long and compiled record. The predator and prey in this case show fluctuations in population over a long period of time. Perhaps these old records will allow us a way to predict population advance and decline better. The bacteria and virus causing diseases of epidemic proportions (the bird flu) can be better understood not just with current modeling but supplemented with models using chaos theory to test population growth (Stewart 264).

Historically what has been taught in Ecology classes is the conventional geometrical exponential growth when a population grows without interference. But, in the case of diseases like A.I.D.S. the conventional approaches to the spread of population growth (or decline in the case of epidemics) are based on constructing specific models of physiological and transport processes involved. To show another approach by the of chaos theory and fractals complements this, by concentrating on empirical observations and trying to extract the underlying dynamic directly. In this type of case, both methods should be taught to analyze this situation (Stewart 267)

Finally, the theories of chaos will play a larger role in all sciences. The role of chaos in Ecology will probably need to be taught in greater length for each applicable subject in the discipline of Ecology.

Historically when teaching about population growth in Ecology, teachers, ecologists and textbooks will begin the argument by stating that population(s) will grow exponentially. That is, each generation will contain a larger number of individuals than the one before it. This kind of growth curve is called a J-shaped curve (Arms p. 331). This is a linear growth function that rises forever upward. Then, ecologists realized the straight upward curve was not realistic. This was first stated by the British economist and philosopher Thomas Malthus about the

human population or the human condition. The idea of the human population following the J-shaped curve could not be true because the human population did not follow the J-shape. Thomas Malthus human population theory took hold for a relatively long time. It lasted almost two centuries. Writing in 1798, Thomas Malthus argued that the human population growth was not always desirable (Dubay p 201). Thomas Malthus pointed out that population tends to increase at geometrical levels (2, 4, 8, 16, 32, .etc), whereas, the food supply does not (Dubay p201). Therefore, Thomas Malthus believed that the human population outgrows its resources and leads to conditions such as famine, war, and other human suffering that reduces the human population. To avoid such things as famine, war, etc., Thomas Malthus advocated practices that would reduce the human population growth rate (Dubay p 201). Thomas Malthus' theory still persists to this day. However, at a closer look at the human population, it may not be true.

Even as late as the year 2000, ecologist and textbooks authors modified Thomas Malthus' model. It is the S-shaped curve. To be more realistic ecologists needed an equation with some extra term that restrains growth when the population becomes large (Gleich p. 62). The most natural function to choose would rise steeply when the population is small, reduce to near zero at intermediate values, and crash downward when the population is very large (Gleich p. 62). By repeating this process, an ecologist can watch a population settle into long term behavior—presuming population reaches some steady state. Ecologists would take the Malthusian version of an equation say; here's a variable representing the productive rate; here's an equation representing the natural death rate; here's an equation representing the additional death rate from starvation or predation and look—the population will rise at this speed until it reaches that level of equilibrium(Gleich p. 62)

Many different equations can work, but, all equations representing the S-shaped curve are linear or modifications of linear equations. What were ecologists trying to explain with these types of S-shaped curves? They were trying to be more realistic in describing population growths. Again, many thought all populations grew similarly to the human population growth rate. Find an equation that fits the S-shaped curve and you have described any number of population growth curves from rabbits to bacteria (Gleich p. 62)

Graphs are created to reflect the growth of populations over time. Usually they go like this. At first the population increases according to maximum growth (sometimes called the biotic potential according to new ecological jargon) or the old J-shaped curve. But, environmental resistance slows the population growth rate as it approaches its equilibrium (carrying capacity). Instead of staying on a straight line (equilibrium), ecology textbooks as late as 2000 state, equilibrium can fluctuate over time. That is, it can rise over carrying capacity (equilibrium) or go under carrying capacity (equilibrium), but, it keeps fluctuating above and below the carrying capacity (equilibrium). This is usually as far as ecologists and

ecology textbooks take the analysis of the S-shaped curve. They do not attempt to describe the fluctuation other than environmental resistance (predators, parasites, diseases, and competition). Still, further environmental resistance includes changes in weather and other natural forces that cause the population to continue to fluctuate over time (Arm p 344).

Is the fluctuation the true or real observation of the data? Or did the ecologist have a different motivation to describe the data? Perhaps, in the back of the ecologists' mind was always the assumption that an erratic string of numbers probably meant the calculators were acting up, or just the lack of accuracy, maybe in the days of the pencil-and-paper arithmetic, mechanical adding machines and numerical exploration never went that far (Gleick p 65).

Why do ecologists or environmental science textbook stop their explanation of the fluctuation of the S-shaped curve of population here? Is it that stable solutions were the interesting ones? Order is its own reward. Or this business of finding an approximate equation and working out the computation was hard, after all (Gleick p 65). No one wanted to waste time on a line of work that was going awry, producing no stability. And no good ecologist ever forgot that his equations were a vastly oversimplified version of the real phenomena. The whole point of oversimplifying was to model regularity. Why go to all the trouble to just see chaos (Gleick p 65)?

Oddly, the flow of numbers begins to misbehave. The numbers do not grow without limits, of course, but, they do not converge to a steady level, (S-shaped) either. If the population kept bouncing back and forth, ecologists' assumed that it was oscillating around some underlying equilibrium. The equilibrium was the important thing. It did not occur to the ecologists that there might be no equilibrium (Gleick p 64).

In what other ways can one measure Chaos in General Science and Ecology? Using the simple logistic equation is a sound method. The simple logistic equation is a formula for approximating the evolution of an animal population over time.(Elert p 1) Many animal species are fertile for a brief time during the year and the young are born in a particular season so that by the time they are ready to eat solid food it will be plentiful. For this reason, the system might be better described by a discrete difference equation than a continuous differential equation. Since not every animal will reproduce (a portion of them are male after all), not every female will be fertile, and not every conception will be successful and not every pregnancy will be successfully carried to term: the population increase will be some fraction of the present population. Therefore, if " A_n " is the number of animals this year " A_{n+1} " is the number next, then " $A_{n+1} = rA_n$ " where " r " is the rate of growth, will approximate the evolution of the population. This model produces exponential growth without limit (Elert p 1).

Since every population is bound by the physical limitations of its surroundings, some allowance must be made to restrict this growth. If there is a carrying-capacity of the environment then population may not exceed that capacity. If it does, the population would become extinct. This can be modeled by multiplying the population by a number that approaches zero as the population approaches its limit. If we normalize the “ A_n ” to this capacity then the multiplier $(1-A_n)$ will suffice and the resulting logistic equation becomes “ $A_{n+1} = rA_n(1-A_n)$ ” or a functional form $f(x) = rx(1-x)$. (Elert p 1)

The logistic equation is parabolic like the quadratic mapping with $f(0) = f(1) = 0$ and a maximum of one quarter “ r ” at one-half. Varying the parameter changes the height of the parabola but leaves the width unchanged. (This is different from the quadratic mapping which kept its overall shape and shifted up or down.) The behavior of the system is determined by following the orbit of the initial seed value. All initial conditions eventually settle into one or three different types of behavior (Elert p1).

1. A fixed situation: The population approaches a stable value. One can do so by approaching asymptotically from one side in a manner something like and over damped harmonic oscillator or asymptotically from both sides like an under damped oscillator. Starting on a seed that is a fixed point is something like starting an SHO at equilibrium with a velocity of zero. The logistic equation differs from the SHO in the existence of eventually fixed points. It is impossible for an SHO to arrive at its equilibrium position in a finite amount of time (although it will get arbitrarily close to it) (Elert p 2).
2. A periodic situation: The population alternates between two or more fixed values. Likewise, it can do so by approaching asymptotically in one direction or from opposite sides in alternating manner. The nature of periodicity is richer in the logistic equation than the SHO. For one thing, periodic orbits can be either stable or unstable. An SHO would never settle into a periodic state for the comfort of equilibrium. Second, periodic with multiple maxima and/or minima can arise only from systems of coupled SHOs (connected or compound pendulum, for example, or vibration in continuous media). Lastly, the periodicity is discrete; that is, there are no intermediate values. (Elert p 2).
3. A Chaotic situation: The population will eventually visit every neighborhood in a subinterval of $(0,1)$. Nested among the points it does visit, there is a countably infinite set of fixed points and periodic points of every period. The points are equivalent to a Cantor middle thirds set and are wildly unstable. It is highly unlikely that any real population would ever begin with one of these values. In addition, chaotic orbits exhibit sensitive dependence on initial conditions such that any two nearby points will diverge in their orbits to any arbitrary separation one chooses (Elert p 2).

The behavior of the logistic equation is more complex than that of the simple harmonic oscillator. The type of orbit depends on the growth rate parameter, but in a manner that does not lend itself to “less than,” “greater than,” “equal to” statements. The best way to visualize the behavior of the orbits as a function of the growth rate is with a bifurcation diagram. Pick a convenient seed value, generate a large number of iteration, discard the first few and plot the rest as a function of the growth factor. For parameter values where the orbit is fixed, the bifurcation diagram will reduce to a single line; for periodic values, a series of lines; and for chaotic values, and a gray wash of dots (Elert p 2).

What are the most prominent features? There are two fixed points for this function: 0 and $1-1/r$, the former being stable on the interval $(-1, +1)$ and the latter on $(1,3)$. A stable 2-cycle begins at $r = 3$ followed by a stable 4-cycle at $r = 1 +$ the square root of 6. The period continues doubling over ever shorter intervals until around $r = 3.5699457\dots$ where the chaotic regime takes over. Within the chaotic regime there are interspersed various windows with periods other than powers of 2, most notably a large 3-cycle window beginning at $r = 1 +$ the square root of 8. When the growth rate exceeds 4, all orbits zoom to infinity and the modeling aspects of this function become useless (Elert p2).

By incorporating into the model some factors limiting growth, one might think that, after an initial expansion phase leading to overpopulation, followed by periods of shrinkage, the population ought to rapidly reach some kind of equilibrium, at which it will either remain relatively constant or fluctuate in a steady cycle. One might be correct. Computer calculations confirm that, as long the growth rate (reflecting the tendency for demographic explosion and overpopulation) remains moderate—with values between 1 and 3 (values smaller than 1 lead to extinction), the population does remain stable. Nothing mysterious occurs. Things get out of control as soon as the growth rate exceeds the value of 3 (corresponding to a population tripling every year), in which case equilibrium is destroyed and the population starts to oscillating between two distinct values from one year to the next. If the growth rate increases even further, a second bifurcation takes place. (Thuan p. 115)

Amazingly, despite the complexity of this behavior, the same values keep recurring on a periodic timetable. Regularity still prevails in complexity. However, even the regularity disappears altogether when the growth rate reaches the critical value of 3.57. At this point, chaos takes over. Periodicity is then swept aside in favor of randomness. The variations in population become completely haphazard. (Thuan p. 115)

In 2006, the very best science uses and must use the computers to analyze data. Computer modeling can show that the fluctuation of the S-shaped curve is not the

result of environmental resistance (overcrowding, predators, parasites, diseases, weather and other natural forces), but, chaos.

Chaos should be taught. It is time to recognize that standard education of a scientist gave the wrong impression. No matter how elaborate linear mathematics could get, it inevitably misled scientists about their overwhelmingly nonlinear world (Gleick p 80). As the number of organisms reaches carrying capacity (a graph of time versus number of organisms) there is usually no equilibrium. The leveling off of the S-shaped rising to carrying capacity overshoots it and falls below. It is not flat nor is this fluctuation, it is chaos.

Furthermore, many environmental science textbooks and ecologists as late as the year 2000 stated that ecosystems, after a disturbance, go back to equilibrium. This, too, is problematic, many ecosystems are permanently damaged. They will never find equilibrium.

Many ecology textbooks state that ecosystems in nature evolve toward a kind of stability. In the theory of Chaos, this could be understood as an initial condition(s). Recovering from a disturbance, seashore, a forest, an estuary will move back toward the state of equilibrium. Stability, equilibrium, and recovery these concepts are similar, but, the problem is in the data and details of analyzed data (Wheelwright p 44).

Take for instance, the Exxon Valdez oil spill of 1989; the strict constructionists were maintaining that Prince William Sound must return to its ecological status before the oil spill-environment clean the wildlife in the very same condition and numbers as before the spill. The Exxon corporation contributed million of dollars to this clean-up of the oil spill. Still, Prince William Sound shows contamination from the oil spill (Wheelwright p 45).

In using chaos to better understand the Exxon Valdez oil spill in Prince William Sound, the environment has not returned to its equilibrium. The biologists today are crunching data to do a statistical analysis of the situation. Ecosystems recover from disturbances, to be sure, but since ordinary flux makes impossible to know what the future might have been, there is firm standard by which to judge the recovery. The absolutes of the balance-of-nature catechism do not stand up to statistical testing. Chaos theory, which rationalizes the seeing disorder of turbulent system, has won a place in ecology (Wheelwright p 45).

Objectives

After finishing the unit students will have achieved the following objectives.

- (1) to have a fundamental comprehension of chaos theory
- (2) to have a basic understanding of the history of the development of chaos theory

- (3) to demonstrate how to use chaos theory to analyze population data
- (4) to be able to construct various types of logistic equations
- (5) to be to put logistic equations into a graphics calculator.
- (6) to be able to print the results of population data showing chaos
- (7) to explain the term mathematical iteration
- (8) to explain four types of infinity
- (9) to draw fundamental curves such as a J-shaped and S-shaped curves
- (10) to draw an oscillating S-shaped curve
- (11) use a personal computer to demonstrate various logistic equations

Students be able to meet the standards by making mathematical models of population data supplied to them during the unit on population. Students will be required to make complete descriptions by making graphs and charts of the population data that were analyzed.

Strategies

The teacher should determine the students' understanding of logistic equations. This could be done showing students some logistic equations on a graph paper, written verbally and on a computer modeling a logistic equation. Students will construct a simple logistic equation. The simple logistic equation will then be put into a folder for viewing at the end of the unit. At the end of the unit the class, students will construct more complex logistic equations reflecting population studies. The class will compare two or three logistic equations and describe what Chaos is and what it is not. The unit will begin with a brief description of Chaos theory. Students will be required to use the classical economist Thomas Malthus' study and description of population theory. Students will be required to make a computer model of Malthus' theory and compare it to the more elaborate Chaos logistic equation. The intentions of the unit are to follow inquiry based methods. The teacher should lead students into a method of discovery. Students will write a brief comparison of Malthus ideas and Chaos theory. A computer model of both should be made. This is a good time to incorporate this unit into the population unit for the study of population growth in the Ecology course. Student shall keep journals of their understanding of Chaos theory in relation to population growth. All finished work should be kept in order to be put in the students' existing portfolios. A final assessment can be given at the end of the unit.

Class activities

Day number one

Procedure

Students shall begin the lesson with a brief discussion on the information contained in the unit. Students can be brought into the discussion by some provoking questions.

1. What is the Chaos theory?
2. When did mathematicians start thinking about Chaos?
3. How do we apply Chaos theory to population studies?
4. Who is Thomas Malthus and what and when did he state his ideas?
5. What population(s) data can we study?
6. Is Chaos theory applicable to all population studies?
7. What is numerical infinity?
8. How many types of infinity are there?

Students can make a concept map of the discussion(s) and question(s). The map can be put on large paper and on each individual students' own notebook or journal.

Analysis

Student will analyze various population growths over time. Students will graph data versus time on both graph paper and a computer. Is there any pattern(s) starting to develop?

Day number two

Procedure

Provide the students with a collection of population data that could possibly be Chaotic or not. Enter the data into the various logistic equation(s). Plot a graph on computer and print the results of population data used. This graphing of various points should done for three days.

Last day on the project

Analysis of the of logistic equations on the computer graphics should be printed at various points to show if and when Chaos occurred or not.

Annotated Bibliography/Resources

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Appendix A

Standards

This unit addresses the following Pennsylvania State Standards that are used by the Pittsburgh Public Schools.

Science and Technology

- 1.) All students will explain how scientific principles of chemical, physical and biological phenomena have developed and relate them to the real-world situations.
- 2.) All students will demonstrate knowledge of basic concepts and principles of physical, chemical, biological and environmental science.

- 3.) All students will explain the relationship among science, technology, and society.
- 4.) All students will construct and evaluate scientific and technological systems using models to explain or predict results.
- 5.) All students develop and apply skills of observation, data collection, analysis, pattern recognition, prediction and scientific reasoning in designing and conducting experiments and solving technological problems.
- 6.) All students evaluate advantages, disadvantage and ethical implications associated with the impact of science and technology on current and future life.

Mathematics

- 1.) All students will use operations (e.g., opposite, reciprocal, absolute value, raising to power, finding roots, finding logarithms).
- 2.) All students will develop and use computation concepts, operations and procedures with real numbers in problem solving situations
- 3.) All students will use estimation to solve problems for which an exact answer is not needed.
- 4.) All students demonstrate skill for using computer spreadsheets and scientific graphing calculators.
- 5.) All students recognize that the degree of precision needed in calculating a number depends on how the results will be used and the instruments used to generate the measure.

Reading, Writing, Speaking and Listening

- 1.) All students read and use a variety of methods to make sense of various kinds of complex texts.
- 2.) All students compose and make oral presentations for each academic area of study that are designed to persuade, inform or describe.
- 3.) All students write for a variety of purposes, including to narrate, inform and persuade, in all subject areas.

