Calculating the Past and Present Using Historical Facts and Modern Technology

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Overview

The purpose of this unit is to serve as a resource for the mathematics and science courses at the high school level. In the Pittsburgh Public Schools, most of the high school curricula are designed so that the standards for learning are incorporated in most courses. This unit contains lessons that meet the academic standards for mathematics, science and technology. These standards reflect the increasing complexity and sophistication that students are expected to achieve as they progress through schools.

This curriculum unit will travel back to the origin of mathematical computation showing how these computations were achieved via mechanical aids. The journey begins 2500 years ago with the abacus and continues through to the present with modern technology**.** The various classroom activities will show how new technology can be integrated into mathematics, history, and science education**.** Many mathematics tasks that took hours to perform during the early years can be performed today in seconds with the new enhanced TI 83 Plus calculator.

Over time, man made some very remarkable discoveries during the 15^{th} through 19^{th} centuries in the domain of methods and machines for simplifying calculations. The work of John Napier and Henry Briggs on the development of logarithms was one of the most important scientific achievements of the 17^{th} century. This curriculum unit will demonstrate how the most common non-electronic calculating device, the slide rule, was used to solve very sophisticated mathematics and science problems during early history.

The lessons provided will give students an opportunity to work with functions and data analysis both nationally and internationally. As students get involved with functions they can describe and determine the outcome of various events. The data analysis activities give students a chance to create regression models, graphs, and tables. In particular, data on the wheat market and cotton industry will give students the opportunity to investigate past performance and make predictions about the future. All problem activities connect to real historical facts and are designed to bring about motivation, active participation, and hands-on experiences.

Student involvement is necessary for the success of this unit. For each of the various lessons, twenty days will be enough time for completing the entire unit. With the families of functions, exponential should be explored first in order to understand the concept behind roots and power. Next the logistic function should be taught to observe the difference between restricted and non-restricted growth and last the logarithmic function will help students to understand the concept behind inverse functions.

Introduction

The framework of my practice involves lecturing, individual explorative class work, and large and small group discussions. The activities I provide for my students are the kind in which students can generate their own ideas and explanations. My overall instructional plan involves making sure conceptual understanding is achieved first so that students communicate mathematically. I believe that students' understanding must be done with consideration; not all students learn the same way. Because of this, it is important to provide the proper tools to attend to the different learning styles.

Today, technology has given educators a true challenge to move our subjects away from the routine and toward new ways of teaching and student learning. In my own classroom, I have found modern technology allows students to investigate mathematics globally. Using the TI 83 Plus calculator as a support device has made a true difference in the way students see and learn mathematics. Student can deal with the mathematics directly instead of just listening. In most of my lessons you will see technology used as a support tool to enhance student learning. Learning is definitely viewed in a much broader perspective than just symbolic representation.

Technology has been a powerful force in the development of civilization. On the whole, technology extends our abilities to change the world: to cut, shape, or put together materials, to move things from one place to another and to reach farther with our hands, voices, and senses. We use technology to try to change the world to suit us better. The changes may relate to survival needs such as food, shelter, or defense, or they may relate to aspirations such as knowledge, art, or control. But the results of changing the world are often complicated and unpredictable (Science Net Links).

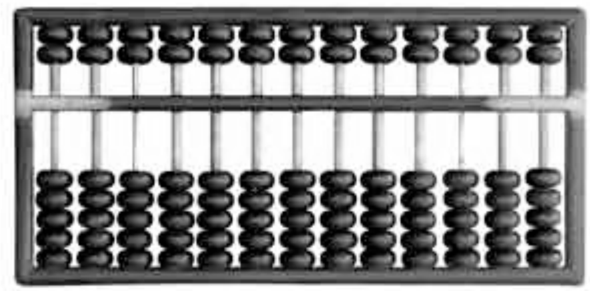
A Brief History of Mechanical Aid (before the twentieth century)

By the middle of the 16th century the first major European exploration of the Americas was well advanced, trade-routes by sea had been established with parts of the Indian sub-continent and the Far East. The growth of merchant trading in Italy and other European sea powers, coupled with the demand for imports from distant parts of the world, created a need for more and more sophisticated mechanical devices. In the mid-16th century there were no suitable mechanisms for constructing detailed maps, for accurately measuring distances at sea, or for precisely determining how long a particular voyage would take to complete. As a result, merchants financing trading expeditions would be faced with ruin if a ship bringing back goods arrived too early or too late. There was very little financial success from all parties (Dunne 1).

The Abacus: The earliest invention of the abacus is credited to the Babylonians. By the sixth century B.C. they were widely used in Greek society. The historian Herodotus (484-424 B.C.) mentions them and left records of complex calculations. It was in the Oriental cultures, principally China and Japan, that the abacus reached its highest level of development. The use of the abacus as an aid to computation reaches its greatest level of sophistication and power in a device which is still very extensively used today in the Far East.

The abacus is a counting device based on the positions of two sets of beads moving on parallel strings. The first set contains five beads on each string and allows counting from 1 to 5, while the second set has only two beads per string representing the numbers 5 to 10. The abacus system is based on a radix of five. Using a radix of five make sense since humans started counting objects on their fingers (Redlin 1).

Heaven beads, each worth 5



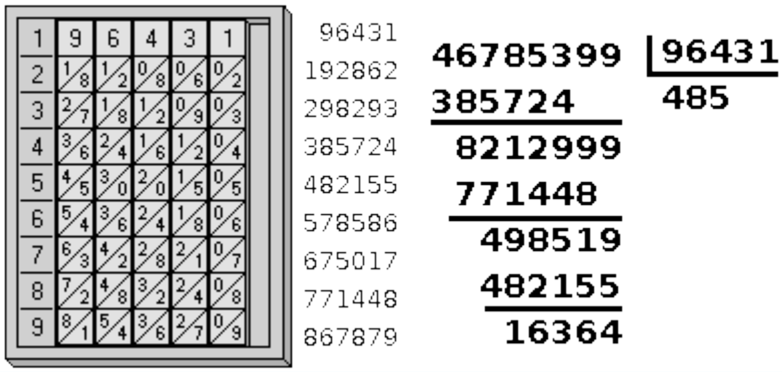
Earth beads, each worth 1

Example: To show the number 9
On the first line, one heaven bead (top deck) would be moved down representing 5 units) and 4 earth beads (bottom deck) would be moved up (each representing 4 units).

To show the number 79, in addition to the beads in the first line used to make the number 9, one heaven bead would be moved down and two each beads would be moved up on the second line, representing 5 tens and 2 tens respectively.

To Add 6 +2, one would move 1 bead from the upper deck down and one bead from the lower deck up; this represents 6. Moving 2 beads from the lower deck (in the same column) up would complete the operation. The answer is then obtained by reading resultant bead positions (Educalc 1).

Napier's bones is a clever multiplication and quotient tool invented in 1614 by mathematician John Napier (1550-1617) of Scotland. The bones are a set of vertical rectangular rods, each one divided in 10 squares. The top square contains a digit and the remaining squares contain the first 9 multiples of the digit. Each multiple has its digits separated by a diagonal line. When a number is constructed by arranging side by side the rods with the corresponding digits on the top, then its multiple from left to right (Redlin 2). Napier's bones were very successful and were widely used in Europe until the mid 1960's. Many different versions were manufactured and employed by accountants, bookkeepers, and others whose work routinely involved computing products of numbers.

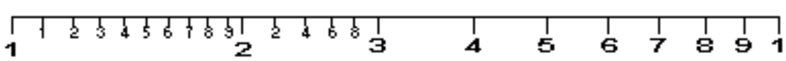


Napier's bones from Wikipedia, the free encyclopedia

The Slide rule (slipstick) was created by William Oughtred (1574-1660) of England in 1633. Logarithms were the basis for this invention. The "slipstick" is used as a mechanical analog computer consisting of at least two finely divided scales (rules) used primarily for multiplication and division, and also for scientific functions such as roots, logs and trigonometry. In early history, the slide rule was commonly used in fields such as science and engineering until the appearance of the electronic calculator (MOHPC).

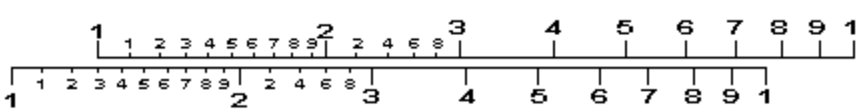
Example of the slide rule

The mathematician had to look up two logs, add them together and then look for the number whose log was the sum. Edmund Gunter soon reduced the effort by drawing a number line in which the positions of numbers were proportional to their logs (MOHPC).

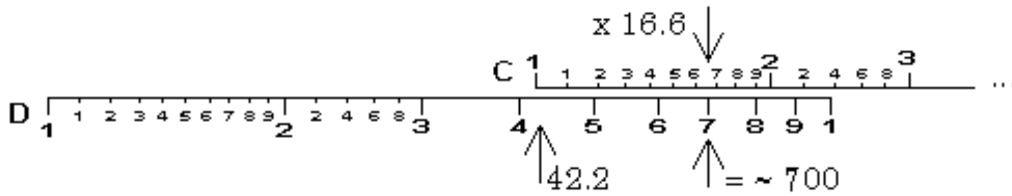


Log 1 = 0 because $10^0 = 1$ log 100= 2 because 10^2

Soon afterwards, William Oughtred simplified things further by taking two Gunter's lines and sliding them relative to each other thus eliminating the dividers.

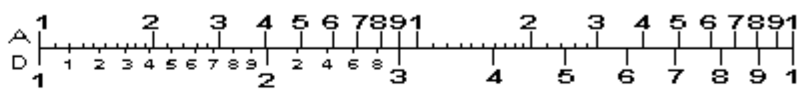


Multiplications with more than a single digit were carried out by making use of the smaller graduations to represent additional digits of decreasing significance. The precision available to the user was directly proportional to the size of the device (or the smallest lines the user could resolve) .The slide rule did not indicate the decimal point. That was done by the user typically by estimation, "common sense" or by computing the characteristic. For example:



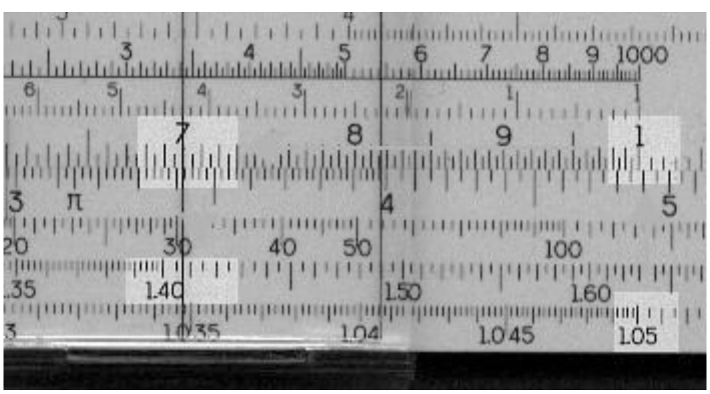
Example: Multiply 42.2 X 16.6= 7.00

These were the squares of the D and C scales respectively. To determine a square root, the user found the number on A and read the root on D. The process was reversed to find a square. The A scale was simply a D scale, reduced to half its length and printed twice (Manley).



Example: $\sqrt{4} = 2$ and the $\sqrt{9} = 3$

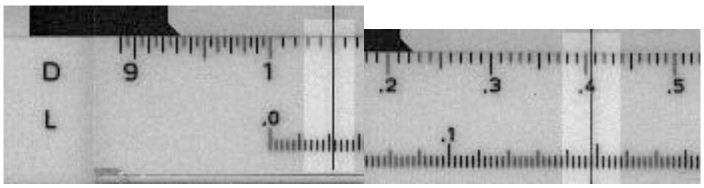
Calculation with log-log scale on slide



Example: 1.05^7

Cursor to 1.05 on LL1
1 on C to cursor
Cursor to 7 on C and the Answer (1.407) under cursor on LL2
Used also the solve the $\log_{1.05}1.407 = 7$

Example: 1.05^7 solve on the scale of logarithms which are more commonly found than log-log scales.



Cursor to 1.05 on C, Read the logarithm of 1.05(.021) on L, Multiply .021 by 7.0 = 0.147 on L. Read answer (1.407) under Cursor on C.

Schickard's Machine was the first known adding machine and was made by Wilhelm Schickard (1592-1635). In 1623, Schickard, a polymath and then professor at the University of Tubingen in Wuerttemberg (now part of Germany) designed and constructed a mechanical device which he called the calculating clock. Able to add and subtract up to six-digit numbers, the artifact was based on the movement of six dented wheels geared through a "mutilated" wheel which with every full turn allowed the wheel to rotate 1/10th of a full turn. An overflow mechanism rang a bell (Redlin 2). "Of all the influences that shape mathematics education, technology stands out as the one with the greatest potential for revolutionary impact: (Harvey, Waits, Demana 75)"

Mechanical Aid during the Twentieth Century

The TI- 83 Series of graphing calculators is manufactured by Texas Instruments. The original TI-83 is itself an upgraded version of the TI-82. Released in 1996, it is one of the most used graphing calculators for students. In addition to the functions present on normal scientific calculators, the TI-83 includes many features including: function graphing, polar/parametric/sequence graphing modes, statistics, trigonometric, algebraic functions and applications.

Texas Instruments replaced the TI-83 with the TI-83 Plus calculator in 1999, which included Flash ROM, enabling the device's operating system to be updated if needed, or for large new Flash Applications to be stored, accessible through a new Apps key.

Example: A typical mathematics problem today on logarithms

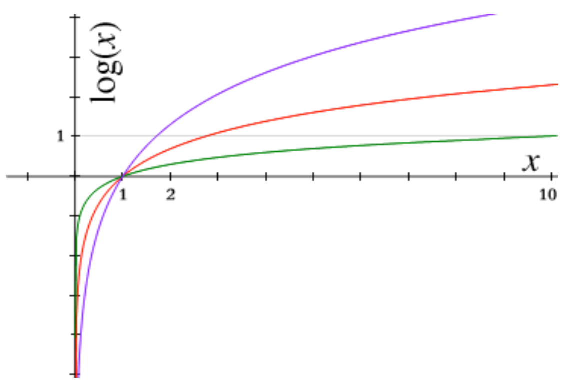
American seismologist Charles Richter developed the Richter scale which is used to measure the amount of energy released at the focus of an earthquake.

1. Compute the earthquake magnitude (strength) R for the given R = log (300/t) + B when the time t is 2 seconds and B (weakening of the seismic wave due to the distance from the epicenter is 4.5)

To solve R = log (300/2) + 4.5 use the base 10 log key by just pressing log 150 which is approximately 2.17609 + 4.5 ~6.67609

2. How many times more severe was the 1978 Mexico City earthquake (R = 7.9) than the 1994 Los Angles earthquake (R= 6.6).

Using the equation for Earthquake Intensity Model, If T and B were the same for the two quakes, we have 7.9 = log a_1 – log T + B and 6.6 = log a_2 – log T + B, so 7.9 – 6.6 = 1.3 = log (a_1/a_2). Then $a_1/a_2 = 10^{1.3}$, so a_1 is about 19.95a_2 – the Mexico City amplitude was about 20 times greater.



The graph of the y = log x function with the TI-83 calculator.

Rationale

This unit is important because it improves students' readiness for calculus and science. Once this unit is completed, students will have developed an understanding of the complete family of exponential, logistic and logarithmic functions. Modern technology, the TI-83 calculator will be used to enhance what they have learned. In particular, population growth and the rumor spread application problems will give students the opportunity to solve very sophisticated logarithmic and exponential equations, become acquainted with function behaviors, and communicate their findings mathematically and graphically.

Knowing the importance of exponential and logarithmic functions, students will be able to analyze and observe their various features (domain, range, intercepts, horizontal asymptote, increasing and decreasing functions, continuous, and limits) using a variety of activities. Logarithms will provide a challenge for students to come to grips with the inverse function concept and allow students to reach a deeper understanding of solving exponential equations. I know that once students can achieve the basic fundamentals involving logarithms and their properties, they will be able to apply the theory behind solving exponential equations.

The activities, Pennsylvania State Population and the High School Rumor provided will help to promote reasoning and thinking. Also, students will see the difference between restricted and unrestricted growth. In allowing connections across content, both functions will be introduced along with true historical facts. With the logistic function, they will see how restricted growth is modeled. Once students become familiar with this concept, they will be able to draw a more reasonable conclusion about different phenomena.

The lesson on data analysis provides an opportunity for students to describe, share their ideas, and generate solutions which are the focus of the National Counsel of Teachers of Mathematics Principles and Standards for teaching and learning. It is important for students to be able to display the data in graphs in order to answer questions about the past and make some kind of prediction about future outcome. I know that the market is very difficult to predict; however, this problem is good because students can learn reasoning statistically. The skills that are acquired in doing this are necessary in order to become informed citizens and intelligent investors.

Objectives:

The students will be able to:

1. Become familiar with an entirely new team of functions: exponential, logistic, and logarithmic. This team will be introduced from the graphical, symbolical, and theoretical frameworks along with real-world application.
2. Use exponential, logistic and logarithmic functions and equations to solve word problems with historical facts.
3. Use exponential, growth, decay, and regression to model real-world problems
4. Apply the properties of logarithms to solve equations.
5. Use the TI-83 Plus graphing calculator to re-express data and make scatter plots.
6. Use the TI-83 Plus graphing calculator to make a complete graph of various functions and analyze their graph completely.
7. Analyze data both national and international over a period of time. Students will gain insight about the trade market from early time to today. Students will investigate the various performances over time and create several kinds of regressions in order to make future predictions.

Strategies

Lessons One**:** Part I and II
Teaching exponential, logarithmic and logistic functions has several challenges. Students must have an understanding of powers, exponents, and roots. To ensure understanding, students must be given a quick review that involves evaluating roots and simplying exponents. Students also experience difficulties graphing exponents and finding reasonable solutions. To ensure that graphing is understood, students must be reminded that most graphs use a large range of y-values due to the behavior of the exponential function. Once students can view the graphing table and can see the y-values, they may be able to create a comfortable window. When approaching equation solving, students must be reminded that simple equations (e.g. $8^x= 32$) can be solved by creating the same base (if possible) or switching from exponential to log. They must also be reminded to keep track of the domain of each expression in the equation; a particular algebraic method may produce solutions that are extraneous. When students have to solve tougher equations, it is important to inform them to pay close attention to how the equation was set-up because this can determine when to switch to exponential or use logs. Informing students about limits is very important; this was a way of providing reasoning. Students should be informed to view the function from left to right in order to see the limit of f(x) as x approaches $\pm\infty$. Students can also be

informed to investigate their table of values to see what value the function will be approaching.

The primary focus of the various lessons is to provide students with connections that help to elicit and build their mathematical thinking. In lesson number one students can become acquainted with an increasing exponential function and a restricted logistic function, from the graphical, symbolical and theoretical frameworks. Students will be asked to investigate all characteristics in order to see all aspects of both graphical curves (the axes, scale, all intercepts, and all asymptotes, entire domain and relevant domain). Students will be made aware of the initial value for $x = 0$ in order to provide connection to the y-intercept. In providing support graphically, students can evaluate both functions for population over time (t) for various years and days. In population growth, a true challenge will come once students have to solve for time (t) to determine which year the population grew to twenty million people. This will allow students to find another alternative besides graphing; students will have to think about how to find an algebraic method. Allowing students to approach solving equations from two perspectives provides awareness. The purpose behind the horizontal asymptote will be addressed and students will be asked, "As time increases, will the population approach a limit?" Students need to apply the theory behind limits in order to understand that some functions may or may not approach a limit. This provides connections to end-behavior and the range as well. Students will have to provide a summary of both functions over its entire domain. This improves students' abilities to communicate their ideas and actually write and talk about the mathematics they are doing.

Lesson Two

To begin this lesson, review how to create a scatter plot, calculate a regression curve and superimpose on the curve with statistic data. For the activities created, students' analyses will be based on three sets of data: The National and International Wheat Trade, World Wheat Market Shares, and United States Cotton Imports and Exports. Students will find various regression curves and models in order to summarize the market behavior over a given time period. Based on the data given, students will make predictions and explain the limitation of those predictions. It is important for students to compare the original data to the model produced in order to see if the model is great for that particular problem. In a summary, students will report their findings with respect to the maximum and minimum value, the zeros, the various intercepts and the end behavior for each model. Students will have to address questions that relate to the Wheat and Cotton Industry. It is important that students explain each of the graphical behaviors as well. Questions that relate to the graphical model will provide an opportunity for students to show connections to both graphs and the data presented. For example, the y-intercept should show the function's initial behavior, the x-intercept will show at what time the market was at a zero (if possible), the maximum value will show the time and the highest value of the industries, the minimum will show the lowest time period. Students will further address questions such as (1) If the Wheat Market continues with the trend, what would be its performance for this year? In 5 years from now? (2) What is predicted to be the end behavior of the Cotton Trade for this year? (3) Since some investors usually look at past performance, consider what happened during the time period of (1990-2000) with the Cotton Market, what does the function graph show during that period?

According to the National Council of Teachers of Mathematic Principles and Standards (2000), it is important for students to become knowledgeable, analytical, and thoughtful consumers of the information and data generated by others. The overall goal for the given activities is to provide students with problems through data analysis so that they can make reasonable and thoughtful decisions. This is considered to be one of the most useful skills for survival. Just as important, students should make predictions based on observing the behavior of functions (mathematical and numerical models) over time. Also, students should be able to recognize that there are some issues that one may not be able to predict. Based on the ambiguity of the question, it may be impossible to interpret the results meaningfully.

Classroom Activities

Lesson One: Part I, Population Growth

Objectives: Number 1-4

Materials needed: TI- 83 calculators

Historical facts:

The Industrial Revolution was born in Great Britain, but once transplanted to the United States it flourished and took a significant new shape. So distinctive was the way in which Americans began to make goods that within a generation it was being called "the American system of manufactures (Pursell 87)".

In Pennsylvania, large areas of the Northern and western parts of the state were unsettled or thinly populated in 1800. By the time of the Civil War, with the exception of the northern tier countries, population was scattered throughout the state. There was increased urbanization, although rural life remained strong and agriculture involved large numbers of people. The immigrant tide continued alter the Civil War and brought about a remarkable change in the composition of the population. While most of the state's pre-1861 population was composed of ethnic groups from northern Europe such as the English, Irish, Scotch-Irish and Germans, the later period brought increased numbers of

Slavic, Italian, Finn, Scandinavian, and Jewish immigrants. At the height of this "new immigration," between 1900 and 1910, the Commonwealth witnessed the largest population increase of any decade in its history. African American migration from the South intensified after 1917, when World War I curtailed European immigration, and again during World War II. By World War II almost five percent of the state's population was African American. In 1940 the Commonwealth was the second largest state in the nation with a population two-third that of New York.

Pennsylvania State Population Growth

Early in its history, the population in Pennsylvania has been increasing due to the migration various settlers. In 1790 the United States Census Bureau reported a total of 434,373 people and in 1900 reported 6,302,115 people present in the state. Assuming the increase has been exponential and increasing at an average rate of 1.0266% annually. **Use the TI 83 calculator to find a complete graph of the given function.**

$$P\ (t)_{1900} = 6302115(1.01026)^t$$

a. Sketch a complete graph of this function, including any asymptotes.
   Label and explain the following:
   - x-axis and y-axis
   - scale on each axis
   - all intercepts
   - all asymptotes

b. State the domain and range of the algebraic function:

c. **Highlight or box** the portion of the graph that is relevant to this **problem situation.**
   How is this portion of the graph different from the domain of the function?

d. Tell how would you will find the growth rate from 1790 to 1900 algebraically and explain what it means according to this problem situation.

e. There is little agreement as to the exact beginning and end of the Baby Boom. In the United States, demographers have put the generation's birth years 1946 to 1964. Use the given model; predict the number of people that were in Pennsylvania in 1950 and 1960. Show and explain how you will find the answer both algebraically and graphically.

f. Prior to the Baby Boom, there was a time period of approximately 20 years in which having children would have been difficult due to World War II and the Great Depression. The Baby Boom reflected the sudden removal of economic and social pressures that kept people from starting families. Because of post Baby Boom period, the average rate dropped to about 0.69%.
The U.S. population grew by more than 205 million people during the century, more than tripling from 76 million in 1900 to 281 million in 2000. In 1970, the census showed that some states lost population. Using the rate of 0.69% which shows a decline in the state's population, about how many people were in Pennsylvania in 1970? Use the model given to show how you will find the answer both algebraically and graphically.

$$P\,(t)_{1970} = 6302115(1.0069)^{t}$$
$$P\,(70)_{1970} = 6302115(1.0069)^{70}$$

g. About how many people were in Pennsylvania in 1995? Use the model given to show and explain how you will find the answer both algebraically and graphically.

h. Based on the function, when will the population grow to twenty million people? Use an average rate of increase of 0.69%. Show or explain how you found your answer.

Solve using log or natural log

$$20{,}000{,}000 = 6302115(1.0069)^{t}$$

$$\text{Log } 20{,}000{,}000 = \text{Log } 6302115(1.0069)^{t}$$

In year 2010 the Pennsylvania population is projected to be 12,449,000 people and 2025 it is projected to be 12,683,000 people (U.S. Census)

i. Based on the model, as the time (years) increase, will the population approach a limit? If yes what is that limit? Explain your reasoning. If no, explain why the population has no limit.

$$\lim_{x \to \pm\infty} P\,(t)_{1900} = 6302115(1.0069)^{t}$$

j. In year 2000 the U.S. Census Bureau reported a total of 12,202,000 people and 2005 reported 12,281,000 people find the growth factor and predict how many people will be in Pennsylvania in the year 2008 and 2015. Show and explain how you will find the answer both algebraically and graphically.

k. Write a summary report describing the Pennsylvania population over a given time period starting with the beginning of the 20th century. Your report should discuss things such as the years when the population was relatively stable or experienced rapid increases, maximum and or minimum population, boundedness, limits, continuity etc. You must support your presentation with a graph of the problem situation.

Pennsylvania had a population of 12.1 million people in 1995. Among the 50 states and District of Columbia, the state ranked as the 5th most populous. By 2000, it is projected to be the 5th most populous with 12.2 million people. By 2025, it is projected to be the 6th most populous with 12.7 million people. Over the three decades, Pennsylvania's total population is expected to increase to 611,000 people. Among the 50 states and District of Columbia, the state's net gain ranks as the 29th largest. Its rate of population change, at 5.1 percent, ranks as the 50th largest. From 1995 to 2000, the state would have a net increase of 130,000 people, which would rank it as the 28th largest net gain in the nation. About 4.6 percent of the nation's population resided in Pennsylvania in 1995, which was ranked 5th largest among the 50 states and District of Columbia, compared with 4.4 percent in 2000 (ranked 5th) and 3.8 percent in 2025 (ranked 6th). Pennsylvania is expected to gain 405 thousand people through international immigration between 1995 and 2025, placing it as the 10th largest among the net international immigration gains among the 50 states and District of Columbia.

The estimated population for Pennsylvania is as follows:
Year 2015 12,449,000 people Year 2025 12,683,000 people
Source: U.S. Census Bureau, Population Estimates Program
Data Set: 2006 Population Estimates

Part II: High School Rumor Problem

Logistic growth functions are used to model real-life quantities whose growth levels off because the rate of growth changes. Use your TI-83 graphing calculator to find a graph for the given function

The Spread of a Rumor

Allderdice High School has about 1700 students. Bob, Dorothy, Kathy, and Mary start a rumor, which spreads logistically so that the given models the number of students who have heard the rumor by the end of t days.

$$S(t) = \frac{1700}{1 + 50e^{-0.2t}}$$

a. Sketch a complete graph of this function, including any asymptotes.
Label the following:

- x-axis and y-axis
- scale on each axis
- all intercepts
- all asymptotes

b. State the domain and range of the algebraic function:

c. How many students will hear the rumor before the end of that day? Show how you can obtain the answer algebraically and graphically.

d. How long does it take for 800 students to hear the rumor? Show how you can obtain the answer algebraically and graphically.

e. In 10 days, how many students would have heard the rumor? Show or explain how you found the answer.

f. Based on the function, is there a limit to the number of students that will hear the rumor? If yes, what is that limit? Explain your reasoning. If no, explain your reasoning.

Lesson Two: Wheat Trade

Objectives: 5-7

Material needed: TI-83 Plus graphing calculator and the data to answer the following questions below:

Part I

Data Analysis: National and International Trade

Quarterly International Trade Report (USDA report December 2005) retrieved 2007

Global wheat production this year is down from last year's record, but will still reach the second highest level in history. Global consumption continues to climb, and is expected to reach another record this year.

Wheat Market (per thousands of Acres)

National Statistics

YEAR	Planted all purposes (thousand acres)	Price per unit (dols/bu)	Value of Production (thousand)
2006	57, 344	4.25	7,721,028
2005	57,229	3.42	7,171,441
2004	59,674	3.40	7,283,324
2003	62,141	3.40	7,929,039
2002	60,318	3.56	5,637,416

Reported: USDA (United States Department of Agriculture)-2007
NASS (National Agriculture Statistics Services)

Listed above are the results from data recorded during the given years (2002-2006). The table displays the actual report from the United States Department of Agriculture about the Wheat Market.

For problems 1-3
Use L1 number of acres and L2 planted value

1. Use the data to draw a scatter plot of the points, and then find a cubic model for the data.
2. Find a cubic model for the planted value as a function of planted acres/thousands, and determine which model is the best fit.

3. Using the cubic regression model determine the planted value for 100,000 acres.

For problems 4-7
Now create a new scatter plot of data that show planted value as a function of year.
L1 year and L2 planted value

4. Find a cubic regression for the data.

5. If the Wheat Market continues with the trend, what would be its performance for this year? In 5 years from now?

6. Using planted value as a function of year, predict the value of wheat in the year 2010, 2015 and 2050.

7. Find the year in which the planted value is expected to rise to $20,000,000,000.

8. Knowing the past behavior of a function is important because future decisions can be based on those performances. Using the cubic regression model what was the value of wheat during the beginning of the 20th Century year 2000?.

Historical Facts (19th and 20th Century).

The accelerated expansion of American agriculture after the Civil War saw continuous growth in wheat acreage and production until the turn of the century and created a large, steady surplus. Great Britain provided by far the most important market for that surplus, just as she did for other major products of America's farms. From 1870 to 1914 Britain absorbed about half of the wheat and flour exported from the United States, and in some year she took as much as two thirds (Rothstein 401).

Widespread crop failures in Russia and India in 1890 and 1891 brought an additional boom to the American trade. Wheat exports reached new heights, and fell sharply during the five years of depression that followed… but from 1893 to 1895 all the major exporting nations enjoyed plentiful yields (Rothstein 404)

Canada and the United States enjoy the largest two-way trading relationship in the world. This relationship includes a dynamic flow of agricultural products in both directions, driven by such factors as comparative advantage, geography, and demographics. In fact, Canada and the United States are each other's best customers for agricultural products (Agri-Food 2005)

Part II
World Situation and Outlook of the Wheat Trade
Global Trade

Percentage of World Wheat-Market Shares

Year	Argentina	Australia	Canada	Europe	United States
1996	5	15	20	18	28
1998	10	17	17	15	30
2000	12	17	16	14	28
2002	6	10	8	18	22
2003	5	10	8	16	24
2004	6	14	14	8	32

Source: Wheat brochure given (2004 Approximation) (Agri-Food 2006)

Use your TI 83 calculator to draw a line plot.

1. Over the last 25 to 30 years, shares of the world's wheat market have changed dramatically. There are many players in the international wheat market, and exporters enter and leave the market as conditions dictate. In a summary, report your findings with respect to the maximum and minimum value, and the end behavior for each model.

2. Using the data above predict the projection for this year and the year 2010 for each of the given countries.

Historical facts of the Wheat Market

Canada's share, which was consistently about 20% for decades, has declined over the last 10 years to average below 17%, the U.S. share of the world market has declined from over 44% in the 1970s, to the most recent 10-year average near 30%.
In the 1980's the European Union took an increasing share of world markets. However, since the mid-1990's Australia (almost 15% of the world market) and Argentina 9% have been exporting more and more wheat, joined most recently by the states of the former Soviet Union and smaller exporters.
As the global wheat market continues to climb, feeding of wheat in the former Soviet Union and Europe is higher due to reduced barley and corn crops. Additionally, there continues to be strong food consumption growth in key markets, especially Nigeria (USDA 2005).

World trade is expected to be down from last year, due primarily to sharply smaller Chinese imports. For exporters, prospects have diverged for Southern Hemisphere suppliers. Argentine production is down significantly.
Canadian exports are expected to be up from last year, in part due to much stronger world durum demand. For spring wheat, though, there are quality concerns again and feeding quality supplies are continuing to be sold into Asian feed markets. Wheat exports from Russia and Ukraine have been very strong so far this year, but could slow somewhat on concerns about smaller crops next year due to reduced winter area.(Agri-Food 2005).

Part III
In this activity you will find a function to model the data for United States Cotton Production import and export over a period of time. Use your TI-83 calculator to create scatter points and find a regression model that best fit the data below, use, linear, quadratic, cubic, quartic, exponential, and logistic or sine.

United States Cotton Production per 1,000 480-Lb Bales 2004/005

Year	Production	Imports	Exports
1990	12,196	2	7,694
1992	17,614	13	6,646
1994	16,134	6	6,862
1996	17,900	408	7,675
1998	18793	13	7,500
2000	16,968	97	6,750
2002	20,303	21	11,000
2004	18,255	50	13,800

Source: USDA-2004

1. Make a prediction for the end behavior of the Cotton Trade imports and exports for this year?
2. Find the imports and exports for years 2007, 2010, 2015 and 2020.
3. Since some investors usually look at past performance, consider what happened during the time period of (1990-2004) with the Cotton Market. Give a complete report of the entire import and export trade for the U.S., report your findings with respect to the maximum and minimum value, the zeros, the various intercepts and the end behavior for each model.

Facts:

World and foreign import and export totals were expanded to include trade among the 12 countries of the former Soviet Union ant the 3 Baltic States.

World production in 2004/05 is forecast at record 102.5 million bales, up 9.5 percent. Area is projected to respond to higher prices seen this year and in some major producing countries production is expected to recover from weather effects experienced last season.
U.S. production for the 2004 crop is forecast at 17.6 million bales down 3.6 percent from the previous crop and in line with the average for the prior 10-year period. The production forecast is based on NASS Reported planting intentions and historical abandonment rates and yields.
Consumption for 2004/05 in the U.S. is forecast to decline by 7.9 percent to 5.8 million bales, as continued downward pressure is put on the U.S. spinning industry as the remaining textile and apparel quotes are phased out. U.S. exports are forecast at 11.5 million bales, down from the record 13.8 million in 2003/04. While higher production in net exporting countries will increase competition, production increases in net importing countries will lower import demand USDA).

Annotated Bibliography/Resources

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Coates's curriculum has excellent problems that connect to real world situation. Most of the activities involve vertical motion that connects to the quadratic function.

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A great resource for algebraic functions, logarithmic and trigonometric

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The National Council of Teachers of Mathematics, (2000). *Principles and Standards for school mathematic.* Reston, VA: The NCTM INC.
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This site gives a complete report of the agricultural industry (NASS).

Appendix A: Content Standards

The Pittsburgh Public Schools have adopted standards that are used throughout the entire district. The standards describe what students should know and be able to do at four grade levels (third, fifth, eights and eleventh). They reflect the increasing complexity and sophistication that students are expected to achieve as they progress through school. The lessons and task in this unit, Calculating the Past and Present Using Historical Facts and Modern Technology, adhere to the following standards:
:

Academic Standards for Mathematics

2.1 Numbers, Number systems and Number Relationships
2.2 Computation and Estimation
2.3 Measurement and Estimation
2.4 Mathematical Reasoning and Connections
2.5 Mathematical Problem Solving and Communication
2.6 Statistics and Data Analysis
2.7 Probability and Predictions
2.8 Algebra and Functions
2.9 Geometry
2.10 Trigonometry
2.11 Concepts of Calculus

Academic Standards for Science and Technology

3.1 Unifying Themes
3.2 Inquiry and Design
3.3 Biological Sciences
3.4 Physical Sciences, chemistry and Physics
3.5 Earth Sciences
3.6 Technology Education
3.7 Technological Devices
3.8 Science, Technology and Human endeavors