Looking for Patterns in Math and Nature

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Overview

This unit was designed to be taught to 7th and 8th grade algebra students and 8th grade pre-algebra students. These students tend to be engaged in the learning process, but at times feel that math is dry and difficult to learn. I developed this unit to help students learn pre-algebra and algebra better and to increase their enjoyment of these subjects. Our Connected Math curriculum emphasizes patterns, however I think that patterns should be more strongly emphasized. Instead of talking about patterns here or there in the curriculum, I want to introduce patterns as a theme in math for the year. If students can develop the habit of looking for patterns every time we start a new unit or a new idea is taught, the students will benefit greatly. Many math ideas are based on patterns. Patterns were used by mathematicians to develop rules for negative numbers and rules for complex numbers. Patterns of graphs for different situations and equations are reflected in the graphs, equations and situations; students when they see the patterns can understand the math better.
Students will also look for patterns in nature. Looking for patterns will help students to see the connection between the way the world works and how mathematics helps to describe the way nature works.

Rationale

I want to use this unit to accomplish two purposes. First I would like to encourage a more enthusiastic attitude toward math on the part of my students. I would like the students to understand some of the exciting ways that math can be used and that math relates to our environment and the world. Second, I want to help students gain a better understanding of math, how it is used, and how it connects to very different ideas and natural phenomena. These purposes seem to coincide more and more as I have thought about them. By showing students connections with real-world phenomena, a world of applications for math is shown to them and the students may see the extraordinary connection and want to learn more.

The concepts and ideas that we covered in our PTI class helped me to think of how I wanted to add some material to the curriculum, both help students learn the material and to engage them in the fascinating world of math. These materials include the nature of theories, Newton's theory of gravitation, complex numbers, and sketching equations.

In our PTI class, we looked at exactly what we mean by theories. Theories are statements that follow known rules and are based on the best knowledge we have at the time. Sometimes theories are just part of a larger picture and subsequently become part of a larger theory that explains more than the original theory did. Theories are not guesses and one theory is not necessarily as good as another. For theories to be valid, they must follow strict rules of being able to account for known results and being able to correctly predict future happenings. Thus theories are not just guesses or possibilities of explanations that cannot be proven, as students sometimes think.

We began looking at Newton's theory of gravity. This led to reflection on how science has not been quantified for that long in terms of civilization. We are used to science being quantified, because we have been viewing science as formulas and numbers for the past 500 years. Newton's theory is the first example of a mathematical statement applying to nature; before this, descriptions in science were given in words without any numbers. One way numbers are important today is that they are used to validate or invalidate different explanations and descriptions of how the world works. Students must understand this to appreciate the necessity of math.

Newton's insight was in realizing that apples falling from trees and the moon going around the earth were examples of the same force. The same underlying description of nature encompasses both objects falling on earth and objects orbiting around the earth. At this time, math had not developed a way to describe the connection between these natural phenomenon. Newton had to develop calculus to describe the connection between gravity and the objects it acted upon. Teaching this important insight may allow a greater understanding of math applications.

Newton's theory works if you are traveling much less than the speed of light with small gravitational fields (no black holes or other large gravitational fields). If you need to account for large gravitational fields, you need general relativity. Newton's theory is incorporated within the theory of general relativity.

The idea that theories are actually the best estimates we have and really not questionable on the parts where they apply well, could be reinforced in the algebra course, when we used the formula for how fast an object is going when it falls. The formula is based on Newton's theory of gravity. This is used in the Algebra course and I want to emphasize how we are using it and applying it to find answers to some of the word problems.

Complex numbers give an illustration of how abstract ideas that mathematicians come up with apart from applications can then become essential to different fields of physics. Complex numbers were developed to ensure that all quadratic equations had solutions. Some quadratic equations don't have solutions within the realm of real numbers, since finding a solution involves taking the square root of a number. Within real numbers, there is no solution to the square root of a negative number. So someone came up with complex numbers, involving a real number component and an imaginary number component (consisting of a number multiplied by the square root of a negative one). Thus complex numbers were developed to satisfy a perceived need by some mathematicians for solutions to quadratic equations. Complex numbers are now integral to many different real life applications – the formulations of the complex Hilbert space which is essential to quantum mechanics, being one of them

I want to emphasize the different applications of prealgebra and algebra problems. Some of the time in class will be spent with more emphasis on real world problems. Also, a few projects will be done that involve real world problems that students solve or make up within specified parameters and then solve. Projects are a good way to allow for diverse needs within the classroom.

When we sketched equations, our teacher was trying to get us to look at the equations and summarize what we knew about the equations without doing any math. The emphasis here was on approximating what the equation would look like, thinking about what rate of increase or decrease it would have, where it would cross the y-axis, if the rate of increase or decrease would stay the same, decrease or increase. The equations we analyzed were more complex than most of my algebra and pre-algebra students could deal with currently. However, basic understanding of equation structure, and approximation skills are important in math. Analyzing equations and how they would look on a graph will help students to understand the equations on a deeper level.

Another part of our PTI class was looking at Fermi problems. These are problems that while we don't know the answer, we could approximate them with information that we have. With Fermi problems we were looking for answers that were loosely correct – within an order of a magnitude of 10. In forming the answers to these problems, you worked out what the variables are that go into the answer. So, in trying to figure out how many piano tuners were in Pittsburgh, one needed to think of what information you would need to find the answer. You would need to think of about how many pianos are in Pittsburgh, about how often each of those pianos gets tuned, about how many pianos a piano tuner can tune in one day, and how many days a year a piano tuner works. Once you approximate these numbers, you can figure out the answer to the problem. This type of problem makes you think of what information you need to solve it and how you can approximate or guess at that information. Once you have solved a Fermi problem, it would be easy to revise your answer based on new information.

The PTI class helped me to realize the importance of using real world problems, of making sure that students make the connection of math and how it applies to real world problems, and that the students need to see how the problems are reflected in the math ideas. While middle school students may not be ready to analyze equations as diverse as Dr. Holman went over with us, I realized how important being able to take information from a situation or equation and be able to sketch a graph and predict what the table will look like. Students analyzing equations and thinking about how to sketch them would help students to think more abstractly and to generalize their knowledge.

Many students see math as uninteresting and of little value. Students don't see connections between math and a future job they might be interested in. The students frequently want to know when and how they might use math. They ask, 'Why should we learn math?' Students have little knowledge of the many ways that math is applied to their lives and that they use many devices that depend on math for the development of these devices. Many jobs use math in ways that the students do not anticipate. I want to broaden students' ideas of when they might use math. Possibly by seeing the patterns in math and how math can be used in very different applications, students will be better able to understand that math covers a very diverse set of information.

Beyond the use of math in possible jobs and how it might relate to devices they may use, math is also a factor in how students may react to material that is presented to them. Much information is presented in the form of numbers and it is important to know how to interpret this information and also to develop a healthy questioning of what information the numbers are actually based on. By developing the habit of looking at patterns in numbers, I hope that students will also begin looking for the causes of the patterns and questioning whether the causes of the patterns are being correctly attributed or whether other causes could also be contributing to the pattern of numbers the students are looking at.

Ian Stewart observes that math is about patterns. It is about seeing patterns in nature, in numbers, and in how the world works. Some patterns are in how numbers relate to each other. Other patterns show up in the natural world, how forces relate to each other, and how numbers relate to nature. I intend to use many different examples of patterns to help students see how math relates to the world and how math concepts relate to each other. As students see patterns more, they may begin to start looking for patterns on their own. As students observe how patterns are used in math to relate concepts to each other, they may begin to look for patterns in new math concepts.

Many mathematical systems that are developed either for one purpose or independently of a specific purpose other than the joy of forming a coherent mathematical system, find other purposes and applications. Stewart in How to Cut a Cake, carves up the Earth and Moon into something they call 'm-pires', which is empires with up to m different countries in them. The empires then must be colored with each empire's countries having all the same colors. Then they are looked at in relation to how the m-pires can be graphed and colored in and looked at with corresponding nodes instead of lines separating them. This does not actually seem to have any relevance to any concrete needs in our world. However, recently these ideas have been used to find practical ways to test complex circuits to find a short circuit among many circuits and what first would have been over 1,00 tests is cut down to four.

Many times real world applications don't seem like they are going to use the math that they do, in fact, use – for example, population trends are predicted using pi . Because we cannot predict what parts of abstract math will find uses in the future, it is important to develop as many as we can. Entirely unexpected connections are made using mathematical ideas that were developed for other purposes or just for the fun of it. Thus it seems important to encourage development of mathematical concepts and connections even when we don't see an immediate use. When students bring up the specificity of pi to circles, I want to tell them about how equations for population trends also use pi.

One of the most important ideas for students to learn about is to look for patterns in math. A pattern can be shown in a list of numbers, and deciding what number(s) come next. Patterns show how mathematical ideas relate to the laws of nature.

Patterns help to show the relationship between different graphs, equations, and tables. Patterns let students see how changes that happen in a representation affect the graph, table, and equation. So when students are learning linear equations, it helps them to be able to see the pattern of change that takes place as b is changed in the equation, $y = mx + b$. Students need to see that m is the slope, and while it represents the change in y over the change in x, they need to get a feel for how the slope changes as m changes. Students would benefit from being given a graph of a function and having to show how the function's graph would change if m or b changed so that they truly understand how m and b effect the linear equation. Complex patterns in nature cannot easily be addressed well until students are looking for patterns on a constant basis in math.

As I think about it now, one of the first activities I would like to see both algebra and prealgebra students do is to look for patterns. One cannot see patterns without being able to add, subtract, multiply, and divide. If doing simple math operations occupies the majority of one's thought processes, a student has less to spare to learn new material. Thus, as an introductory activity it also serves to let the student know the importance of being able to do the simple math operations. Looking for patterns gives students the chance to practice simple calculations.

I will review adding, subtracting, multiplying, and dividing integers in terms of patterns. This will help students see how mathematicians try to make coherent systems out of the way we use numbers. Hopefully it will also give students more to connect the way integers work together, so they will be better able to remember how to do the operations. Being able to do algebra and find slopes depends very heavily on being able to consistently add, subtract, multiply, and divide positive and negative numbers. Looking at applying operations to integers in terms of patterns should help my students to remember the operations and make fewer mistakes.

Searching for patterns seems to be an essential part of math. While both our prealgebra and algebra curriculums have it as part of the curriculum, it does not get the emphasis students need to make it an important part of how they understand math. I am hoping by bringing it up at the beginning of the year and making it a theme of the year, that students will be looking for patterns more often and be able to begin seeing the importance of noticing patterns.

After looking for patterns in lists of numbers, I will introduce looking for patterns in two variables, as in linear equations. One idea that is often missed is that the pattern of two variables, (1, 3), (2, 5), (3, 7), is the same as the pattern of (1, 3), (3, 7), (5, 11), Many of my students do not immediately see that these follow the same pattern and if we connected the points with a line, that the lines would be identical. I want to make sure that students recognize these as the same pattern. This will take making up some worksheets for students to have some practice in identifying equivalent patterns. Also, I want students to try to make up equivalent patterns that might not at first be obviously equivalent patterns.

Within linear equations, I want the students to abstract patterns about slopes and y-intercepts. I want them to describe how changing m in the linear equation, $y = mx + b$, effects the graph and table of the equation. Students need to see how changing b in the equation results in changes in the graph and table of the equation. While our books have a lesson on this, I want to enlarge on that lesson and extend it. The lesson asks students to draw conclusions from just 5 diverse equations. I think the equations need to have fewer differences. One way to do this would be to have students select ways to change the equations and then observe and record the changes in the graph and what the changes would mean in terms of an actual situation.

Because of the importance of slopes and y-intercepts, I think that I will have each student do a project on changes that could happen in a scenario that would result in changes to the applicable equation. Furthermore, how changes in the equation effects the graph and table of the equation. Students will need several different scenarios that they can choose to make their equations from.

Fibonacci numbers will be introduced as a pattern of numbers that have many interesting characteristics. The fact that the Fibonacci are so prevalent in nature will be explored.

I will also introduce the golden ratio, which, as well as showing interesting and pleasing proportions that we can explore, is another irrational number. Since students have little experience with irrational numbers other than pi, it will be beneficial to have them exposed to another irrational number.

I will check with the art teacher to see if she could bring up some ways that the golden ratio shows up in art. I don't yet understand the connection of the golden ratio or the Fibonacci numbers to music, but I intend to see if that could be presented as well.

I want to bring in some special problems where math has been used to found solutions for, where the math was not developed to be the solution for that problem. The idea will be to show that there are unintended good consequences for mankind using math. Both when math is developed for one thing and then someone finds out that this same method applies to a very different type of problem, and when math is being viewed purely theoretically and then it is found that the math can be used to solve problems that had not even been thought of at the time the math was being developed.

Objectives

First, students will be able to identify patterns in lists of numbers and in the relationship of two variables to each other. Students will begin to approach new material by looking for patterns so they can take ownership of the material. Students will be able to express in writing and in words patterns that they observe. Students will be able to see how the equation for gravity is incorporated in different problems they do. Students will see how the Fibonacci sequence is seen in the world of nature.

Next, students will be able to understand patterns in applications of prealgebra and algebra problems. Students will be made aware of real world problems and will spend some time looking for applications of math to the real world.

Also, students will have a project to create real world problems within specified parameters that they solve and create a project board showing the problem and their solution. The solution would need to state how the problem is reflected in the graph, the equation and the table. This project will allow for the diverse needs of different students in the classroom.

Students will read about the Fibonacci numbers and look for examples of the Fibonacci numbers in nature and in art. Students will know what the Golden Ratio is and look for it in art pictures.

Strategies

Enable students to identify patterns in lists of numbers by giving them practice with identifying patterns.

Have students make up their own patterns for other students to identify.

Have students use patterns to understand positive and negative numbers and the results of different operations on the numbers.

Help students to develop strategies for identifying patterns in the relationship of two variables to each other. Have students realize that the same pattern between two variables can look different on two different tables by presenting them with three tables at a time and having them identify which two are the same; then have students match up tables that show the same relationship between the two variables.

Emphasize patterns in applications of prealgebra and algebra problems, by having students looking at how equations are written and what each non-variable number in the equation can change the table and graph that go with the equation. Have students look at how changes in the equation would be represented in a word problem.

Bring in real world problems to help students realize that math can be applied to problems they may encounter.

Have students create or find a real world problem and use it to make a table, graph, and equation that represent the problem. Have students explain how the table, graph, and equation are represented in the problem they have chosen. Students will explain how each different part of the problem shows up in each form of mathematical representation.

Use the Fibonacci series to introduce more patterns to students and allow them to find instances of the Fibonacci series or the Golden Ratio in nature.

Classroom Activities

First, I will go over patterns of numbers. I will give students opportunities to develop their ability to discover patterns in lists of numbers. Gradually I'll make the patterns more complicated. At first, I will use only positive integers. Patterns to present will be multiplying by 5, 2, 7, and 3. Making the patterns more complicated, I want students to be able to recognize multiplying by different real numbers while not starting with 0. More complicated patterns, like adding or multiplying by increasing numbers. Challenge problems would involve more variations, including decreasing numbers. Students will make up patterns for each other to discover.

Next I will introduce again (they had this in 7^{th} grade) adding positive and negative integers. I think that pointing out the pattern of how adding negative integers is an extension of adding positive integers. Students will be asked to notice that as the number being added to five decreases by ones, the answer also decreases by ones.

5 + 5 = 10
5 + 4 =9
5 +3 = 8
5 + 2 = 7
5 + 1 = 6
5 + 0 = 5
5 + (-1) = 4
5 + (-2) = 3

This may help students to understand and remember how adding integers works. While adding positive and negative numbers, students can also use the number line, which serves as another way for students to be able to see what happens when we add integers.

Then we will look at patterns of subtracting positive and negative integers. Students will look at the pattern made as 5, 4, 3, 2, 1, 0, -1, -2 are subtracted from five. As the number subtracted from five decreases by one, the answer increases by one.

5 - 5 = 0
5 - 4 = 1
5 - 3 = 2
5 - 2 = 3
5 - 1 = 4
5 - 0 = 5
5 - (-1) = 6
5 - (-2) = 7

Students will also write an explanation of what happens so they have put these patterns into words. Emphasis will be on the way that we can tell what will happen from the patterns formed. Many students find this complicated but when they see the patterns that form, it will help them to understand how negative numbers work much better. Adding and subtracting positive and negative integers is an important skill that will be used frequently in algebra. I really want to emphasize the importance of being able to quickly add, subtract, multiply, and divide any positive and negative integers.

Lastly, multiplication and division will be observed as numbers increase and decrease. Patterns of what happens as numbers increase and decrease will give students a better understanding of how integers work when they are multiplied and divided.

$^{+}5\ \text{X}\ (^{+}5) = (^{+}25)$
$^{+}5\ \text{X}\ (^{+}4) = (^{+}20)$
$^{+}5\ \text{X}\ (^{+}3) = (^{+}15)$
$^{+}5\ \text{X}\ (^{+}2) = (^{+}10)$
$^{+}5\ \text{X}\ (^{+}1) = (^{+}5)$
$^{+}5\ \text{X}\ (0) = (0)$
$^{+}5\ \text{X}\ (^{-}1) = (^{-}5)$
$^{+}5\ \text{X}\ (^{-}2) = (^{-}10)$
$^{+}5\ \text{X}\ (^{-}3) = (^{-}15)$.

As they observe these patterns, students will write an explanation of why a negative time a positive is negative. Students will then be able to extend this to figure out what negative times negative will be. This exercise will help students to remember what happens when positive and negative integers are multiplied or divided.

After this, I will give students more practice identifying patterns, this time with both positive and negative integers. Again, I will slowly make the patterns more complicated.

The first pre-algebra book is about linear equations. As we do this unit, the emphasis on patterns will help students to make sense of the unit, to have a deeper understanding of linear equations.

Students will use the equation, $y = \mathrm{m}\,x + \mathrm{b}$ to look at what happens when we change m and b. We will begin with the equation, $y = x$. After making a table and graph for it, I will ask students to think of a situation where that equation may apply. One possible situation is how much it will cost in dollars (y) for x items. I think that looking at equations of the form, $y = \mathrm{m}\,x$ first without the b will help students to understand the relationship between the rate of increase (m) and the slope or steepness of the line on a graph. They are directly related.

Students will next change the b in the equation, $y = \mathrm{m}\,x + \mathrm{b}$. Having students change the value of b and graph it should help them to see how b is the y-intercept. Students will be asked to think of some situation where their graph might apply.

I also want students to try sketching what they think a graph will look like even without making a table of values. I think that once students see how the values of m and b change the students will be able to sketch a graph of the equations without first making a table.

The same strategies will again be used later in the year when we begin to look at parabolas. Parabola equations take the form, $y = \mathrm{a}\,x^2 + \mathrm{b\,x} + \mathrm{c}$. I would first have students look at graphs with equations, $y = \mathrm{a}\,x^2 + \mathrm{c}$. After the students find that the c is the y-intercept and the value of a relates to how quickly the parabola increases in value. As students can see the effects of changing non-variable parts of the equations, it will help them better understand how a graph should look, even when they are making a table first. This would serve as a way for them to check their work. The algebra students would also work with equations of the form, $y = \mathrm{a}\,x^2 + \mathrm{b\,x} + \mathrm{c}$.

Students will have a project to make or select a real world problem and show a table, a graph, and an equation for their problem. The students will tell how the problem situation is reflected in the table, the graph, and the equation. This will give students a chance to show what they have learned and reinforce the concepts.

The difficulty of the problems selected will allow for differentiation. I will encourage those students who have a better grasp of the mathematics to use more difficult or complicated situations and equations.

Annotated Bibliography/Resources

Cohen, I. B., The Triumph of Numbers. W. Norton and Company, Inc., New York, 2005
This book looks at the history of how numbers have been used and how they have shaped societies. It looks at the use of statistics in history, how the use of statistics affected history, at how numbers began to be used in science, and how the use of numbers in science affected the world.

Peterson, Molly Malone, 'Art and Mathematics in the Elementary Curriculum', 1994, Paper presented to California Sate University, Fullerton, for partial fulfillment of MS degree in Education
This paper tells how one can use math and reinforce ideas from math in the art classroom in both elementary and middle school. It also reviews the importance of math and art to each other and how teaching art can help students learn math.

Sarukkai, Sundar, "Revisiting the 'unreasonable effectiveness' of mathematics, Current Science, Vol. 88, No. 3, 10 February 2005
This paper considers what exactly we mean by mathematics, what we mean by applying mathematics in science and also what we mean by 'unreasonable effectiveness."

Stein, Sherman K., Strength in Numbers; Discovering the Joy and Power of Mathematics in Everyday Life, John Wiley and Sons, Inc., New York, 1996
Stein describes how math is relevant in our daily lives and describes the beauty of math and how it can be used.

Stewart, Ian, How to Cut a Cake, Oxford University Press, Oxford, 2006
In this book, Stewart describes different mathematical recreations. After describing a part of math that was developed as an area of interest to play around with, he then describes how that math is used in practical applications.

Stewart, Ian, Life's Other Secret; The new mathematics of the living world, John Wiley and Sons, Inc., New York, 1998
This book looks at how math is part of the world, how math is seen in such diverse things as shells, and the stripes of a tiger.

Stewart, Ian, Nature's Numbers, Basic Books, New York, 1995

Stewart describes how mathematics is used to describe nature.

Stewart, Ian, The Problems of Mathematics, Oxford University Press, New York, Oxford, 1987

Wigner, Eugene, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," In Communications in Pure and Applied Mathematics, vol. 13, No. I (February 1960). New York: John Wiley and Sons, Inc., 1960
Wigner looks at how mathematics can be applied so effectively to describe our physical world and how many ideas in math have applications that are not anticipated by the person(s) developing the mathematical ideas.

Appendix-Content Standards

M8.A.3.3.1 Add, subtract, multiply, and/or divide integers, fractions and/or decimals with and without a calculator (straight computations or word problems)

M8.C.3.1.1 Plot, locate, or identify ordered pairs on a coordinate plane

M8.D.1.1.1 Continue a numeric or algebraic pattern that could be extended infinitely (pattern must show 3 repetitions – may include up to 2 operations, squares and square roots)

M8.D.1.1.2 Find Missing elements in numeric, geometric, or graphic patterns and/or functions (may be given a table or rule – pattern must show 3 repetitions)

M8.D.1.1.3 Write/state the rule of a function (given elements in an input-output table, chart or list)

M8.D.2.1.3 Determine the value of an algebraic expression by simplifying and/or substituting a value for the variable.

M8.D.2.2.1 Match a written situation to its numeric and/or algebraic expression, equation or inequality (up to two variables in equations or expressions – one variable with inequalities).

M8.D.2.2.2 Write and solve an equation for a given problem situation (one variable only)

M8.D.4.1.1 Graph a linear function based on an x/y table (integers only).

M8.D.4.1.2 Match the graph of a linear function to its x/y table (integers only).

M8.D.4.1.2 Match an inequality to its graph on a number line (integers only).

M8.D.4.1.4 Match the linear equation (y = mx + b form) to the x/y table (integers only in the table).