Population and Exponential Growth

Barbara Misechok
Student Achievement Center

1. Overview

Mathematics is a form of communication. Math is more than just formulas, numbers, operations and symbols. Math is one of the ways of making sense of the world. It is a part of everyday living not a topic in isolation. Yet, many educators try to separate mathematics from communication making math a dry topic where students learn formulas and how to plug numbers into the formulas to solve problems. Actually math and literature enrich each other. The focus of this prospectus will be to show the benefits of the marriage between math and literature.

This unit will use the novel, Among the Hidden, to introduce mathematical operations. This novel takes place in the future under a totalitarian government. Each family is permitted only two children to limit the population of the country. Some people have a third child that they hide from the population police. The third child is called a shadow child. Several mathematical concepts are presented in this novel. I plan to pull these concepts out for mathematical exploration and instruction. One key concept is the idea of exponential growth. The novel is taught in our district at the middle school level. This lesson will be appropriate for middle school as it uses a middle school novel and the connected math book. I plan to use it with my high school student with special needs that receive instruction in the resource room. The District's special education department has asked the resource room teacher to use the connected math series for students with special needs.

2. Rationale

The national math standards recognize the importance of communication in mathematics enough to include communication as one of the standards. The National Council of Teachers of Mathematics (NCTM) states that communication is a critical component of math education. The goal of teaching mathematics is for the student to gain an understanding of the concepts of math. Linking math with literature will help students better understand math. Research shows that the more learning is connected to prior learning, the more likely we are to remember it. Literature helps link math for the students. The concepts are linked to the story not just to a rule or formula.

When students encounter mathematics in the context of a story, students are forced to think about the mathematics. Students work through the concept as they read the story, which leads the students to owning the math. With the math embedded in the story, the student sees how math is used to communicate life experiences. Therefore, a direct link between math the real life is developed through the use of literature. Students do not have to think when they might encounter the concept. The link is apparent. Research indicates that students are better able to make the connection between math ideas and their personal experiences when the math is presented using literature (Murphy, 2000).

Literature provides the opportunity to use more modalities in teaching. Using more modalities leads to better understanding and retention by students. Murphy found that math that is taught using literature is able to accommodate students with different learning styles (Murphy, 2000). Visual learners are able to see the math through visualizing the story and by looking at the text of the novel. Auditory learners are being introduced to the concept through hearing the story.

Each story has many applications to math class. (Welchman-Trischler, 1992) The story can have manipulatives that can be used as in the story or in other ways in the classroom. The book might be used to pose an interesting question that students will work cooperatively to solve. The story might explain the concept or just introduce the concept so the teacher can teach it. Sometimes the book might be used to review a skill that has been learned and provide a context for the skill in the real world.

Students may become more interested in math when the concept is initially presented in literature. This leads to more motivation to learn. Research backs up this idea that literature might ignite student's curiosity in math (Usnick & McCarthy, 1998). When the student relates to the characters and plot of a story, the math concept becomes interesting. As the reader becomes intertwined with the action in the story, the reader also becomes intertwined with the math. There is something familiar about reading a story. It is not natural to look at a formula and plug in numbers.

American students are losing ground to other nations in math scores. Literature may be a way to help American students gain on the standardized testing. Research points out that one of the results of using literature to teach mathematics is that students become better critical thinkers (Murphy, 2000). Being critical thinkers is key to being a strong scholar of math. Another key area that is improved when literature is used in conjunction with math is in problem solving. Jacob and Rack found students' ability to be problem solvers is increased with the use of literature in the math curriculum (Jacob & Rack, 1997). Being able to be a

problem solver is needed to be successful not only in school but in your work life after high school.

When math is integrated with literature, it can provide a way for parents to better help their children. Often parents do not remember or were not taught the math they are trying to help their children with. If the math is in the literature, parents are better able to help the students at home. Research provides validity for this idea (Hartog & Brosnan, 2003). Parents are better able to help with homework that is literature based. This provides a framework for the parent to understand the assignment thereby increasing their comfort level with assisting with homework.

With all the benefits of using literature for teaching math, it is not surprising that is it becoming very popular for younger students. Not as much material is available for older students. The research would apply to all aged children so the benefits are there for the older students. I plan to design material that will be appropriate for the older students with special needs.

My classroom has students who are not able to be successful in the general education classroom. Teaching mathematics using literature may help my students better understand the idea. The math concept is not a stand-alone item to remember. Instead the concept is part of a story that the student can link in their mind. The brain of a student with a learning disability often does not work in the same way as a student without the disability. This method may help the students make the link in their brain.

Additionally, my students are typically low skilled in reading and mathematics. Teaching math with literature gives the student more exposure to literature and reading. The terms used in trade books for math are often hard for my students to understand. The story format should help with understanding. I could find no specific research about the topic of teaching math through literature for students with special needs, but based on my experience working with the students for the past eight years, I believe this approach will enhance the learning of my students.

The curriculum for the Pittsburgh Public School is one of discovery in learning. Students are accountable for their learning instead of being passive recipients of learning. This model of linking communication and mathematics fits with the curriculum of the Pittsburgh Public Schools. Instead of a lecture to introduce a topic, the students learn about the topic by reading the story. This generates natural curiosity about the mathematics topic for the student. The mathematics topic is a student discovery model based on information in the novel.

3. Objectives

For my project, I plan to use assignments in English class as a springboard to math lessons. Since I teach both English and Math in the resource room I have the ability to link the disciplines. Most of the students who are in the resource room for math are also in the resource room for English. We can read the novel in English class then raise math questions to explore and solve in math class.

The novel, Among the Hidden, written by Margaret Peterson Haddix has many ideas that can lead to mathematics instruction. The novel is about a futuristic society where a totalitarian government limits the number of children that each family can have. Once parents have two children, they have to get rid of any additional children. The young man telling the story is a third child. He has to hide from the outside world or the population police will take him away. Farmers are told what crops and how much they can raise. The government rations foods as well as other essentials.

One of the texts in the connected math curriculum is the text, Growing, Growing, Growing Exponential Relationships that includes the topic of exponential growth. Since population grows exponentially, the text connects with the population growth issues at the core of the theme of the novel. The lessons in the text will involve reading and creating tables and graphs leading to an understanding of exponential growth through discovery in cooperative learning situations. The idea of rationing food so that each person gets the 1500 calories permitted by the government in the novel could lead to predicting and measuring lessons. It would also have applications in the consumer science curriculum. Other math applications may be found as the student raise math questions from the novel. This interaction between the disciplines should help students see the connection of math and literature as well as the link between math and the real world.

Population growth is a natural topic to introduce the idea of exponential mathematics. The Stevens Institute of Technology has developed a series of activities that the students perform on-line that explore the environmental and mathematical aspects of population growth. This will be the starting point for this topic. The students will work in cooperative learning groups to complete these activities. The students will use archived census and demographic data as well as current population estimates from the U.S. Census Bureau to learn about population growth. The exercises will help the students study the implications of a changing population. Students will examine linear, quadratic and exponential models in this project. The complete unit consists of nine interactive lessons that the students can do independently. Given the complexity of some of the lessons, I do not plan to incorporate all nine lessons for my students. We will complete only the first two lessons and the last three lessons in the series. The first lesson involves the

population explosion and requires the students to examine the population using the population clock on line. The second lesson focuses on factors that are involved in population growth. The third lesson relates to constant growth rate patterns. Students will examine linear and exponential growth rates. The fourth lesson is a continuous growth rate model. And the fifth and final lesson in this group is the impact of a growing population on the world resources.

Exponential growth is covered in the Growing, Growing, Growing Exponential Relationships text in the connected math series being used in the Pittsburgh Public Schools for middle school students and high school students with special needs. Students will complete a series of investigation. The first investigation is a hands-on activity where students will cut papers to make ballots for an election. Next students will investigate growth of money in a kingdom called Montarek. These activities give students several variations of exponential growth of coins. It allows students to explore several patterns of growth with doubling and tripling on squares. The next investigations are more real life applications. The first one utilizes population growth of rabbits in Australia. This investigation shows how scientists use exponential growth to research and solve problems they encounter. The second investigation looks at growth of collectable coins over time. It introduces the ides of compound growth.

The knowledge the students have gained from the population unit will lead to the exponential mathematics lesson found in the connected math book. The objectives for the math investigation in the Growing, Growing, Growing Exponential Relationships Book as well as the population unit are for the students:

- To develop an understanding of basic exponential growth patterns
- To identify exponential patterns in tables graphs and equations
- To solve problems involving exponential growth
- To express a number in both exponential form and standard form.
- To compare and contrast growth exponentially and linearly
- To explore problems involving exponents and exponential growth
- To calculate the growth factor in an exponential model
- Use data to determine growth factors and produce representations of an exponential population model
- To explore compound growth as it relates to increases in the value of an asset
- To explore the factors that impact population growth
- To increase research skills to investigate population growth
- To visualize the world population
- To estimate the population based on specific assumptions
- To successfully work in cooperative learning situations

- To develop presentation skills
- To increase computer skills

4. Strategies

Students will work in small groups to perform the explorations in this unit. In the beginning the students will be working on the internet to explore the world's population using the Stevens Institute of Technology's on-line activities.

The first two lessons introduce students to population growth. The U.S. Census Bureau's World Population Clock is available on-line to provide students with the real time estimates of the world's current population. This is not the actual population because there is no way of knowing exactly what the population is at any given instant in time. The clock is an estimated population of the world based on current mathematical models. Students will use the clock to complete exercises in determining by how many people the population grows each second, minute and week. Next they will compare the weekly increase in population to the population of the United States. This comparison is performed to help the student get a better feel for the large numbers. The number alone with nothing for comparison may be meaningless for the students.

Next, the students will move to examine historical population growth. Students will consider some past events that may have influenced population growth during that time. This will lead to the students constructing their own graphs of world population growth using the data provided by the U.S. Census Bureau.

The last part of the first two lessons is for the students to inspect a chart that shows areas of the world that have had rapid population growth in the last decade. Students will use the chart to find the regions that have experienced the most rapid growth. Each team will speculate about why this might have happened. It is not just chance that one area might grow much more quickly while another area is declining in population.

The students will move to a lesson on predicting exponential growth rates with a constant growth rate. Students will use the formula for exponential growth rate with a constant rate to develop an equation to determine the US growth rate in 1999. This year's model will be used to predict other years. The students will compare their predictions with other noted predictions. Students will discuss the accuracy of their model and limitations to using it for long term predictions.

The next lesson will require the students to make prediction about population growth in the US with a continuous change exponential model. Students will now work with the formula for a continuous change model. The students will again develop the equation for 1999 and use this formula to predict other years. The same process will be used for comparing their predictions to other known predictions. Students will discuss the accuracy of this equation and limitations to the use of the model.

The final lesson in the Stevens Institute group will be Lesson 9 where the students will be looking at the relationship between population growth and the world resources. Students will identify the factors that will limit population growth. Next, the students will look at how the human population is impacting resource consumption. This will lead to consideration of how many people the world can support.

These sets of lessons from the Stevens Institute provide the students with direct real world application for exponential growth. Students will be able to discuss population data and answer questions that they generated in the beginning of the discussion of the novel, Among the Hidden. They will understand that math is useful in the real world.

The class will begin working in the connected math text to further enhance their understanding of exponential growth. The students will begin with a hand-on activity cutting a paper to make ballots for an election. The set of activities involve a fictitious kingdom where a peasant has saved the king's daughter and the king wants to reward the peasant. The students look at a number of scenarios for paying the reward. Each scenario helps the students see the rate of growth of the rubas given specific constraints.

Next the students will put what they have learned to use in more real life situation. Homework will look at the number of ancestors of a student. The class work will involve an investigation of rabbit reproduction rates in Australia that requires examining exponential growth.

The final activity utilizes coin value to explore exponential growth. This activity will require the students to use what they have learned about exponential growth to compile data about the value of a coin over time. The students will learn about compound interest.

5. Classroom Activities

First, the students develop questions they have about the ideas presented in this novel. Some of the questions might be:

- Could the world get too crowded to be able to feed the population?
- Are we in any danger of that happening soon?
- Why are people hungry today?
- How fast is the population growing?
- Is this different from past growth?

Next, the class will work in teams to get some answers to these questions. The class will begin with the exercises in the Stevens Institute of Technology's on-line activities described above. This will let students see first hand how the population is growing in the world. Students will see how this compares to the past. The students will learn how to make predictions using the formulas for exponential growth both with constant growth and with changing growth rates. Then, students will perform exercises to connect the population with the resources on the earth. This should enable students to answer the questions they have generated or at least make reasonable assumptions about the answers.

Each team will go to the internet site to begin the exploration. Worksheets will be provided to the team for the first two activities. (See Attachment A for a copy of the worksheets) For the first activity, the teams will use the population clock to estimate the world population growth at given intervals and record their results. The U.S. Census Bureau's State and County Quick Fact will be used to find the most recent population estimates for Pennsylvania and Allegheny County. The teams will need to compare Allegheny County to the number of people the world population grows by each week. The next exercise requires the teams to look at historical estimates of the world population to see when the world's population started to suddenly increase and think about what might be the cause for this. The teams will check the World Hyper History site or other sources to see what may have influenced population growth. Next the teams will examine the Population Growth Rate Map to determine areas of the world where the population has been growing most rapidly and make conjectures as to why this is happening. After all teams have completed the assignment, each team will report back to the class what they have learned. We will discuss the world population and the population in Allegheny County in comparison to the world population.

For the second activity the teams will look at the factors that make up the population, birth and death rates, to make conjectures about that might cause a high or low rate. (See Attachment A for a copy of the worksheet) The teams will examine charts of birth and death rates making statements about the world's growth

rate in recent years. Next the teams will look more in-depth at two factors that impact birth and death rates, fertility rates and life expectancy. Students will use the International Database (IDB) Summary of Demographic Data database to estimate the total fertility rates and life expectancy in the United States. The teams will explore how fertility rates affect birth rates and ultimately affect the population growth rate. Next they will examine life expectancy to answer similar questions about its impact on death rates and the population growth rate. Migration of people across regional boundaries and emigration to other regions will be examined as to how it impacts population rates.

The next tasks for each group will compare exponential growth with constant growth rates. Each team will complete the activity independently. (See Attachment A for a copy of the worksheet) The students will be introduced to the formula for exponential growth with a constant growth rate. From the United States Census Bureau's Historical National Population Estimates, 1990 to 1999, the teams will record the estimated national population in 1999 and the estimated average annual percent change (growth rate given in percent) for that year. Next student will use this data to give an equation to express the population as a function of *t*, the number of years after 1999. The teams will use the equation to predict the population in 2000, the current year, and 2010. Next, the team will compare your results to the estimated values given in the International Database (IND) Summary of Demographic Data for the United States as well as the U.S. Population Clock. Using the U.S. census data Population, Housing Units, Area Measurements, and Density: 1790-1991 that you previously assembled and the average annual growth rates that you calculated, record the national population for any year prior to 1970 and the average growth rate for that year. Assuming that the growth rate remained at this value, students will be asked to give an equation to express the population as a function of *t*, the number of years after your selected year. Then each group will report their results to the class and we will discuss these results. The procedure will be repeated for the activity for continuous population growth. From the U.S. Census Bureau's Historical National Population Estimates, 1990 to 1999, the team will record the national population for 1999 and the average annual percent change (growth rate given in percent) for that year and give an equation to express the population as a function of *t*, the number of years after 1999. Using the equation the teams will predict the population in 2000, the current year, and 2010. Next they will compare their results to the estimated values given in the International Database (IND) Summary of Demographic Data for the United States as well as the U.S. Population Clock. The teams will need to decided which model they think is better and support why.

The final activity for this section requires the teams to consider the impact that a growing population has on our world resources. (See Attachment A for a copy of the worksheet) Student will look at an experiment where a scientist places one

bacterium in a Petri dish at 9:00 am. The bacteria can reproduce at a rate that doubles its population every minute. The scientist observes that the Petri dish is completely full at 10:00 am. The students will answer questions about the observations including how the experiment is alike and different than human population. The teams will consider what areas of the world are most populated. And make conjectures as to why do you think these areas have so many people based on what they know about these areas. Students will look at how population size affects resources. Student will consider idea of carrying capacity of the earth and decide if the earth has reached its carrying capacity yet. Then students will research an issue related to population growth in a country of your choosing and explain how that country's population growth impacts the country as well as the world. Each team will report back to the group. This can be made into a more formal activity requiring the students to prepare posters or PowerPoint presentations for the class. It would be a great portfolio item for the student. It culminates the learning of the students on the population issue while requiring the students to use their knowledge of exponential growth.

The next set of lessons is investigations that relate to exponential growth found in the Growing, Growing, And Growing Exponential Relationships. The book is an algebra book from the connected math series. These activities are performed in small groups. The first activity is about making ballots for an election where the student to repeatedly cut a piece of paper in half and count the number of pieces of paper. This exercise shows first hand how exponential growth work. This activity helps the students visualize the growth.

The next class investigation is about a fictitious kingdom where the king wants to reward a peasant for savings his daughter's life. The peasant asks for rubas, the least valuable coin in the kingdom; however the peasant wants one ruba on the first square of a chess board with the number of rubas doubling on the next square until the end of the board of 64 squares. The king agrees. The students have to see how many rubas the king will need to pay as a reward by making a chart and examining the data for patterns. Students can work in groups to do these activities. The results will be reported back to the class and results posted on the wall. The queen tells the king that he is giving away all the money in the kingdom. So he makes a second offer. The class will do the same activities with the new offer. Once again the results will be discussed by the class and data posted. The queen helps the king and she makes a counter offer different than the two offers of the king. The students examine at the new offer to see which plan is better for the kingdom. Projections of the cost to the kingdom will be computed by the teams. The students will decide if the peasant should accept the queen's offer. The results of this activity will be posted on the wall as well.

The accountants help the king develop an offer that is reasonable for the kingdom. This offer has the ruba increase by five on each square. The students will examine this offer. They will work again in teams to compile and analyze the data. As a group the class will see how the offer is acceptable for the kingdom. The peasant does not think this offer is as good as the other offer and refused. One final offer from the king involves starting with a very large number and increases by one million for each square. Students will perform calculations for this scenario. The class will examine the results that are posted from the previous offers to see how they are different. Why are the other offers not good while this one is acceptable to the resources of the kingdom? The students will independently write an ending for the story.

The students will be assigned an application of this concept of exponential growth for homework. This problem asks the students to determine the number of ancestors back for 10 generations. The students have to make a chart, table, equation and answer questions about their finding. The homework will be compared and discussed in the class the following day.

The third investigation done is class will be looking at rabbit reproduction in Australia. Rabbits are not indigenous to Australia and are causing problems for farmers. Students develop a formula to see how many rabbits are in the country given a specific number of years after their introduction by the British. The students will work in groups to compile charts and graphs to analyze the data. What are the implications of the data? The groups will come together to discuss their findings. The class will discuss the officials programs to control the population growth and answer questions about the efforts.

The final investigation that will be done as a group will be on investing for the future. The student will consider the value of a coin collection at various times based on a given rate of increase in value. This will show students another direct use of exponential growth in the real world and use a pattern of change referred to as compound growth. This final investigation will be done in groups with the same protocol. The teams will compile data, scrutinize the data, and make conjectures based on the data. Teams will report back to the class their conclusions.

Attachment A

Group Members Names: _____
_____

Activity #1
Population Explosion

1. Look at the U.S. Census Bureau's World Population Clock. This clock gives an up to the second estimate of the world population.

By how much does the population grow in 1 minute? _____

How many people is that per second? _____

How many people is that per week? _____

2. Using the U.S. Census Bureau's State and County Quick Facts, find the most recent population estimates for your state and county.

How does your county population compare to the number of people the world population grows by each week?

_____
_____
_____

Are you surprised by the results?

_____
_____
_____

3. Look at this graph of Historical Estimates of World Population. In what year did the world population start to suddenly increase? _____

What do you think might be the cause for this?

_____
_____
_____

How might historical events or the quality of life during different historical periods have influenced population growth? Check the World HyperHistory site or other sources to see what may have influenced population growth.

_____
_____
_____

4. According to this Population Growth Rate Map, in what areas of the world has the population been growing most rapidly? _____

Why do you think this is?

_____
_____
_____

Group Members Names: _____
_____

Activity #2
Factors in Population Growth

The natural increase of a population depends on the number of births and deaths. If the number of births is greater than the number of deaths at any given point in time there will be natural increase in the number of people. Typically, the growth rate of a population is given in terms of the birth rate (number of births per 1000 people per year) and death rate (number of deaths per 1000 people per year).

1. Write an equation that expresses the growth rate of a population in terms of the birth rate and death rate.

_____

2. What circumstances might result in a high birth rate for a population?

_____
_____
_____

A low birth rate?

_____
_____
_____

3. What circumstances might result a high death rate for a population?

_____
_____
_____

A low death rate?

_____
_____
_____

4. Look at these charts of Birth and Death Rates. If both birth rates and death rates are declining worldwide, why is the world's population still increasing?

_____
_____
_____
_____

Based on this birth rate and death rate information, what could you say about the world's growth rate in recent years?

_____
_____
_____
_____

5. There are many factors that contribute to birth rates and death rates. Fertility rates and life expectancy are just two examples. Fertility rate is the average number of children born to women in a given population. Look at the summary demographic data for the U.S. in the IDB Summary Demographic Data database. In 2000, what was the estimated total fertility rate and life expectancy in the U.S.?_____

6. Look at historical information for U.S. birth rates and fertility rates between 1920 and 1999. How might fertility rate affect the birth rate?

   _____
   _____
   _____

How might it ultimately affect the population growth rate?

_____
_____
_____
_____

7. Look at historical information for U.S. life expectancy at birth between 1900 and 2000 (all races, both sexes). How might life expectancy affect the death rate?

   _____
   _____
   _____

How might it ultimately affect the population growth rate?

_____
_____
_____

8. For countries, states, and regions, the population growth rate is also affected by the rate at which people migrate across regional boundaries. Net migration is the difference between immigration (movement into a region) and emigration (movement out of region.) Write an equation that expresses the growth rate of a population in terms of the birth rate, death rate, immigration rate and emigration rate.

_____

9. Can you think of any other factors that might affect the growth rate of a country?

_____
_____
_____
_____

Group Members Names: _____
_____

Activity #3
Exponential Models
Constant Growth Rate Model

When looking at population growth over a short period of time it may appear to follow a linear function.

While a linear function can be used to model population growth that has a constant increase or decrease in the number of people, an exponential function can be used to model population growth that has a constant percentage change in population.

Since the average annual percent change in a population (growth rate) is often relatively constant during a short period of time, it is not uncommon to fit an exponential model to population data.

Exponential Function
Constant Growth Rate
Model

$$f(t) = ab^t$$

f(t) = population after *t* years

a = initial value

b = base or growth factor

t = time in years

This exponential model can be used to predict population during a period when the population growth rate remains constant.

1. From the U.S. Census Bureau's Historical National Population Estimates, 1900 to 1999, record the estimated national population in 1999 and the estimated average annual percent change (growth rate given in percent) for that year.

   _____

2. Assuming that the growth rate remains constant, give an equation to express the population as a function of *t*, the number of years after 1999. Remember:
   f(t) = Population *t* years after 1999
   a = Population in 1999
   b = Growth factor = 1 + growth rate in decimal form
   t = number of years after 1999

   Example: In July 1964 the population was 191,888,791 and the growth rate was +1.39 % or 0.0139.
   The growth factor = 1 + 0.0139 = 1.0139
   This means that the population in 1964 was approximately 1.0139 times the population in 1963. An equation to find the population *t* years after 1964 would be:
   $f(t) = (191{,}888{,}791)(1.0139)^t$

   _____

3. Using the equation you determined for 1999 data, predict the population in 2000, the current year, and 2010.

   _____
   _____

4. Compare your results to the estimated values given in the International Database (IDB) Summary Demographic Data for the U.S. as well as the U.S. Population Clock. How close were your results? Why might they be different?

   _____

5. Using the U.S. census data Population, Housing Units, Area Measurements, and Density: 1790-1990 that you previously assembled and the average annual growth rates that you calculated, record the national population for any year prior to 1970 and the average growth rate for that year. Assuming that the growth rate remained at this value, give an equation to express the population as a function of *t*, the number of years after your selected year.

6. Using this equation, predict the population in 2000, the current year, and 2010.

7. Compare your results to the estimated values given in the International Database (IDB) Summary Demographic Data for the U.S. as well as the U.S. Population Clock. How close were your results? Why might they be different?

8. From what you've learned, is this exponential model good for predicting population in the short term?

   In the long term? What about over thousands of years? Explain your reasoning.

Group Members Names: _____
_____

Activity #4
Exponential Models
Continuous Change Model

There are other functions that can be used to model exponential growth. If a population grows (or decays) continuously over a period of time such that the rate of change of population is proportional to the total population, then a continuous change model for exponential growth can be used. This model is similar to the model used to determine how much a savings account grows when the interest is compounded continuously.

Exponential Function Continuous Change Model

$$A(t) = Pe^{rt}$$

A(t) = amount of population after *t* years
P = initial Population
e = exponential constant
r = annual growth rate
t = time in years

This exponential model can be used to predict population during a period when the growth of a population is continuous. The constant growth rate model used in Activity 4 does not assume continuous growth.

1. From the U.S. Census Bureau's Historical National Population Estimates, 1900 to 1999, record the national population for 1999 and the average annual percent change (growth rate given in percent) for that year.

   _____

2. Assuming that the growth rate remains constant, give an equation to express the population as a function of *t*, the number of years after 1999.

   _____

   Remember:
   A(t) = Population *t* years after 1999
   P = Population in 1999
   e = 2.718....

r = Annual growth rate in decimal form
t = number of years after 1999

Example: In July 1964 the population was 191,888,791 and the annual growth rate was +1.39 % or 0.0139.
An equation to find the population t years after 1964 would be:

$$A(t) = (191{,}888{,}791)\, e^{(0.0139)t}$$

3. Using the equation you determined for 1999 data, predict the population in 2000, the current year, and 2010.

   _____
   _____
   _____

4. Compare your results to the estimated values given in the International Database (IDB) Summary Demographic Data for the U.S. as well as the U.S. Population Clock. How close were your results? Why might they be different?

   _____
   _____
   _____

5. Do you think this continuous change model is any better than the constant growth model for predicting population in the long term? Support your answer by showing what today's predicted population would be if the continuous change model was used for a year in the past, assuming that the growth rate remained at the same value as in that year.

   _____
   _____
   _____
   _____

6. Using the continuous change model, predict when the U.S. population will reach 300 million, assuming the growth rate remains at its present rate. When will it reach 350 million?

   _____
   _____
   _____

   When will the population double in size from its present value?_____

   Why might your results differ from those given in the International Database (IDB) Summary Demographic Data ?

   _____
   _____

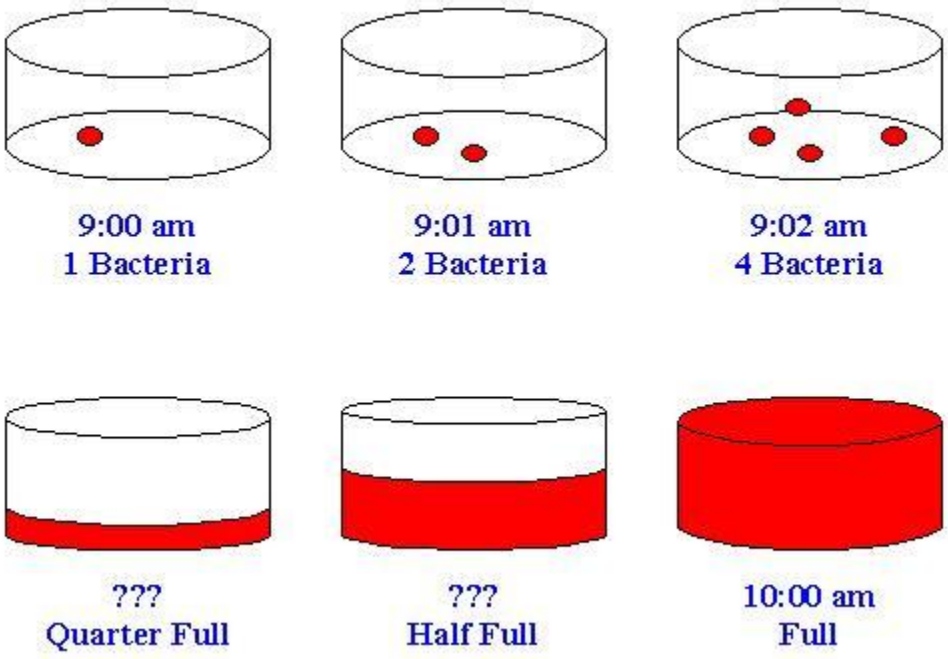
_____
_____

Group Members Names: _____
_____

Activity #5
Impact of a Growing Population

Consider the impact that a growing population has on our world resources.

1. A scientist places one bacterium in a Petri dish at 9:00 am. The bacteria can reproduce at a rate that doubles its population every minute. The scientist observes that the Petri dish is completely full at 10:00 am.



- When was the dish half full? _____Quarter full? _____
- What volume will the bacteria population occupy at 10:02 am? _____
- At what time do you think the scientist realized there would be a problem? _____
- In what ways is this example similar to human population growth? _____

_____
_____
_____

- In what ways is this example different from human population growth?

_____
_____
_____
_____

2. What are the limiting factors in an environment that will control the growth of most populations of organisms?

_____
_____
_____
_____

3. What areas of the world are most populated? Why do you think these areas have so many people? The links below may help.

- Visible Earth Images - Population
- World Population Density Map

_____
_____
_____
_____

4. How does population size affect the resources used by a country? How does population size affect other environmental conditions? Are there other factors besides population size that can have an impact on resource consumption and environmental conditions in a country? Explore the links below (or other sources) to answer these questions.

- Earth Trends - World Resources Institute
- Visible Earth Images

_____
_____
_____
_____

5. What is meant by carrying capacity?

_____
_____
_____
_____

Do you think the earth has reached its carrying capacity yet?

Why are there so many differing opinions about this?

6. Research an issue related to population growth in a country of your choosing and explain how that country's population growth impacts the country as well as the world.

6. Annotated Bibliography/Resources

Hartog, Martin D. and Brosnan, Patricia. (2003). Doing Mathematics with Your Child. ERIC Digest. http://www.ericdigests.org/1995-1/child.htm

Hartog, M.D., & Brosnan, P.A. (1994). Doing mathematics with your child. ERIC/CSMEE Digest. Columbus, OH: ERIC Clearing-house for Science, Mathematics, and Environmental Education. (ED 372 967)

Haddix, Margaret Peterson. (1998). *Among the Hidden*. New York. Aladdin Press.

Jacobs, A. & Rak, S. (1997). Mathematics and literature-A winning combination. "Teaching Children Mathematics," 4 (3), 156-57. [EJ 556 192]

Lappan, Fey, Fitzgerald, Friel, Phillips. (1998). *Connected Math: Growing, Growing, Growing: Exponential Relationships.* Michigan State University. Dale Seymour Publications.

Murphy, S. J. (2000). Children's books about math: Trade books that teach. "New Advocate," 13 (4), 365-74. [EJ 617 808]

Rosemaire Kolstad, L.D. Briggs, Karen Whale. Education, Vol. 116, 1996. (Kolstad, Briggs & Whalen 1996)

Steven Institute of Technology. (1999). “Population Growth”. Center for Innovation in Engineering and science Education. Available on line at http://www.k12science.org/curriculum/popgrowthproj/index.html

Usnick, V. & McCarthy, J. (1998). Turning adolescents onto mathematics through literature. "Middle School Journal," 29 (4), 50-54. [EJ 615 455]

Welchman-Tischler, R. (1992). "How to use children's literature to teach mathematics." Reston, VA: National Council of Teachers of Mathematics. Available online at http://watt.enc.org/online/ENC2285/ 2285.html.

Appendix-Standards

National Science Education Standards

- Content Standard C: As a result of the activities, all students should develop an understanding of the interdependence of organisms.
- Content Standard F: As a result of the activities, all students should develop an understanding of population growth, natural resources, and environmental quality.

NCTM Curriculum and Evaluation Standards for School Mathematics

- Number and Operations Standard
  - Understand numbers, ways of representing numbers, relationships among numbers, and number systems
  - Understand meanings of operations and how they relate to one another
  - Compute fluently and make reasonable estimates

- Algebra Standard
  - Understand patterns, relations, and functions
  - Represent and analyze mathematical situations and structures using algebraic symbols
  - Use mathematical models to represent and understand quantitative relationships
  - Analyze change in various contexts

- Data Analysis and Probability Standard
  - Formulate questions that can be addressed with data and collect, organize,
    and display relevant data to answer them
  - Select and use appropriate statistical methods to analyze data
  - Develop and evaluate inferences and predictions that are based on data

- Problem Solving Standard
  - build new mathematical knowledge through problem solving
  - solve problems that arise in mathematics and in other contexts
  - apply and adapt a variety of appropriate strategies to solve problems
  - Monitor and reflect on the process of mathematical problem solving.

- Communication Standard
  - organize and consolidate their mathematical thinking through communication;
  - communicate their mathematical thinking coherently and clearly to peers, teachers, and others;

    - analyze and evaluate the mathematical thinking and strategies of others;
    - Use the language of mathematics to express mathematical ideas precisely.

- Connection Standard
    - recognize and use connections among mathematical ideas;
    - understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
    - Recognize and apply mathematics in contexts outside of mathematics.

- Representation Standard
    - create and use representations to organize, record, and communicate mathematical ideas;
    - select, apply, and translate among mathematical representations to solve problems;
    - Use representations to model and interpret physical, social, and mathematical phenomena.

National Educational Technology Standards (NETS)

- Standard 4: Technology Communication Tools
    - Students use telecommunications to collaborate, publish, and interact with peers, experts, and other audiences.

- Standard 5: Technology Research Tools
    - Students use technology to locate, evaluate, and collect information from a variety of sources.
    - Students use technology tools to process data and report results.

- Standard 6: Technology Problem-solving and Decision-making Tools
    - Students use technology resources for solving problems and making informed decisions.
    - Students employ technology in the development of strategies for solving problems in the real world.